Computer Graphics - Exercise 2
Reminder of Linear Algebra Basics

20/03/2003

Question 1

Let V be a vector space with inner product. Prove that if $\{v_1, v_2, ..., v_k\} \in V$ is a set of (pairwise) orthogonal vectors then $\{v_1, v_2, \ldots, v_k\}$ are linearly independent.

Question 2

Prove that if A is an orthogonal matrix then $\det A = \pm 1$.

Question 3

True or not true? A linear operator $A : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is 1-1 $\Leftrightarrow$ a matrix representing A with respect to some basis of $\mathbb{R}^n$ is non-singular.

Question 4

Let A be a square matrix. Prove that if $\lambda_1, \lambda_2, \ldots, \lambda_k$ are *distinct* eigenvalues of A then corresponding eigenvectors are linearly independent.

Question 5

Prove that if A is a symmetric matrix and λ, μ are its eigenvalues ($\lambda \neq \mu$) then corresponding eigenvectors are orthogonal (i.e. if $Av = \lambda v$ and $Aw = \mu w$ then $< v, w >= 0$).