G22.3033-008, Spring 2010

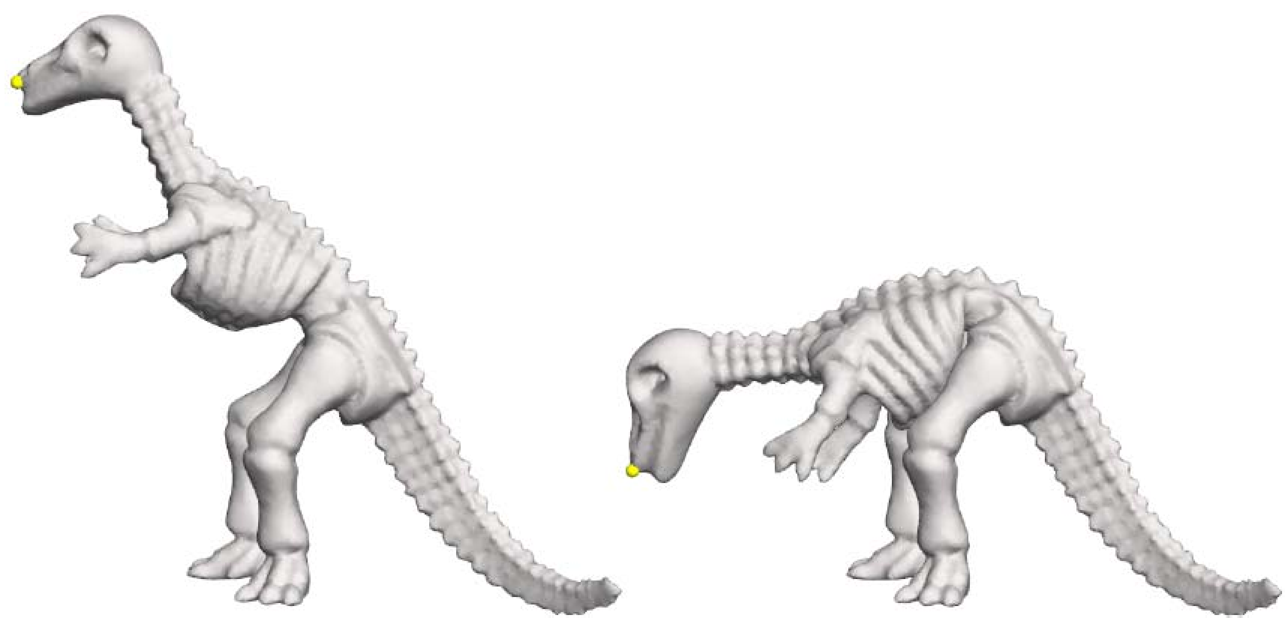
Geometric Modeling

As-rigid-as-possible surface modeling

As-rigid-as-possible surface deformation

Sorkine and Alexa 2007

- Smooth effect on the large scale
- As-rigid-as-possible effect on the small scale (preserves details)



Modeling ARAP detail preservation

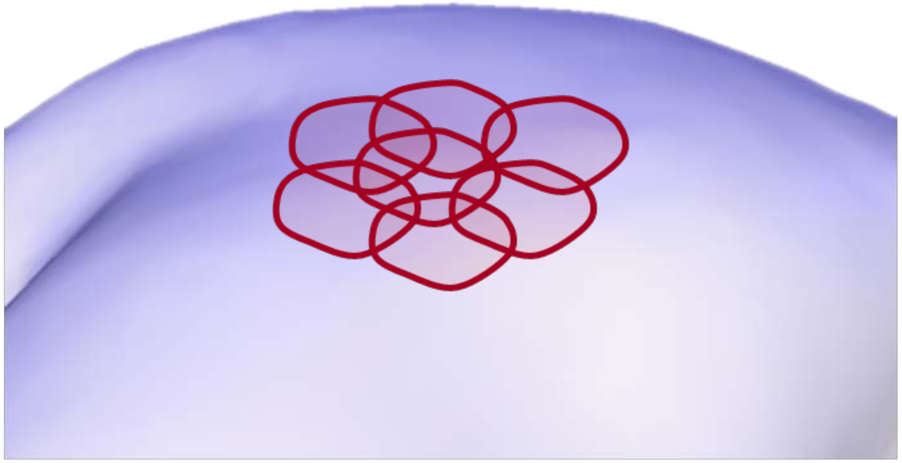
- Previous work: Laplacian editing and its variants

$$\min_{\mathbf{v}'} \sum_{i=1}^{n} \left\| L(\mathbf{v}_i') - R_i \boldsymbol{\delta}_i \right\|^2 \qquad s.t.\ \mathbf{v}_j' = \mathbf{c}_j,\ j \in C$$

- Concentrated on making the optimization linear by "inventing" the right rotations or optimizing their linearized version

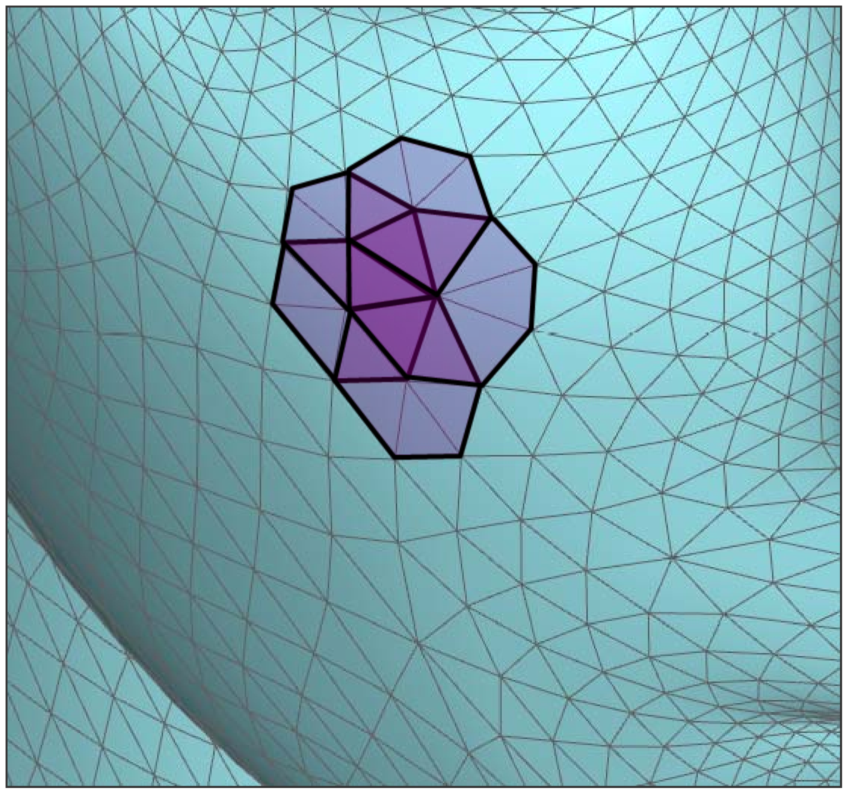
Direct ARAP modeling

- We actually may want to preserve the shapes of cells covering the surface



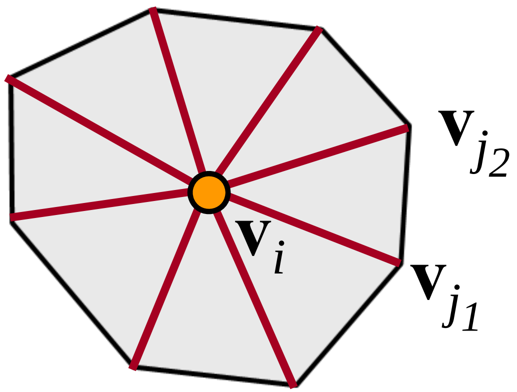
Direct ARAP modeling

- Let's look at cells on a mesh



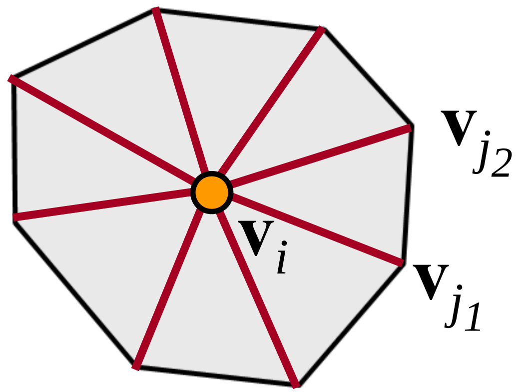
Direct ARAP modeling

- Ask all the star edges to transform rigidly, then the shape of the cell is preserved



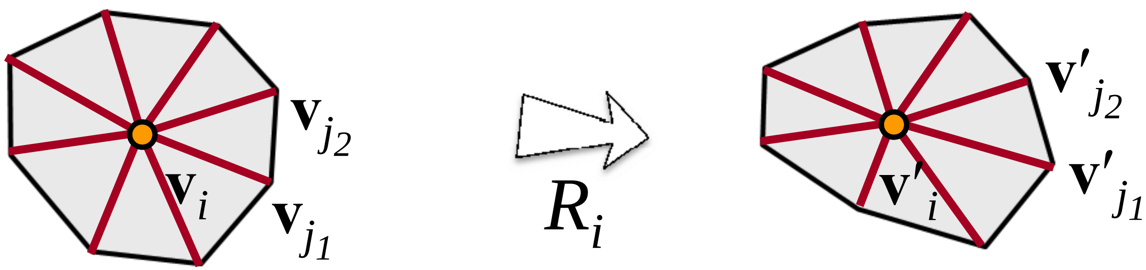
Direct ARAP modeling

- Cell energy: $\min \sum_{j \in N(i)} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i (\mathbf{v}_i - \mathbf{v}_j) \right\|^2$



Direct ARAP modeling

- If $\mathbf{v}$, $\mathbf{v}'$ are known then R_i is uniquely defined



- It’s the shape matching problem!
  - Build covariance matrix $\mathrm{S} = \mathrm{V}\mathrm{V}'^{\mathrm{T}}$
  - SVD: $\mathrm{S} = \mathrm{U}\Sigma\mathrm{P}^{\mathrm{T}}$
  - $R_i = \mathrm{U}\mathrm{P}^{\mathrm{T}}$

R_i is a non-linear function of $\mathbf{v}'$

Direct ARAP modeling

- Can formulate overall energy of the deformation:

$$\min_{\mathbf{v}'} \sum_{i=1}^{n} \sum_{j \in N(i)} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i (\mathbf{v}_i - \mathbf{v}_j) \right\|^2$$

$$s.t.\ \mathbf{v}'_j = \mathbf{c}_j,\ j \in C$$

Energy minimization

- Alternating iterations
  - Given initial guess $\mathbf{v}'_0$, find optimal rotations R_i
    - This is a per-cell task! We already showed how to define R_i when $\mathbf{v}, \mathbf{v}'$ are known
  - Given the R_i (fixed), minimize the energy by finding new $\mathbf{v}'$

$$\min_{\mathbf{v}'} \sum_{i=1}^{n} \sum_{j \in N(i)} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i (\mathbf{v}_i - \mathbf{v}_j) \right\|^2$$

Energy minimization

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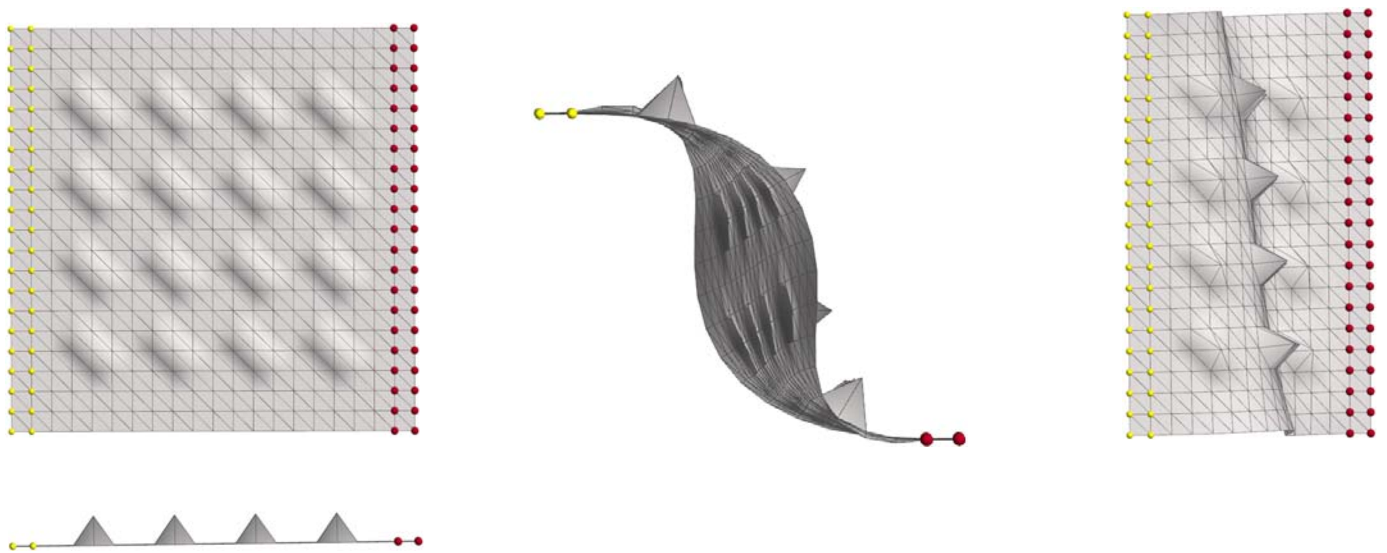
$$L\mathbf{v}' = \mathbf{b}$$

The big advantage

- Each iteration decreases the energy (or at least guarantees not to increase it!)
- The matrix L stays fixed!
  - Precompute Cholesky factorization
  - Just back-substitute each iteration (+ the SVD computations)

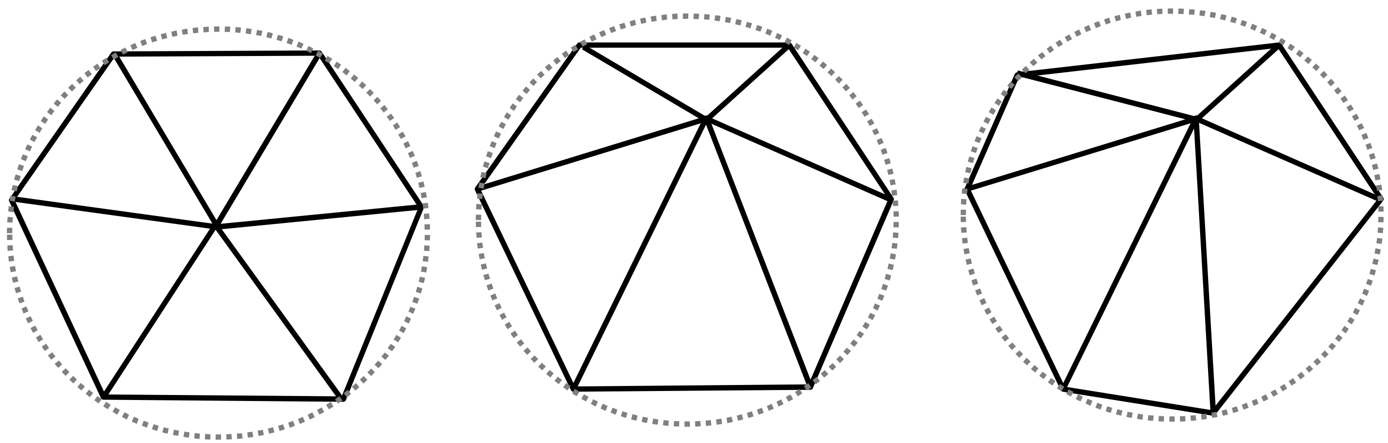
The importance of proper weighting

- If we use uniform Laplacian L



The importance of proper weighting

- The problem: need to compensate for varying shapes of the 1-ring

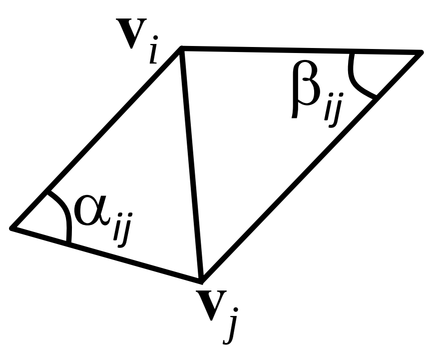


$$E_{cell} = \sum_{j \in N(i)} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i (\mathbf{v}_i - \mathbf{v}_j) \right\|^2$$

Use cotan weights

- Add cotangent weights [Pinkall and Polthier 93]

$$E_{cell} = \sum_{j \in N(i)} w_{ij} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i (\mathbf{v}_i - \mathbf{v}_j) \right\|^2$$

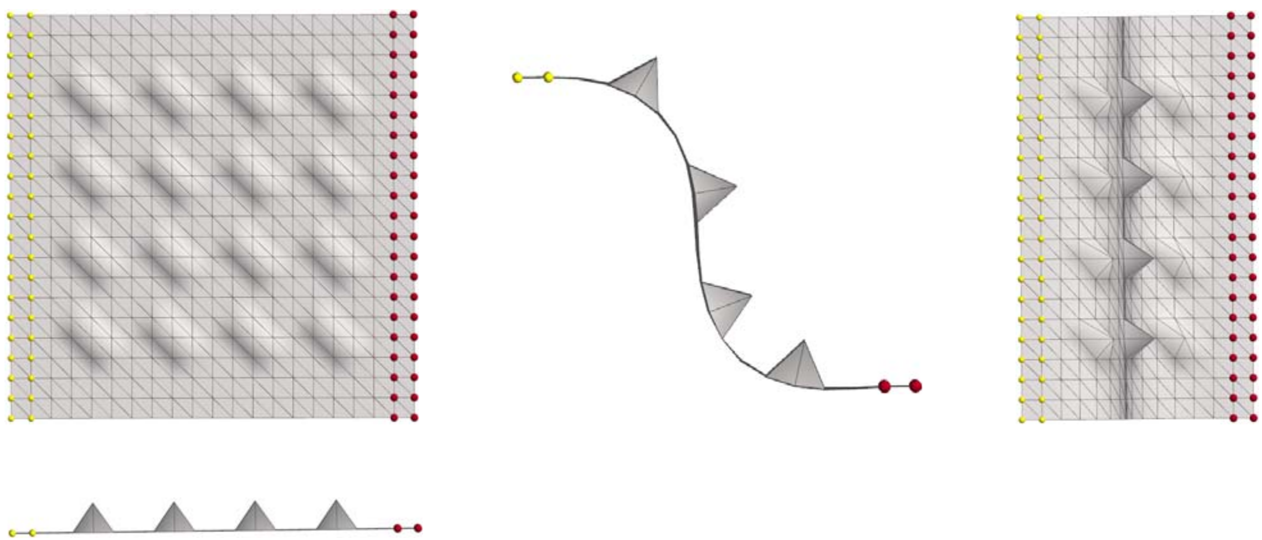


$$w_{ij} = \frac{1}{2}\left(\cot \alpha_{ij} + \cot \beta_{ij}\right)$$

Use cotan weights

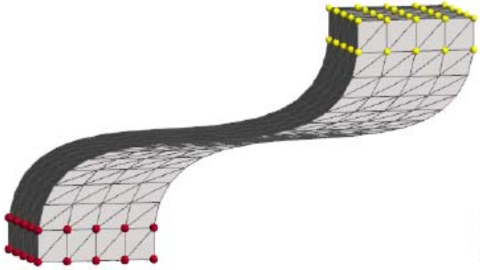
- This gives symmetric results

$$E_{cell} = \sum_{j \in N(i)} w_{ij} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i (\mathbf{v}_i - \mathbf{v}_j) \right\|^2$$

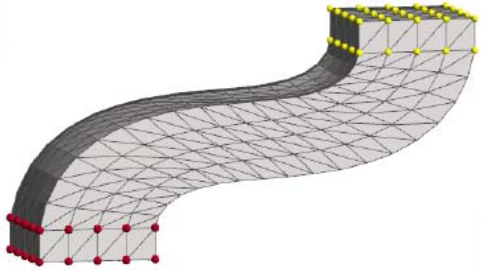


Results

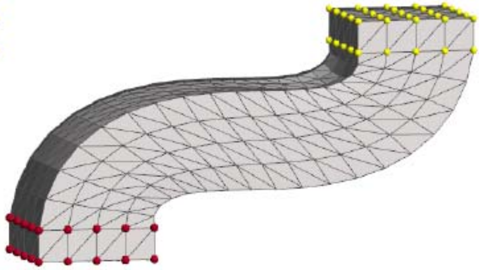
- Can start from naïve Laplacian editing as initial guess and iterate



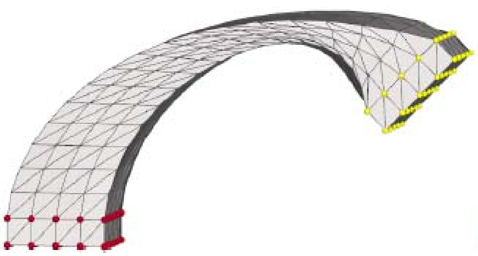
initial guess



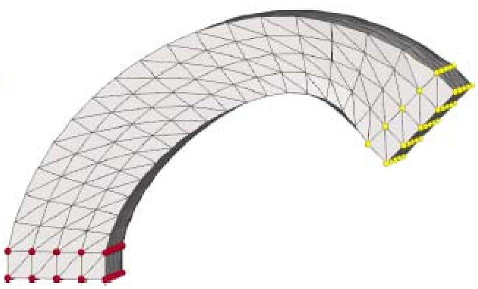
1 iteration



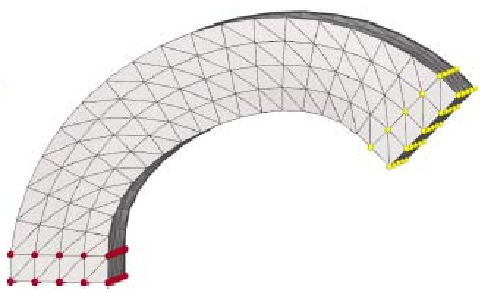
2 iterations



initial guess



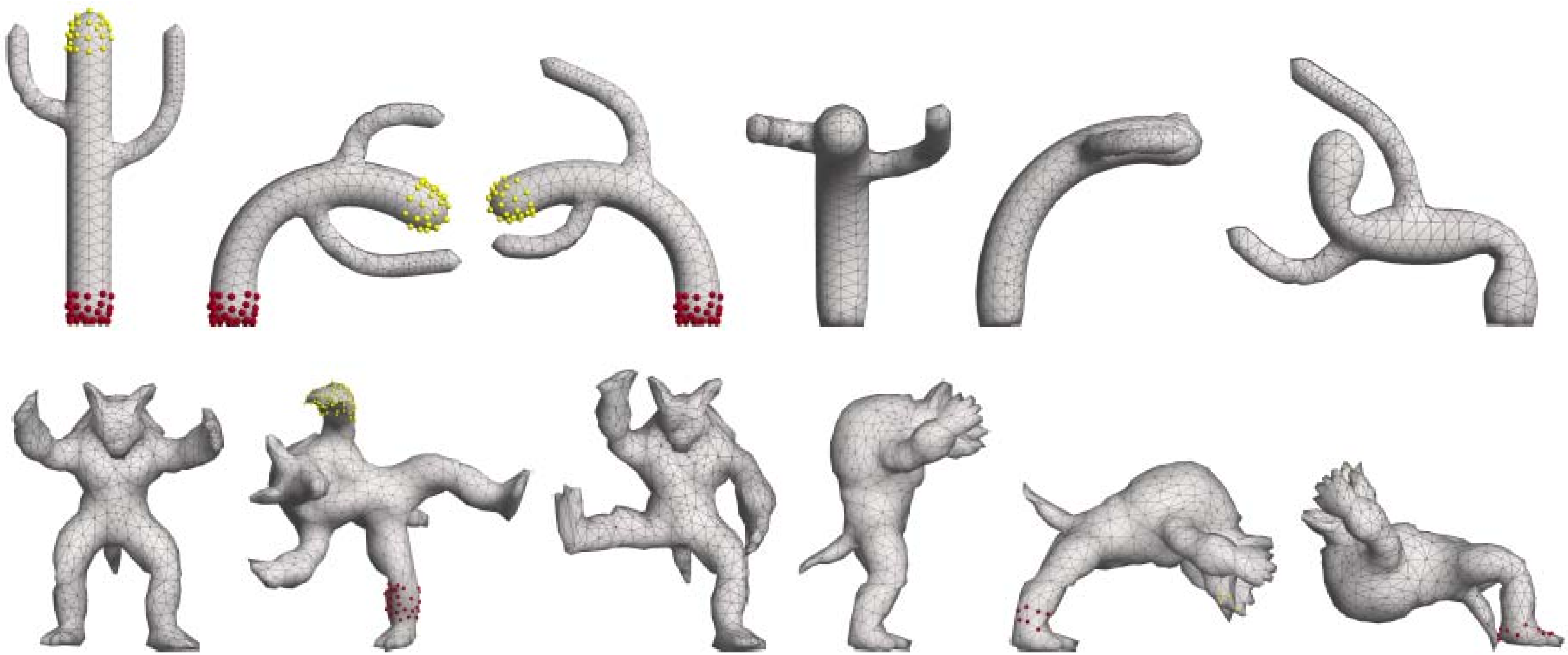
1 iterations



4 iterations

Results

- Faster convergence when we start from the previous frame



Issues

- Works fine on small meshes
- On larger meshes: slow convergence
  - Each iteration is more expensive of course
  - Need more iterations because the conditioning of the system becomes worse as the matrix grows
- Implement multi-res strategy?
- Also: material stiffness depends on the 1-ring size (lots of wrinkles for fine meshes)

More issues

- This technique is good for preserving edge length (relative error very small)
- No notion of volume, however
  - Essentially, thin shells for the poor
- Can extend to volumetric meshes