Here we extend our discussion from Section 5.3 regarding the comparison of different monotonicity graphs. We know that the same set of selected extrema can be represented by a large number of different monotonicity graphs. We are interested in understanding how different, or how similar, the converged reconstruction results can be with respect to different monotonicity graphs.

To do so, we made the following experiment. We constructed 100 random monotonicity graphs for the 2D vorticity data set (Figures 3 and 5 in the paper). They all describe the same set of selected extrema that we used throughout the paper for this data set. The construction of random monotonicity graphs is possible using our extrema cancellation method from Section 3.2.2: we start with a random function and mark all cells of the cell complex as critical. Then we remove all extrema except for the selected ones, which gives us a random, but feasible monotonicity graph.

Figure 1 shows the results as histograms. Compare this to the numbers for the vorticity data set in Table 3 in the paper. Not surprisingly, all optimizations needed significantly more iterations to converge than for our well-constructed monotonicity graphs. Also, the final energy levels are consistently higher. However, the histogram of the normalized $L^2$ distance shows that the majority of optimizations converged to results that are reasonably close to the “topology” result from Table 3 in the paper. Note that the “random” optimizations started in the first iteration with energy levels of usually above $5 \cdot 10^5$ and $L^2$ distances of close to 1.

We conclude that our iterative convexification from Section 4.2 is crucial to achieving consistent optimization results. In fact, the experiment suggests that the choice for a specific monotonicity graph construction method can be guided by implementational effort and computation time – the iterative convexification seems to be able to make up for unfavorable start conditions.
Figure 1: Results for reconstructing the 2D vorticity data set using 100 random monotonicity graphs, which all describe the same set of extrema.