G22.3033-008, Spring 2010

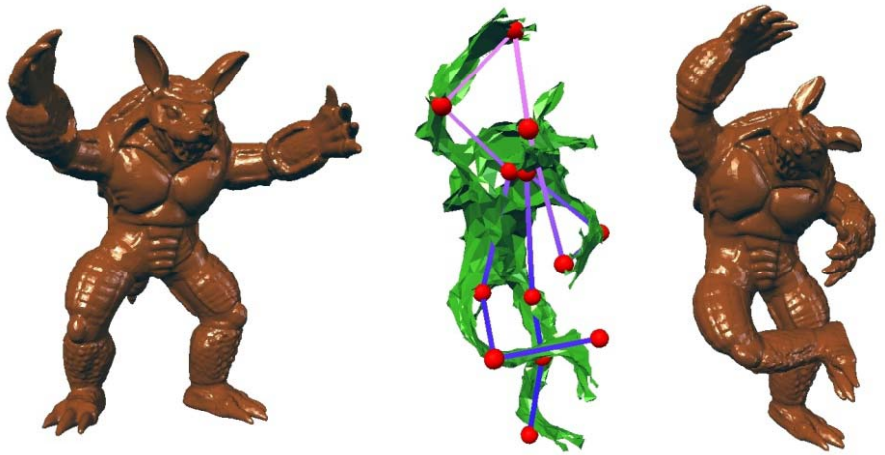
Geometric Modeling

Shape deformation intro

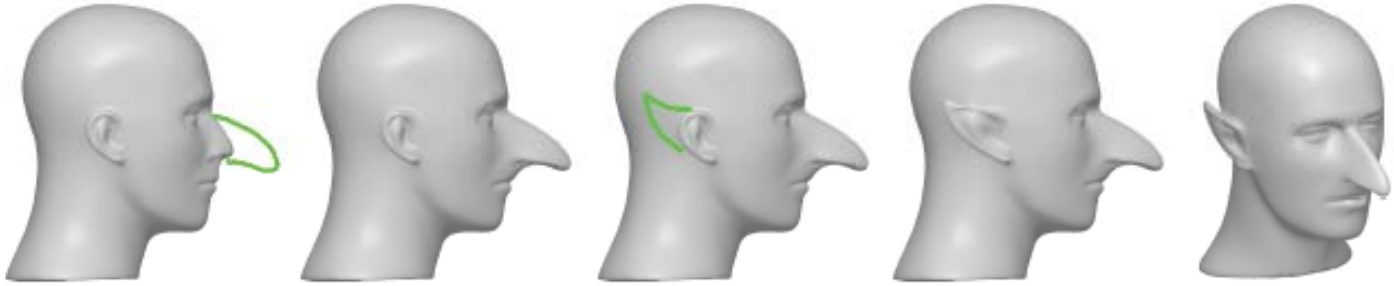
Surface vs. space deformations

Why shape deformation?

- Animation



- Editing



- Simulation

Parametric curves and surfaces

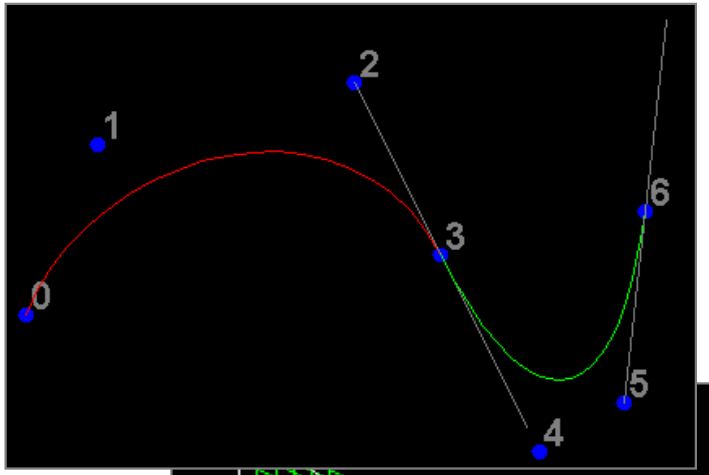
Deformation by control point manipulation

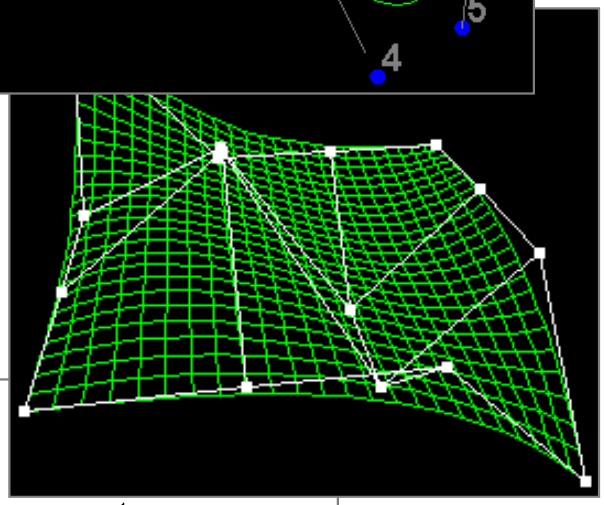
- Some online demos

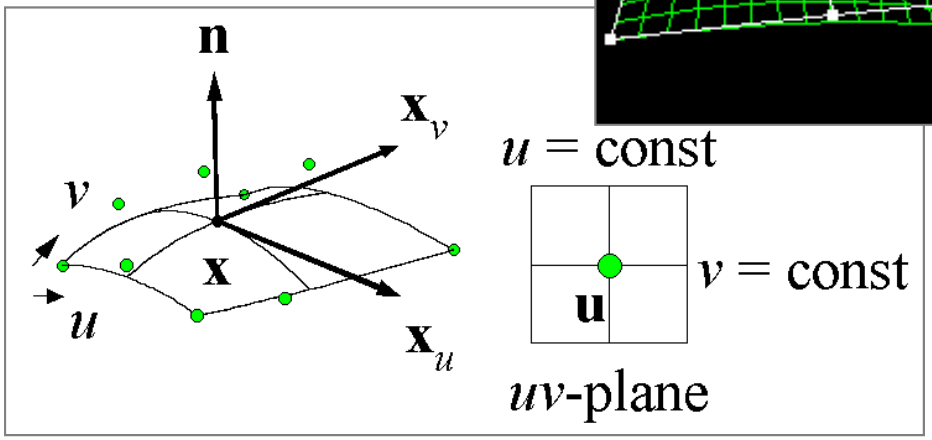
- http://www.cs.princeton.edu/~min/cs426/jar/bezier.html

- http://www.nbb.cornell.edu/neurobio/land/OldStudentProjects/cs490-96to97/anson/BezierPatchApplet/

- http://wwwvis.informatik.uni-stuttgart.de/~kraus/LiveGraphics3D/cagd/



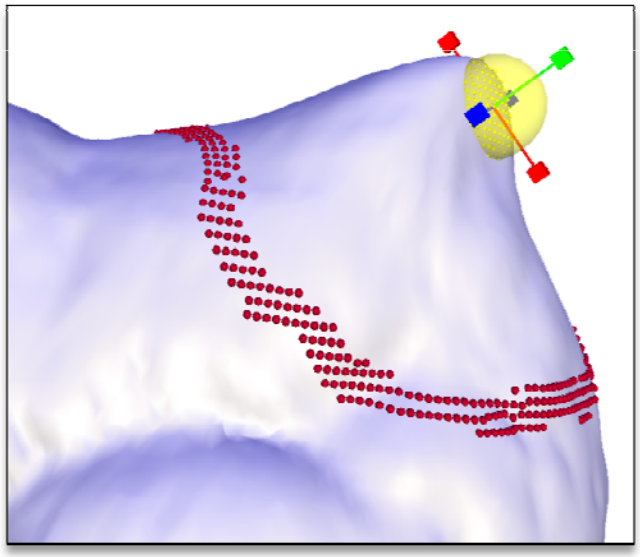




Mesh/shape deformation

Basic idea

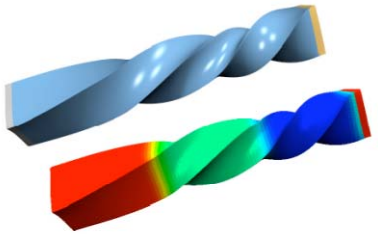
- Naïve method: dragging single vertices
- Smarter:
  - Create a small set of control parameters
  - Introduce a small set of deformation handles
    - Makes deformation editing easier
    - Introduces a trade-off between degrees of freedom and simplicity of the deformation task



Mesh/shape deformation

Commonly used paradigms

- **Surface based deformation**
  - Laplacian surface editing and other surface-based energy minimization approaches
  - Physically motivated:
    Laplacian preservation ≈ bending/stretching resistance

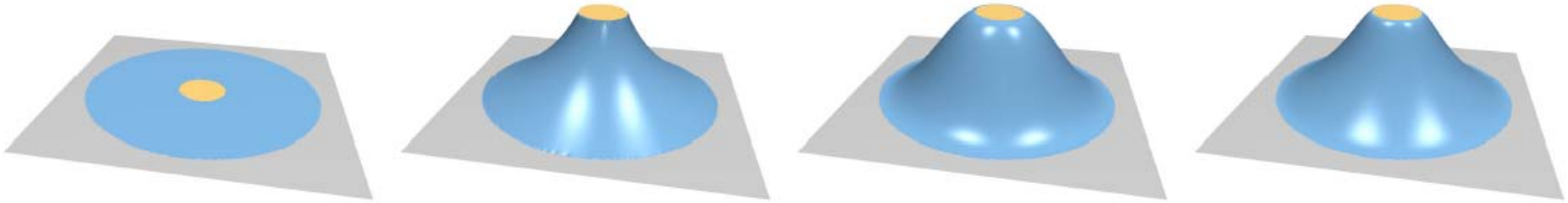


- **Space deformation**
  - Deforms some 2D/3D space using a *cage*
  - Deformation propagation to all points in the space
  - Independent of shape representation

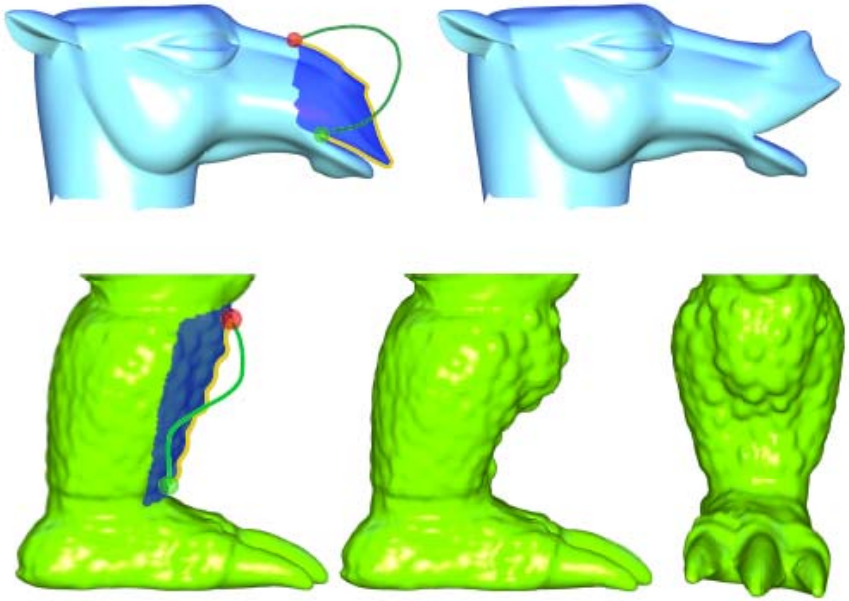
Surface-based deformations

Examples

- Region of interest (ROI) + affine deformation of handle with variable boundary continuity



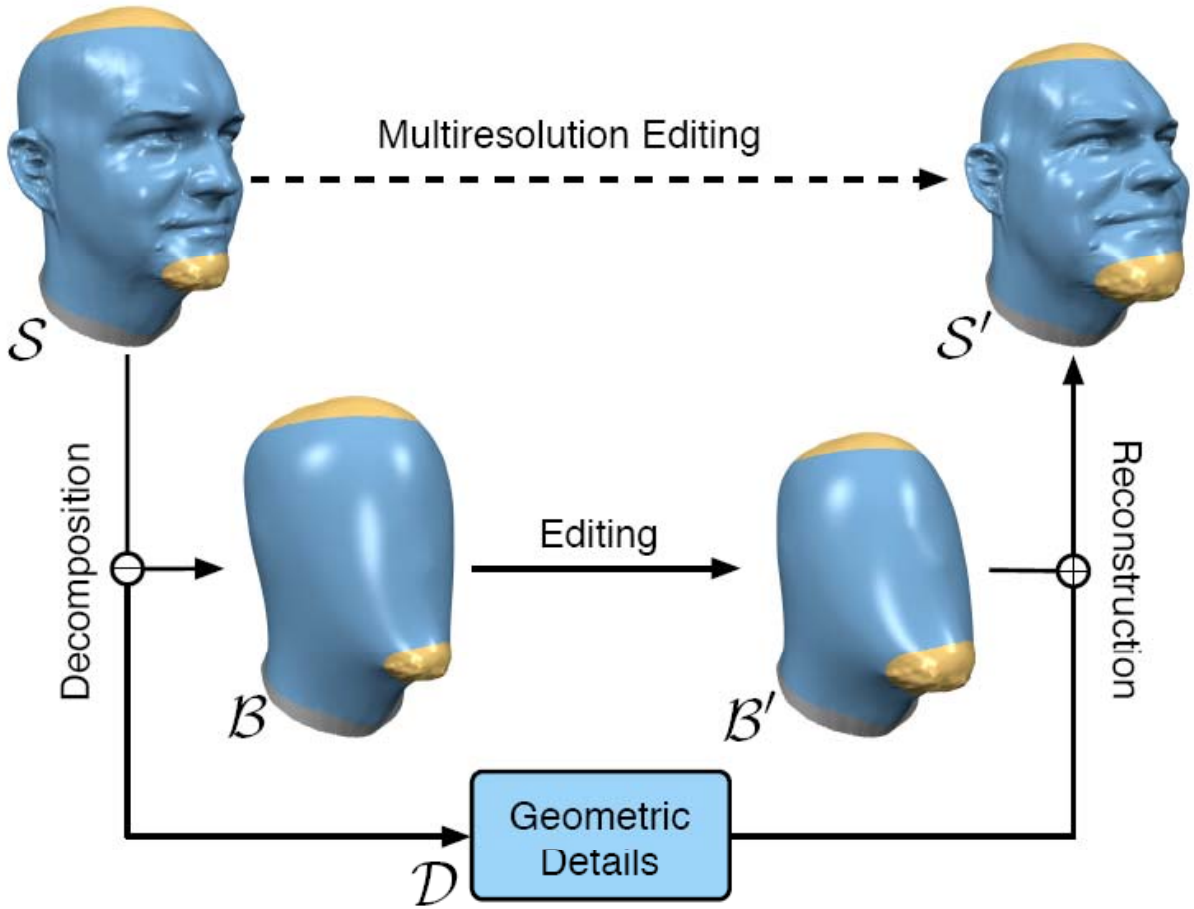
- Intuitive sketch-based deformation interfaces



Surface-based deformations

Examples

- Multi-resolution mesh editing



Surface-based deformations

General framework

- Find a mesh that optimizes some objective functional and satisfies modeling constraints

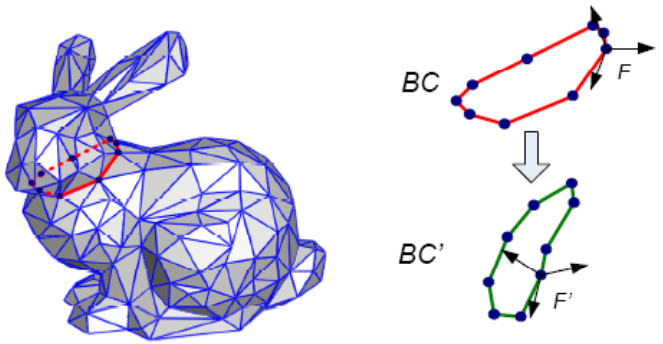
$$\mathbf{x}' = \arg\min_{\mathbf{x}'} F(\mathbf{x}') \qquad \text{s.t. } \mathbf{x}'_i = \mathbf{c}_i$$

Surface-based deformations

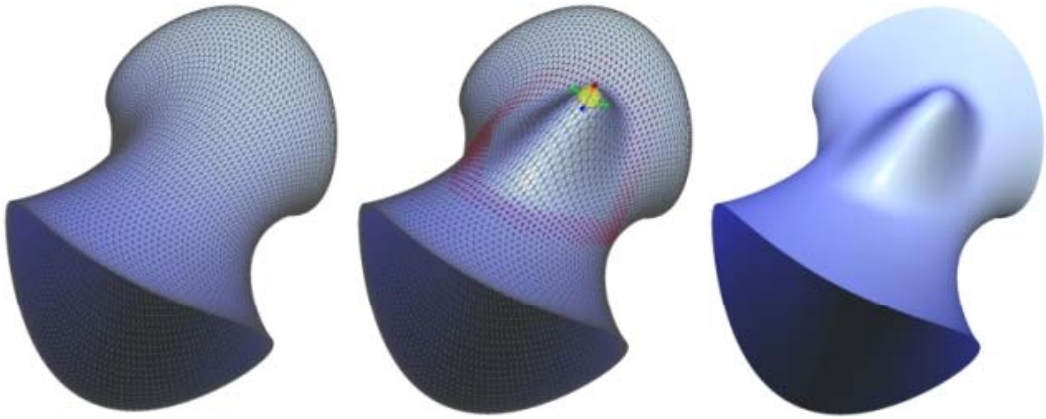
Linear methods

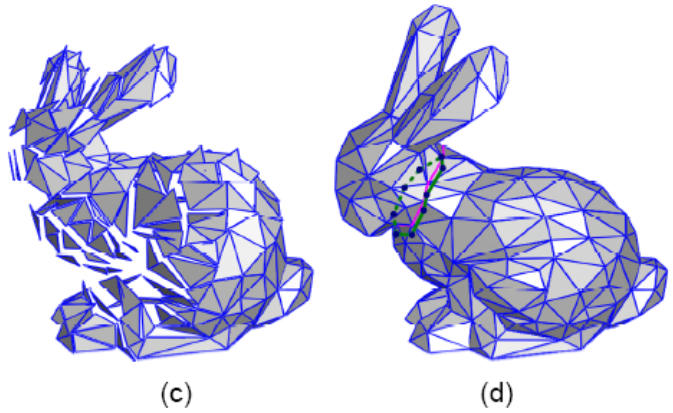
- (2D) As rigid as possible shape manipulation

- Triangle gradient methods



- Laplacian surface editing

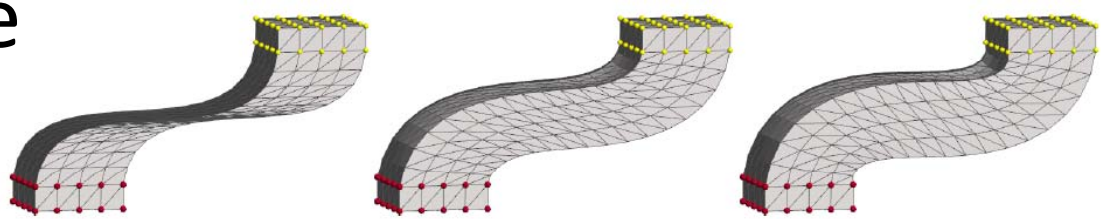




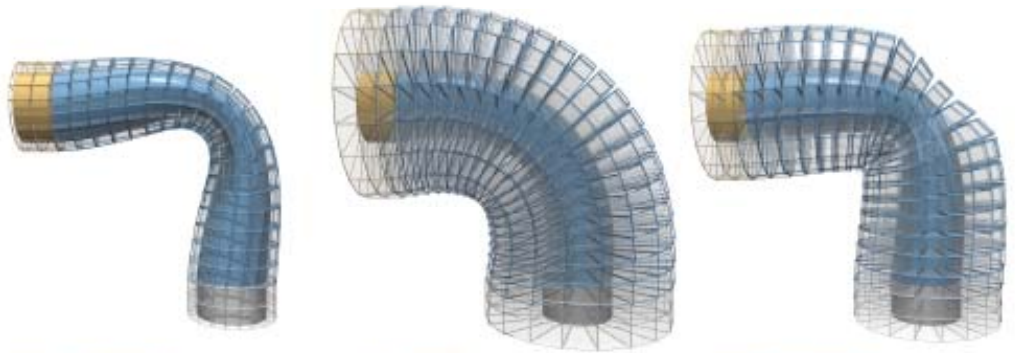
Surface-based deformations

Nonlinear methods

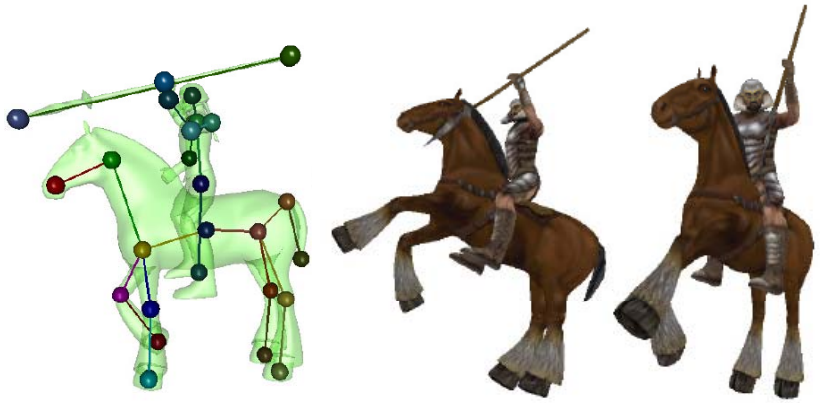
- As rigid as possible surface modeling



- PriMo



- Mesh Puppetry



Surface-based deformations

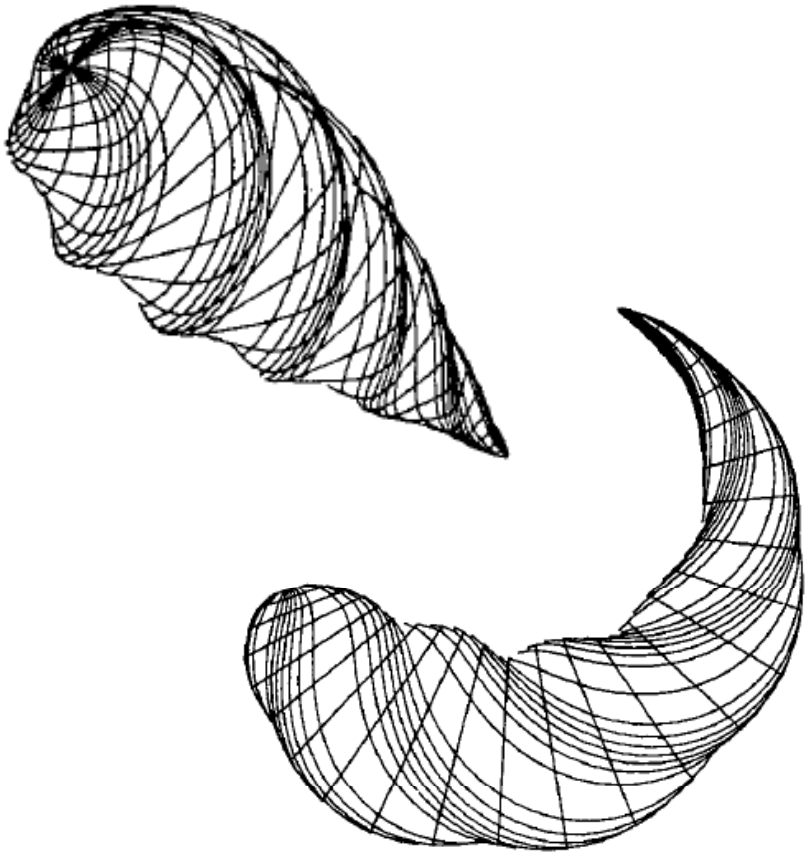
Summary

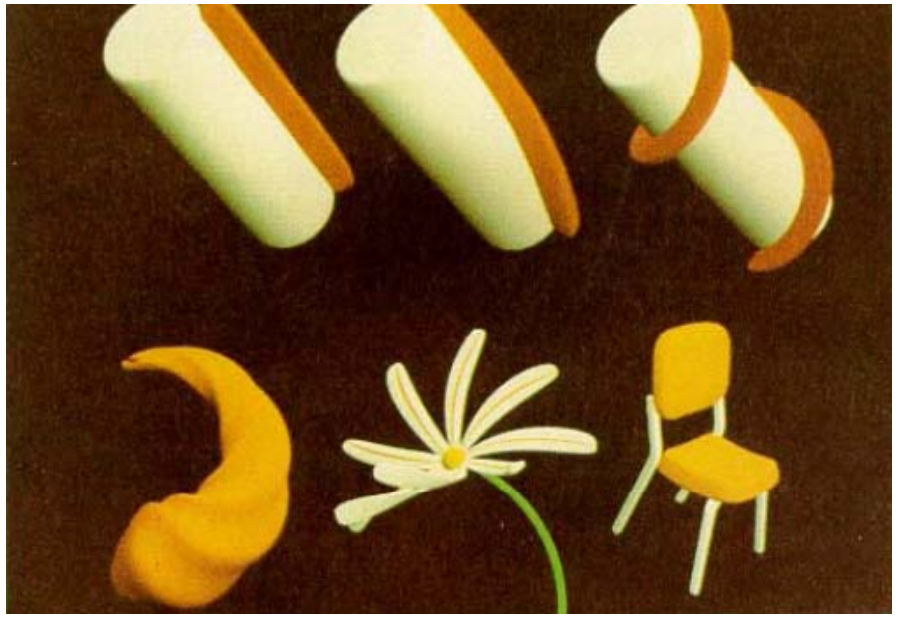
- Objective functional in the mesh elements (vertices)
- Complexity depends on the mesh size
- Linear methods:
  - Solve a global linear system on the mesh
  - Usually suffer from some artifacts
- Nonlinear methods
  - Fewer artifacts but slower, and harder to implement

Space deformations

Early seminal work in computer graphics

- Global and local deformation of solids [Barr 1984]

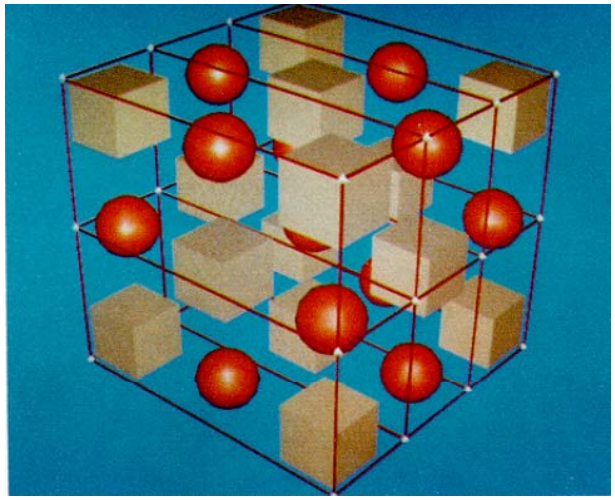


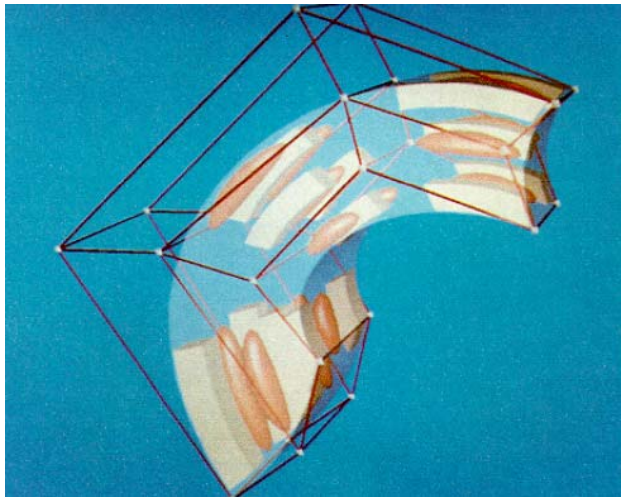


Space deformations

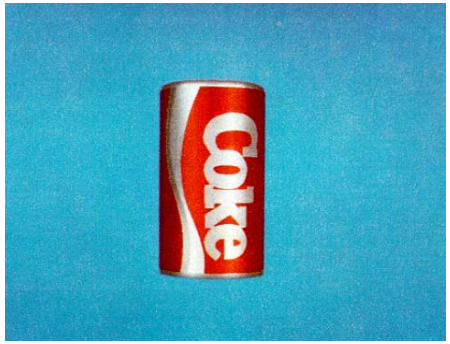
Early seminal work in computer graphics

- Free form deformations [Sederberg and Parry 1986]
  - Uses trivariate tensor product polynomial basis





  - Can be designed to be volume preserving



$$\mathbf{F}(x,y,z) = (F(x,y,z), G(x,y,z), H(x,y,z))$$

then the Jacobian is the determinant

$$Jac(\mathbf{F}) = \begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \\ \frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} & \frac{\partial H}{\partial z} \end{vmatrix}$$

Space deformations

Basic idea

- Design a set of coordinates for all points in R^d w.r.t. the cage vertices
  - Each point $\mathbf{x}$ can be represented as a weighted sum of cage points

$$\mathbf{x} = \sum_{i=1}^{k} w_i(\mathbf{x}) \cdot \mathbf{p}_i$$

  - When the cage changes the coords stay the same, substitute the new cage geometry:

$$\mathbf{x}' = \sum_{i=1}^{k} w_i(\mathbf{x}) \cdot \mathbf{p}'_i$$

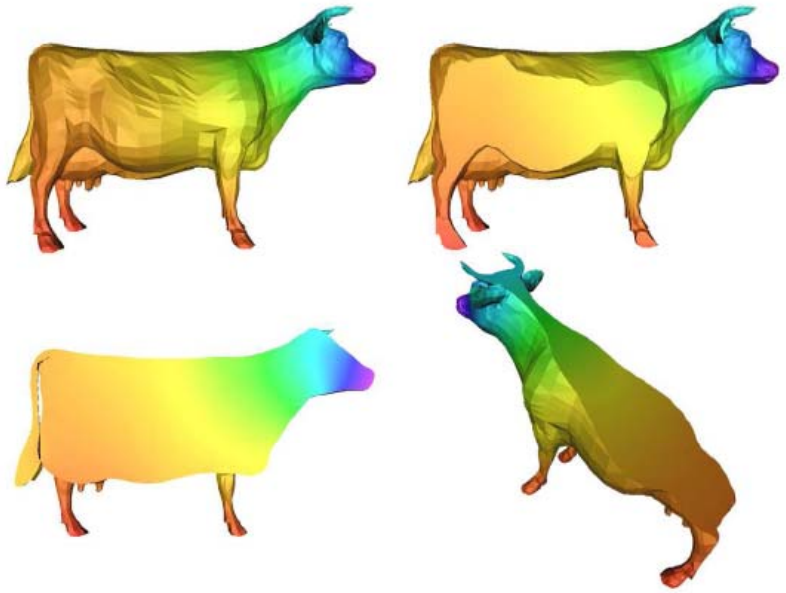
Space deformations

Basic idea

- Design a set of coordinates for all points in R^d w.r.t. the cage vertices
  - Each point $\mathbf{x}$ can be represented as a weighted sum of cage points

$$\mathbf{x} = \sum_{i=1}^{k} w_i(\mathbf{x}) \cdot \mathbf{p}_i$$

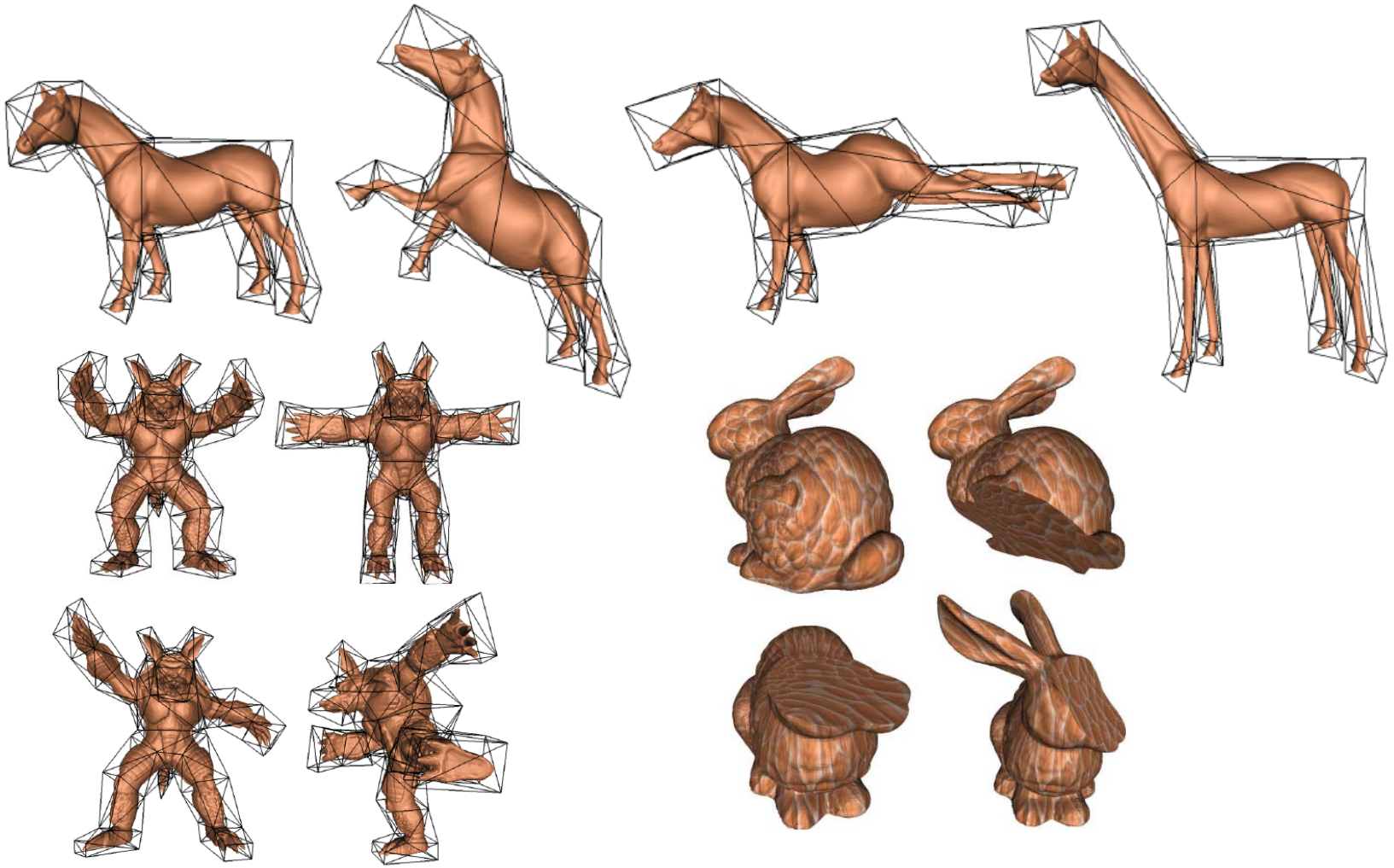
  - The coordinates are smoothly varying and guarantee continuity inside the volume



Space deformations

Examples

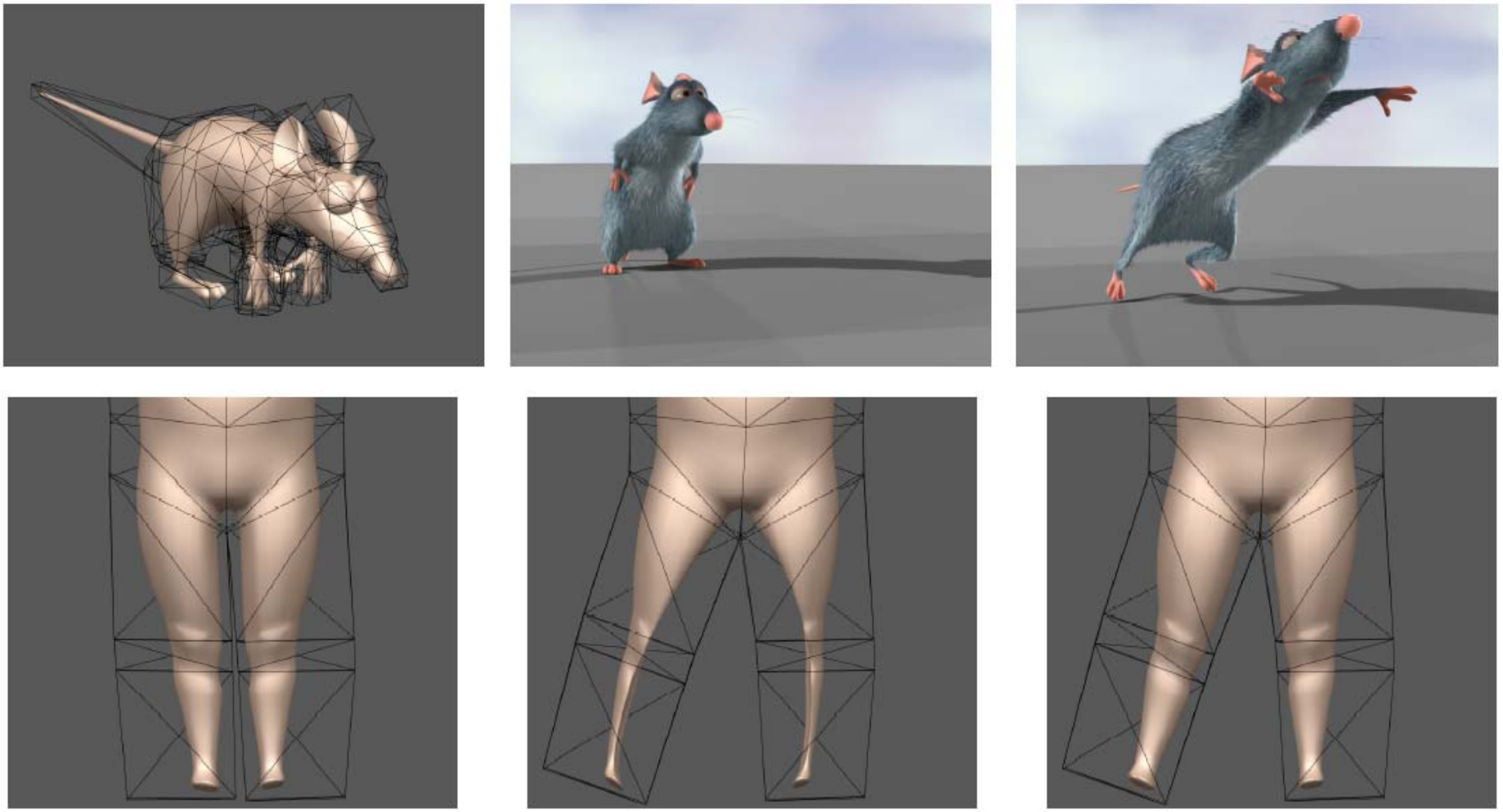
- Mean value coordinates for closed tri meshes



Space deformations

Examples

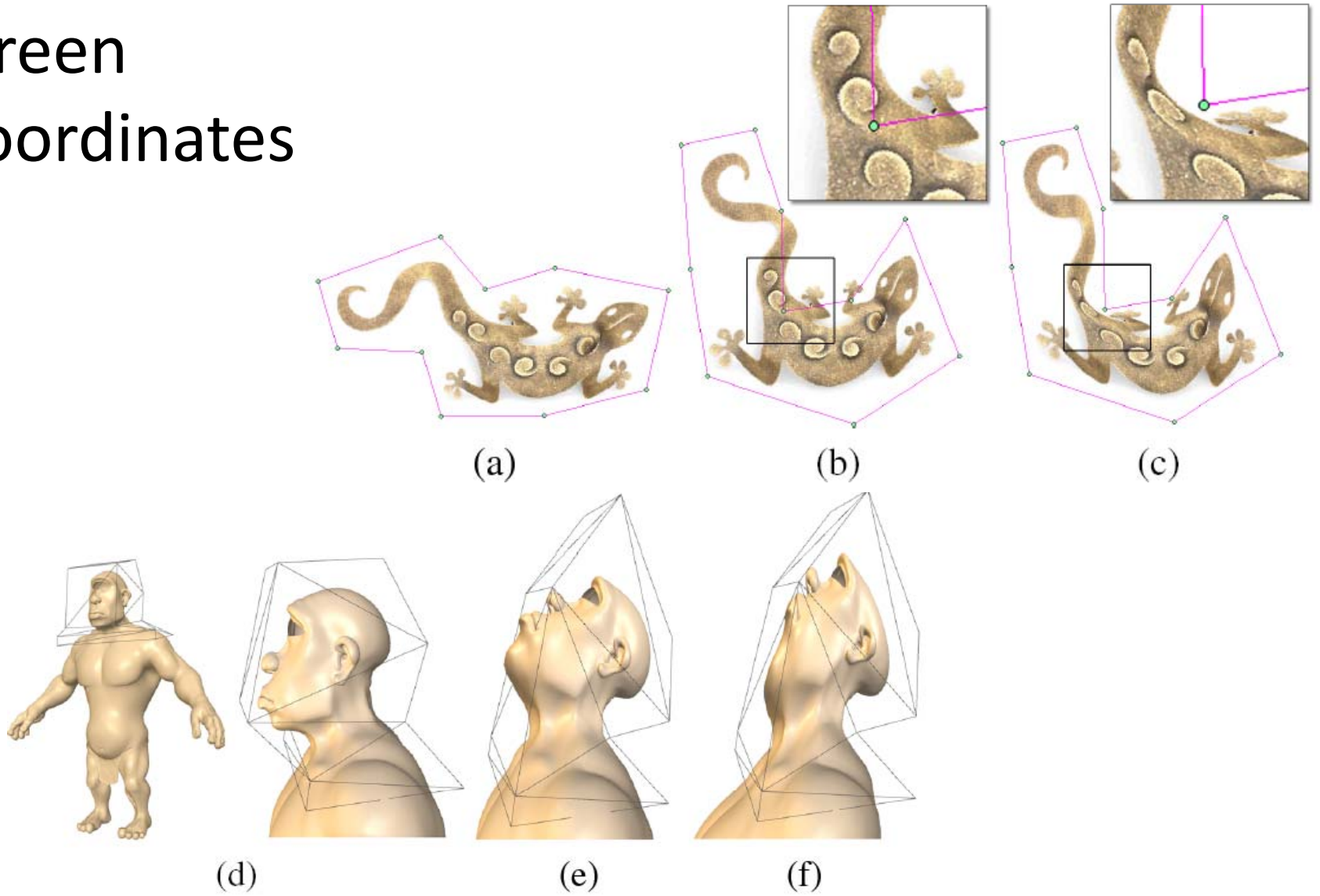
- Harmonic coordinates



Space deformations

Examples

- Green coordinates



Space deformations

Summary

- Complexity depends mainly on the cage; linear in the number of mesh elements
- Can handle disconnected components or even just point sets
- Harder to control the surface properties since the whole space warps