3D Geometry for Computer Graphics - Exercise 2

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1. Let V be a vector space with inner product. Prove that if $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k\} \in V$ is a set of (pairwise) orthogonal vectors then $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ are linearly independent.
2. Prove that if A is an orthogonal matrix then $\det A = \pm 1$.
3. True or not true? A linear operator $A : \mathbb{R}^n \to \mathbb{R}^n$ is 1-1 $\Leftrightarrow$ a matrix representing A with respect to some basis of $\mathbb{R}^n$ is non-singular.
4. Let A be a square matrix. Prove that if $\lambda_1, \lambda_2, \ldots, \lambda_k$ are *distinct* eigenvalues of A then corresponding eigenvectors are linearly independent.
5. Prove that if A is a symmetric matrix and λ, μ are its eigenvalues ($\lambda \neq \mu$) then corresponding eigenvectors are orthogonal (i.e. if $A\mathbf{v} = \lambda\mathbf{v}$ and $A\mathbf{w} = \mu\mathbf{w}$ then $< \mathbf{v}, \mathbf{w} >= 0$).
6. **Computer-science question**: Suppose you are implementing a matrix library. You want to be prepared to handle operations on big matrices. As mentioned in class, if A is a matrix and $\mathbf{b}$ is a column-vector, then the entries of $A\mathbf{b}$ are scalar products of $\mathbf{b}$ with the rows of A. Alternatively, we can look at $A\mathbf{b}$ as a linear combination of the columns of A, where the coefficients of the linear combination are the entries of $\mathbf{b}$. When is the first interpretation more computationally-advantageous, and when is the second? Hint: matrices are usually stored in the computer as arrays of numbers. They can be stored by rows or by columns. Think what happens when the matrix A is very big, in terms of access to its elements during the computation of $A\mathbf{b}$.