3D Geometry for Computer Graphics - Exercise 1

04/03/2004

Question 1

Let S_1 and S_2 be two line segments. The distance between the segments is defined as follows:

$$\text{dist}(S_1, S_2) = \min_{\mathbf{p}\in S_1,\ \mathbf{q}\in S_2} \|\mathbf{p} - \mathbf{q}\|.$$

(a) Derive a formula for dist(S_1, S_2) as function of the endpoints of the two segments. Assume that the segments lie in the plane.

(b) Same as (a) but for arbitrary segments in the 3D space.

Question 2

Three points A, B, C in the plane are said to be a *left-turn* if the counter-clockwise angle from the vector $\mathbf{v} = B - A$ to the vector $\mathbf{w} = C - A$ is less than π (see Figure 1). Given a coordinate system in the plane, it can be easily verified that if $A = (A_x, A_y)$, $B = (B_x, B_y)$ and $C = (C_x, C_y)$, then (A, B, C) is a left-turn iff:

$$L(A, B, C) := (B_x - A_x)(C_y - A_y) - (B_y - A_y)(C_x - A_x) > 0.$$

(a) Given two segments AB and CD in the plane. When do AB and CD intersect each other? Find a condition using left-turn tests on the endpoints (A, B, C and D).

(b) Given a closed convex polygon $P_1P_2...P_n$ in the plane (i.e. the edges of the polygon are the segments P_iP_{i+1} for $i = 1, 2, ..., n$) and a point Q. How to decide whether Q is inside the polygon or not using left-turn tests?

(c) Given an arbitrary (not necessarily convex) closed polygon $P_1P_2...P_n$ in the plane and a point Q. How to decide whether Q is inside the polygon or not?

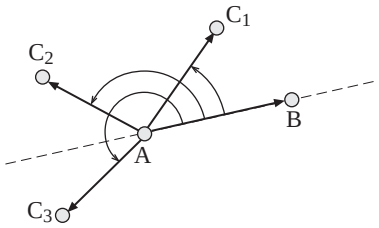

Figure 1: (A, B, C_1) and (A, B, C_2) are left-turns while (A, B, C_3) is not (it is a right-turn).