

3D Geometry for Computer Graphics

Exercise 4 – selected solutions

2. By now we have learned two techniques to fit a linear subspace to a given set of points in \mathbb{R}^n (to fit a line to a set of points in the plane, to fit a plane to a set of points in \mathbb{R}^3 , etc.). The first technique was PCA and the second was least-squares fit. Explain the difference between the two methods, in terms of the error they minimize (and hence the output they produce) and in terms of the computational effort.

Answer: Linear subspaces found by PCA minimize the orthogonal distance between the input points and the subspace. The least-squares fit, on the other hand, minimizes the sum of squares of the offsets along the chosen axis (see Figure 1).

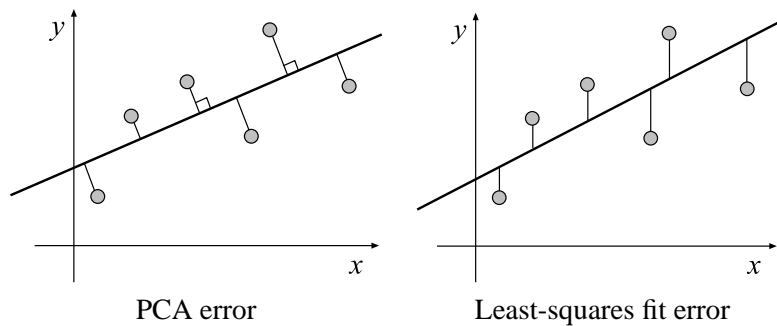


Figure 1: PCA and least-squares fitting minimize different error functionals.

Computationally, PCA requires solving an eigendecomposition problem, which can be expensive for large dimensions. Least-squares fitting requires solving a linear system.