3D Geometry for Computer Graphics - Exercise 1 Solutions

Solution to Question 1

(a) The distance will be attained between one segment's endpoint and the other segment. Denote the endpoints of S_1 by $\mathbf{p}_1$ and $\mathbf{p}_2$ and the endpoints of S_2 by $\mathbf{q}_1$ and $\mathbf{q}_2$. Denote by d_1 the distance between $\mathbf{p}_1$ and S_2.

$$d_1^2 = \left\{ \begin{array}{ll} \|\mathbf{p}_1 - \mathbf{q}_1\|^2 - \frac{<\mathbf{p}_1-\mathbf{q}_1, \mathbf{q}_2-\mathbf{q}_1>^2}{\|\mathbf{q}_2-\mathbf{q}_1\|^2} & \text{if } t \in [0,1] \text{ where } t = \frac{<\mathbf{p}_1-\mathbf{q}_1, \mathbf{q}_2-\mathbf{q}_1>}{\|\mathbf{q}_2-\mathbf{q}_1\|^2} \\ & \\ \min\{\|\mathbf{p}_1 - \mathbf{q}_1\|^2, \|\mathbf{p}_1 - \mathbf{q}_2\|^2\} & \text{otherwise} \end{array} \right\}$$

This means that the distance between the point $\mathbf{p}_1$ and the segment S_2 is the length of the perpendicular line casted from $\mathbf{p}_1$ onto S_2, if the other end of this line falls inside S_2 (i.e. if the parameter t is between 0 and 1); otherwise it's the minimal distance to the endpoints of S_2.

We compute d_2, the distance between $\mathbf{p}_2$ and S_2, in exactly the same way. Same for the distances between $\mathbf{q}_1$ and $\mathbf{q}_2$ and the segment S_1 (denoted by d_1' and d_2').

$$\text{dist}(S_1, S_2) = \min\{d_1, d_2, d_1', d_2'\}.$$

(b) In 3D, we need to check all the options we had in 2D (meaning, the distances between the endpoints of one segment to the other segment). In addition, we have another possibility to get shortest distance: the distance between the 3D lines passing through S_1 and S_2, respectively. Let's denote those lines by l_1 and l_2. We compute the distance $d = \text{dist}(l_1, l_2)$ as learned in class, and check that the perpendicular segment that attains this distance is actually touching both S_1 and S_2. This means verifying that $\tilde{s} \in [0,1]$ and $\tilde{t} \in [0,1]$, where $\tilde{s}$ and $\tilde{t}$ are the parameters, as shown in class.

Solution to Question 2

(a) AB intersects CD if A and B are on the opposite sides of the line through CD, and C and D are on the opposite sides of the line through AB. This is expressed by the following:

$$L(C, D, A) \cdot L(C, D, B) < 0 \text{ and } L(A, B, C) \cdot L(A, B, D) < 0.$$

(b) If the polygon is convex and Q is inside, then all the left-turn tests $L(P_i, P_{i+1}, Q)$ should be the same (either all negative or all positive).

(c) If Q is inside the polygon, then any ray we shoot from Q will intersect the polygon an odd number of times (because the ray starts inside the polygon and eventually must exit it). If Q is outside, then any ray shot from Q will intersect the polygon an even number of times (if the ray enters the polygon, it will also exit it). Thus, we need to find a point outside the polygon (say, $\tilde{Q} = (\min_{i=1,\dots n}(P_i)_x - \varepsilon, \min_{i=1,\dots n}(P_i)_y - \varepsilon)$) and to check the parity of the number of intersections between the segment $Q\tilde{Q}$ and the polygon.