

Bounded Biharmonic Weights for Real-Time Deformation

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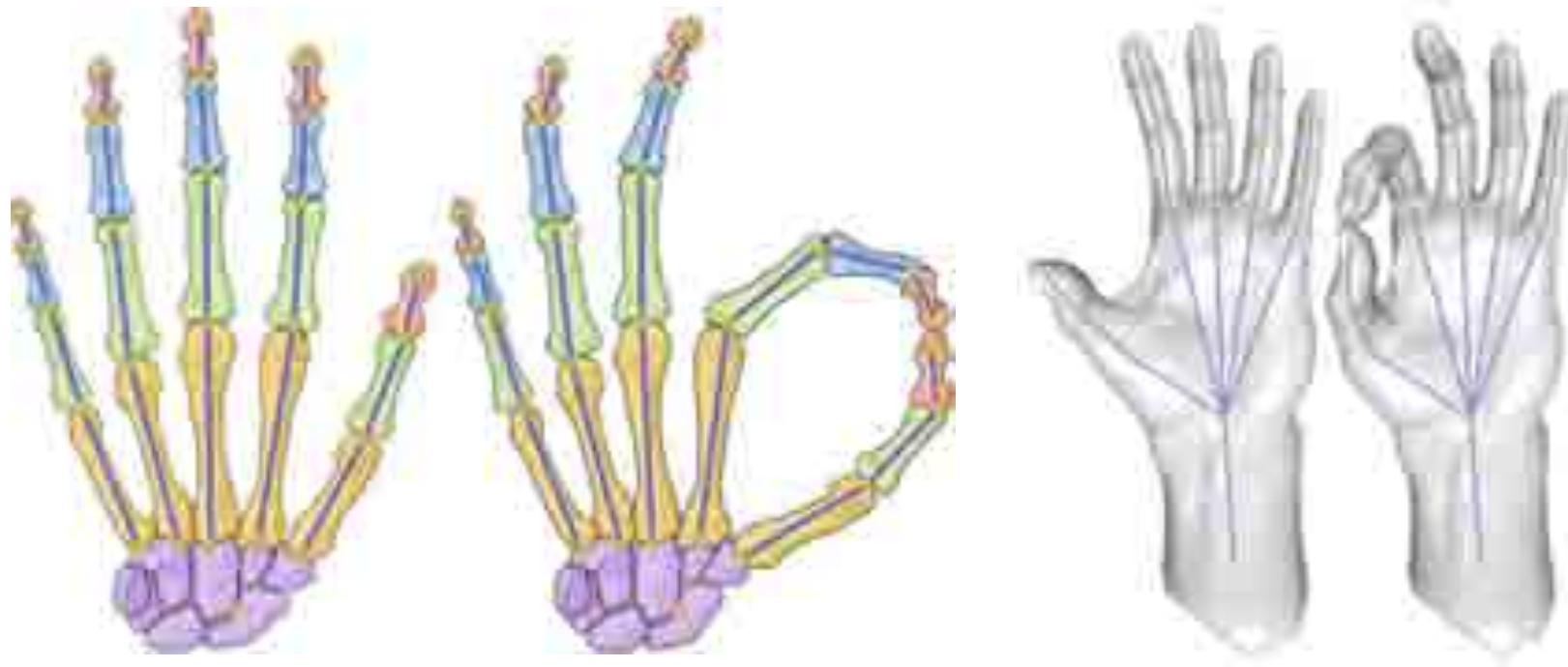


September 14, 2011



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Real-time performance critical for interactive design and animation



Real-time performance critical for interactive design and animation



Recent works provide high quality results at cost of pose time performance

Typical stress test of high quality deformation system require pose-time optimization
[Botsch et al. 2006]

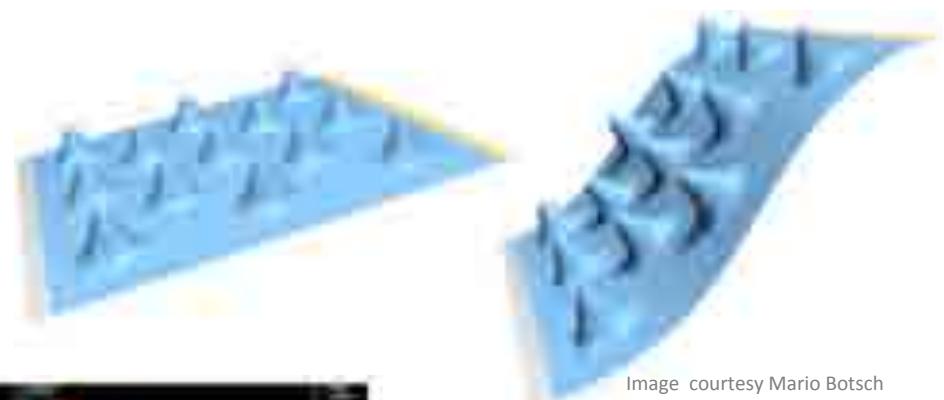


Image courtesy Mario Botsch



Physically accurate muscle systems require off-line simulation
[Teran et al. 2005]

Image courtesy Joseph Teran

Linear Blend Skinning can't match quality
but makes up in real-time performance

load shape



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Linear Blend Skinning can't match quality
but makes up in real-time performance

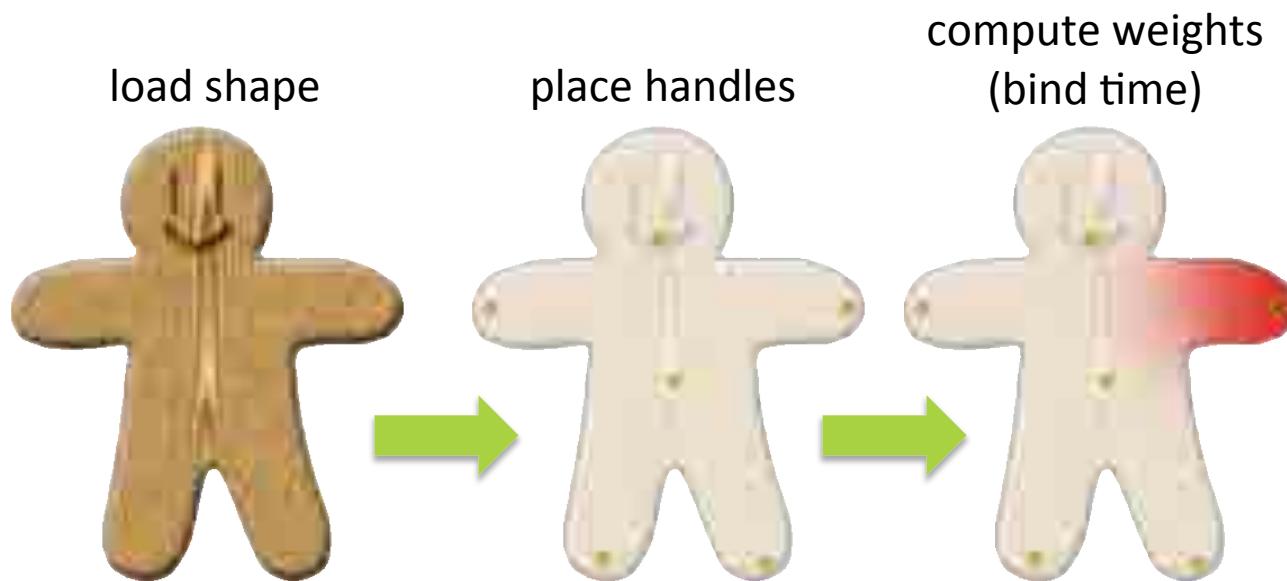
load shape



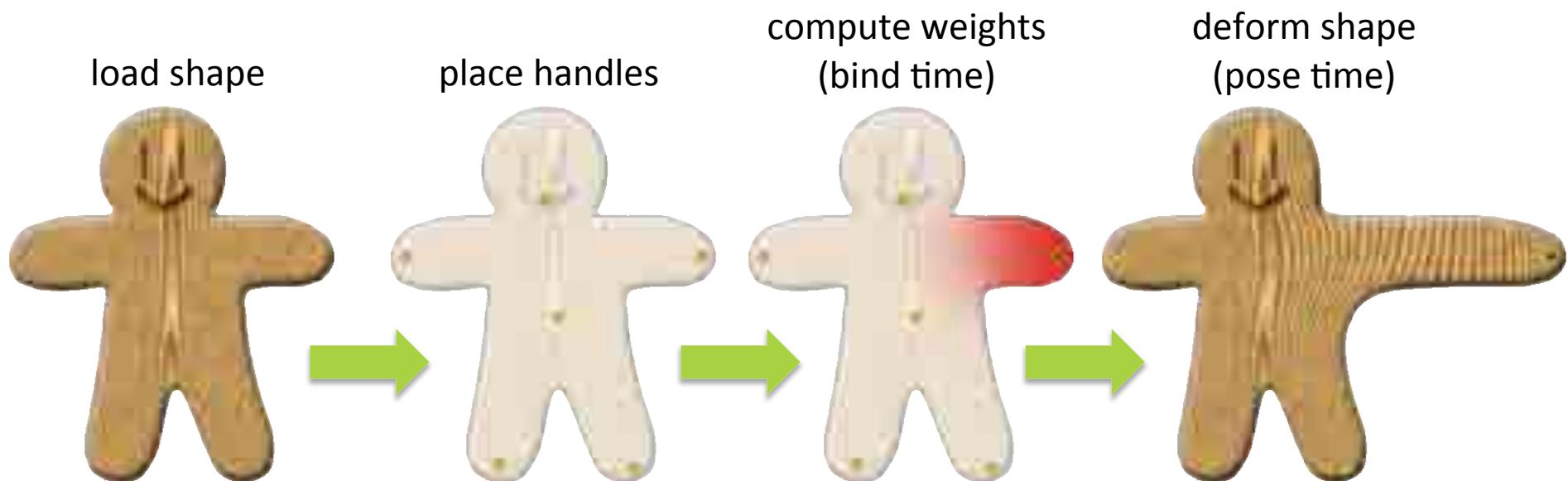
place handles



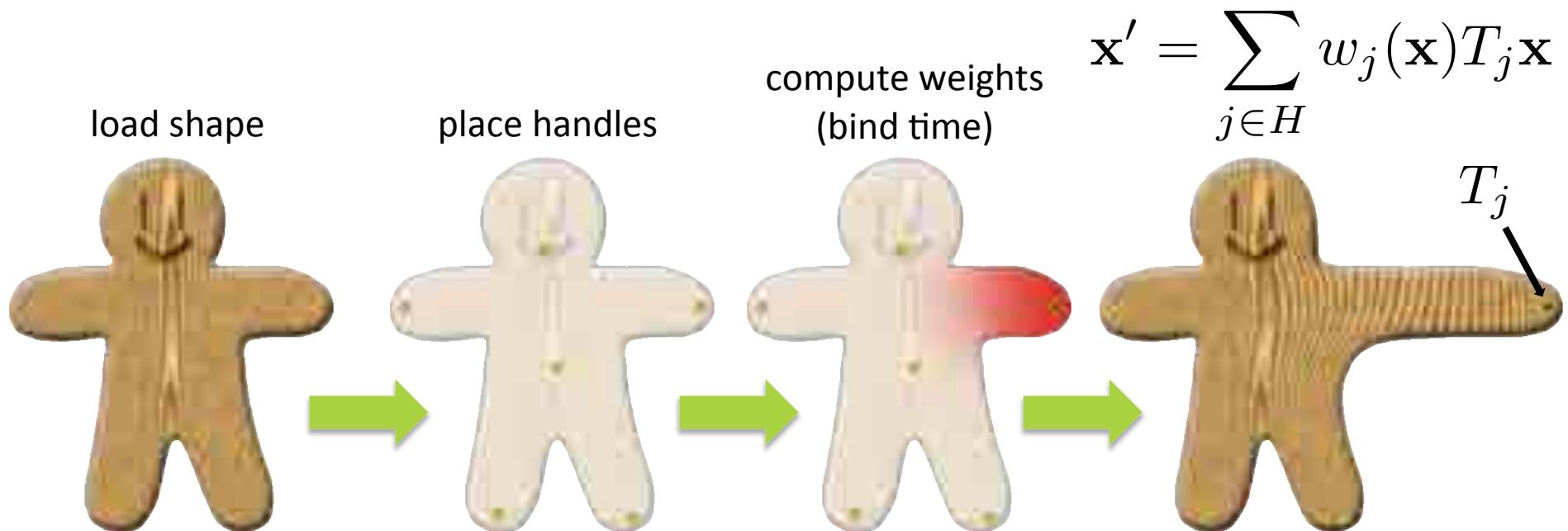
Linear Blend Skinning can't match quality but makes up in real-time performance



Linear Blend Skinning can't match quality but makes up in real-time performance



Linear Blend Skinning can't match quality but makes up in real-time performance



Warping and deformation tasks require different user interactions

$$\mathbf{x}' = \sum_{j \in H} w_j(\mathbf{x}) T_j \mathbf{x}$$



points

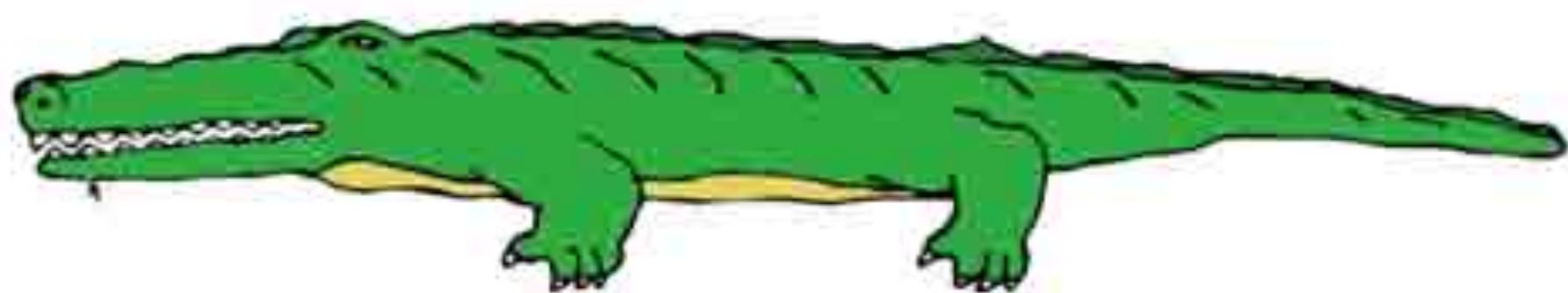


skeletons

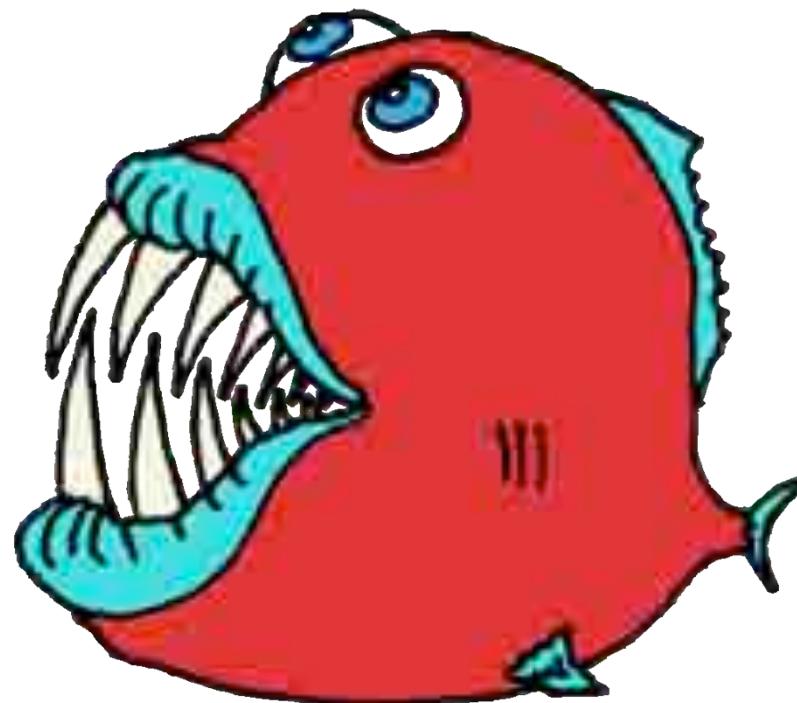


cages

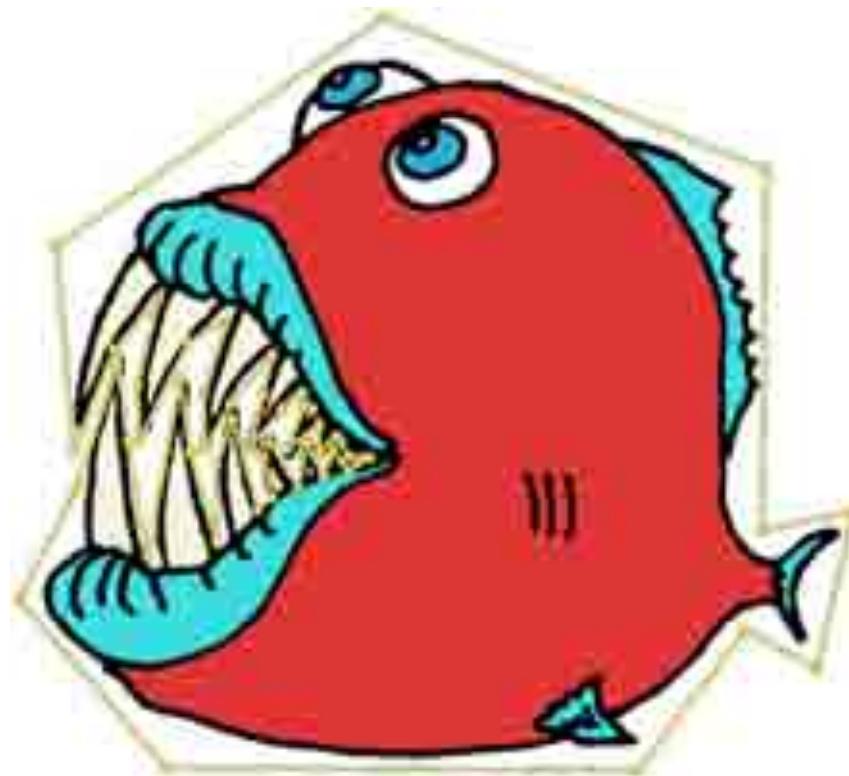
Each handle type has a specific task, more than just *different modeling metaphor*



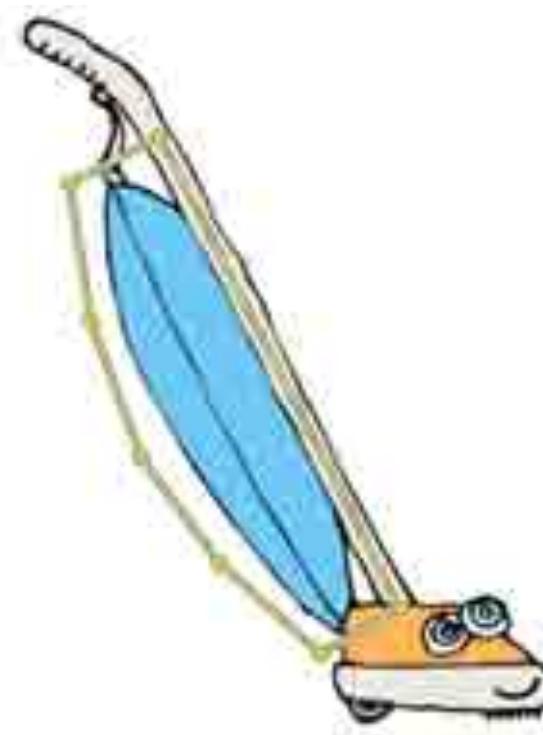
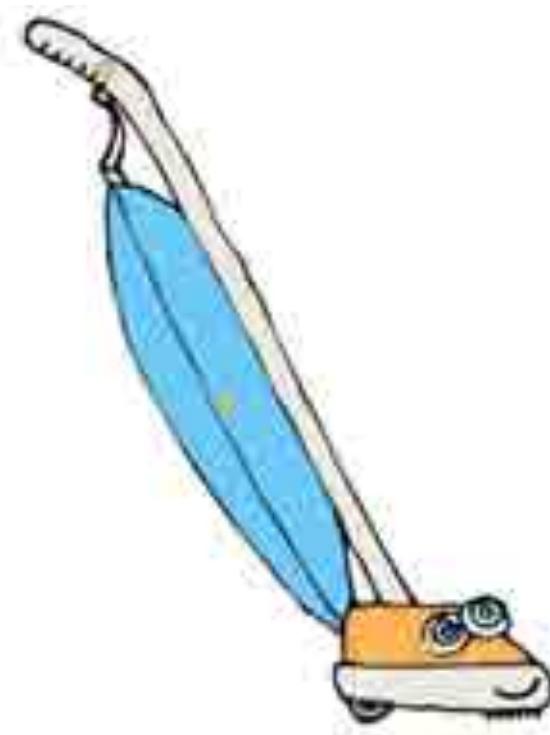
Cages can often be tedious to build and control



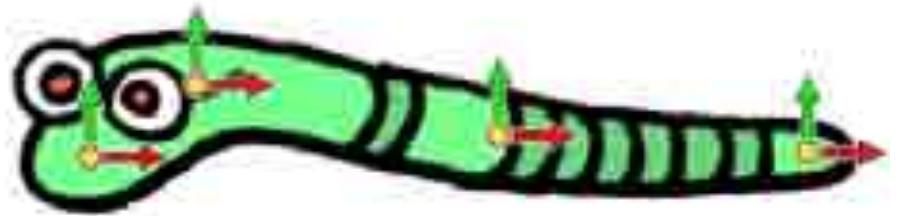
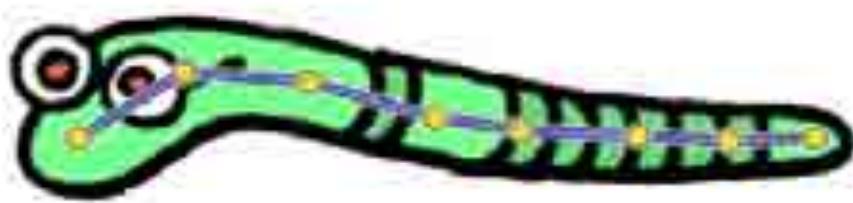
Cages can often be tedious to build and control



Points can only provide crude scaling



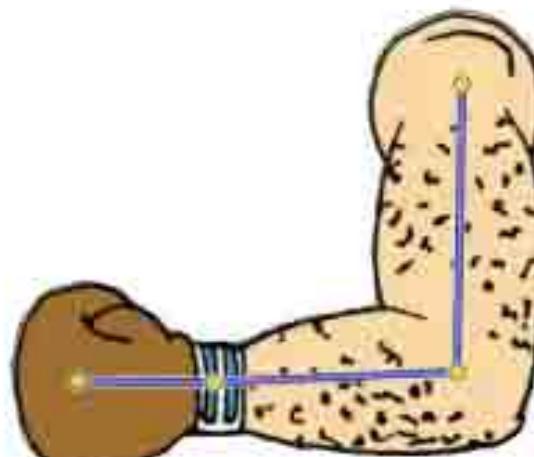
Skeletons may be too rigid or too cumbersome



We want to compute weights that unify points, skeletons and cages



Weights should be smooth,
shape-aware, positive and *intuitive*



Weights must be smooth everywhere, *especially* at handles



Our method



Extension of Harmonic Coordinates
[Joshi et al. 2005]

Weights must be smooth everywhere,
especially at handles



Our method



Extension of Harmonic Coordinates
[Joshi et al. 2005]

Shape-awareness ensures respect of domain's features



Our method



Non-shape-aware methods
e.g. [Schaefer et al. 2006]

Shape-awareness ensures respect of domain's features



Our method



Non-shape-aware methods
e.g. [Schaefer et al. 2006]

Non-negative weights are necessary for intuitive response

Our method



Unconstrained biharmonic
[Botsch and Kobbelt 2004]

Weights must maintain other simple, but important properties

$$\sum_{j \in H} w_j(\mathbf{x}) = 1$$

Partition of unity

$$w_j \Big|_{H_k} = \delta_{jk}$$

w_j is linear along cage faces

Interpolation of handles

Weights must maintain other simple, but important properties

$$\sum_{j \in H} w_j(\mathbf{x}) = 1$$

Partition of unity

Set of handles, aka mesh vertices under handles

$$w_j \Big|_{H_k} = \delta_{jk}$$

w_j is linear along cage faces

Interpolation of handles

Kronecker's delta

Previous techniques only partially satisfy properties

2

	Harmonic Coordinates [Joshi et al. 2005]	Unconstrained biharmonic [Botsch and Kobbelt 2004]	Shepard interpolation [Shepard 1968]	Natural neighbors [Sibson 1981]
Smoothness	-	✓	✓	-
Non-negativity	✓	-	✓	✓
Shape-aware	✓	✓	-	-
Locality, sparsity	-	-	-	✓

Previous techniques only partially satisfy properties

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$$\Delta w_j = 0$$

Previous techniques only partially satisfy properties

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	Harmonic Coordinates [Joshi et al. 2005]	Unconstrained biharmonic [Botsch and Kobbelt 2004]	Shepard interpolation [Shepard 1968]	Natural neighbors [Sibson 1981]
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$$\Delta^2 w_j = 0$$

Previous techniques only partially satisfy properties

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Shape-aware	✓	✓	-	-
Locality, sparsity	-	-	-	✓

Inverse distance,
weighted least-squares

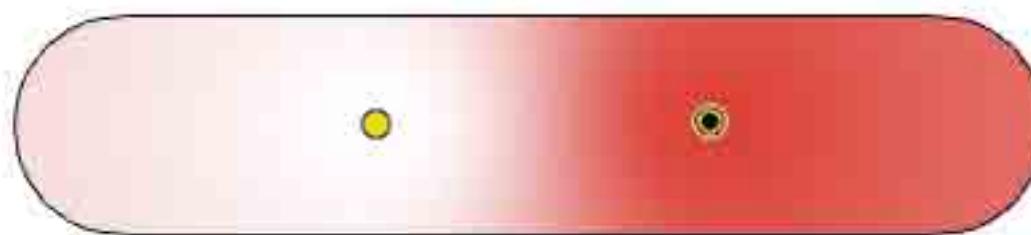
Inverse distance methods inherently suffer from *fall-off effect*



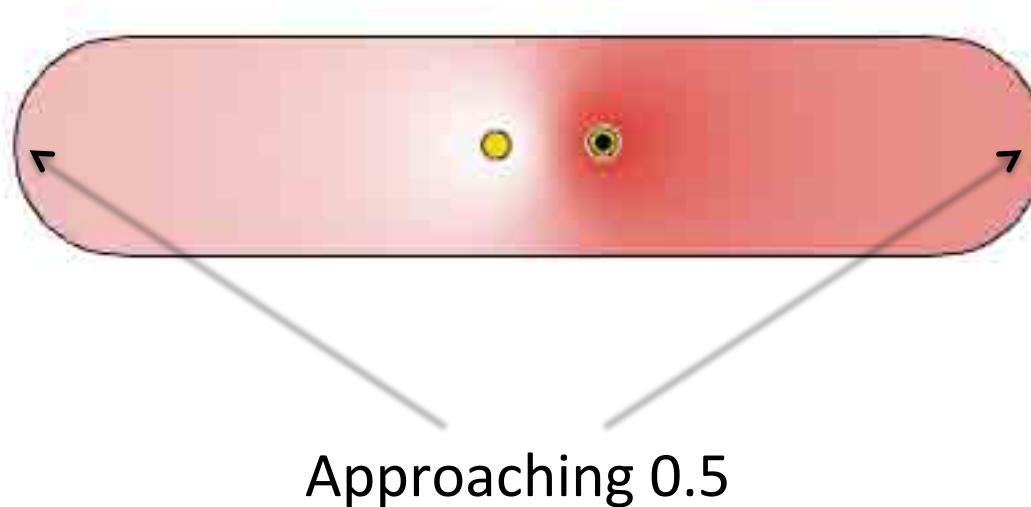
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Smoothness	-	✓	✓	-
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Shape-aware	✓	✓	-	-
Locality, sparsity	-	-	-	✓

How to support bones and cages?
How to make shape-aware?

Previous techniques only partially satisfy properties

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Smoothness	-	✓	✓	-
Non-negativity	✓	-	✓	✓
Shape-aware	✓	✓	-	-
Locality, sparsity	-	-	-	✓

$$\Delta^2 w_j = 0$$

Bounded biharmonic weights enforce properties as constraints to minimization

$$\arg \min_{w_j} \frac{1}{2} \int_{\Omega} \|\Delta w_j\|^2 dV$$

$$w_j \Big|_{H_k} = \delta_{jk}$$

w_j is linear along cage faces

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Constant inequality constraints
 $0 \leq w_j(\mathbf{x}) \leq 1$

Partition of unity

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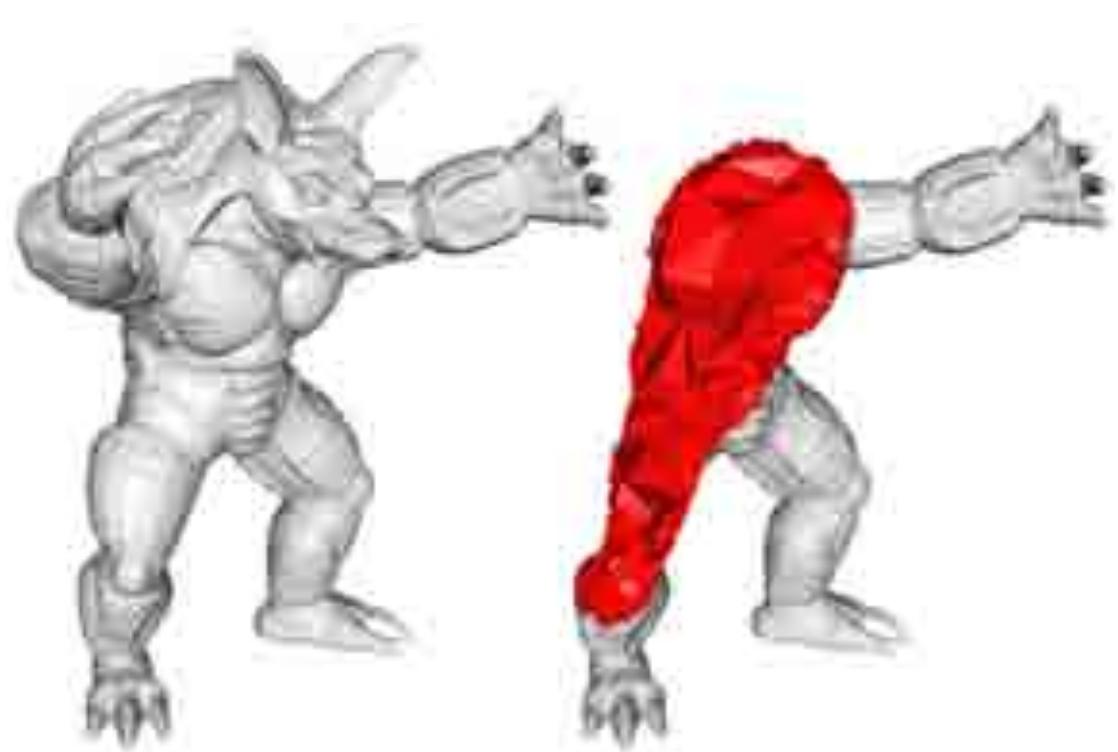
Solve independently and
normalize

$$w_j(\mathbf{x}) = \frac{w_j(\mathbf{x})}{\sum_{i \in H_k} w_i(\mathbf{x})}$$

Weights optimized as precomputation at bind-time

$$\arg \min_{w_j} \frac{1}{2} \int_{\Omega} \|\Delta w_j\|^2 dV$$
$$w_j|_{H_k} = \delta_{jk}$$

w_j is linear along cage faces
 $0 \leq w_j(\mathbf{x}) \leq 1$



FEM discretization

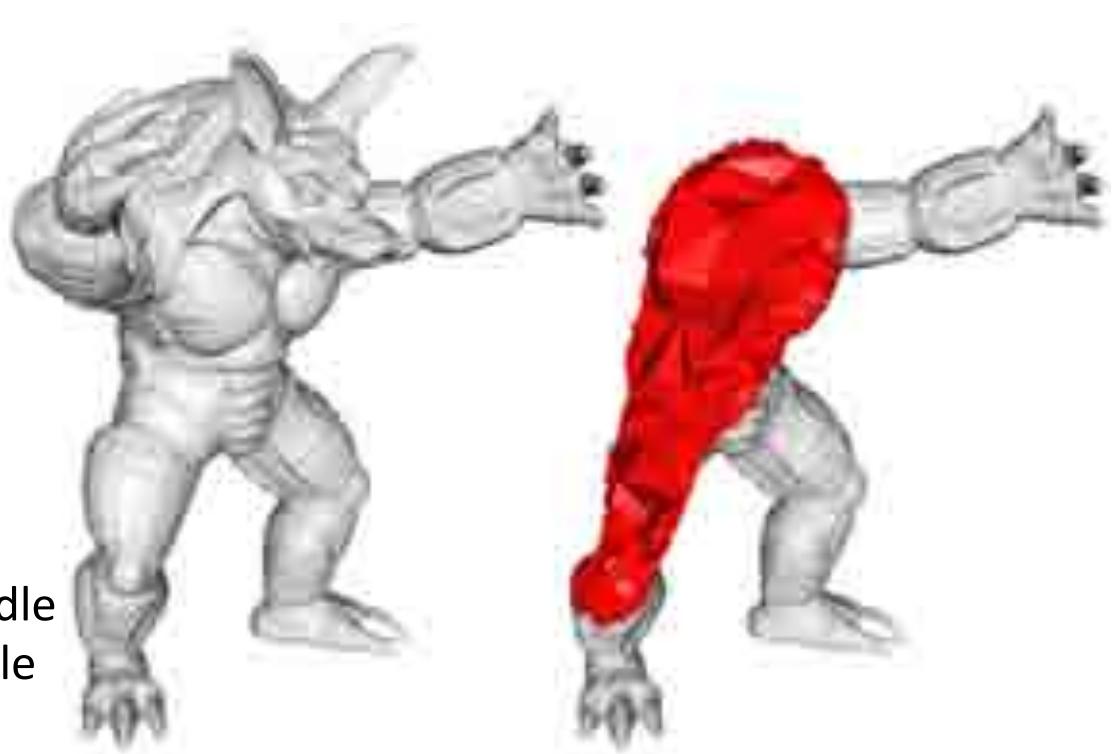
2D → Triangle mesh

3D → Tet mesh

Weights optimized as precomputation at bind-time

$$\arg \min_{w_j} \frac{1}{2} \int_{\Omega} \|\Delta w_j\|^2 dV$$
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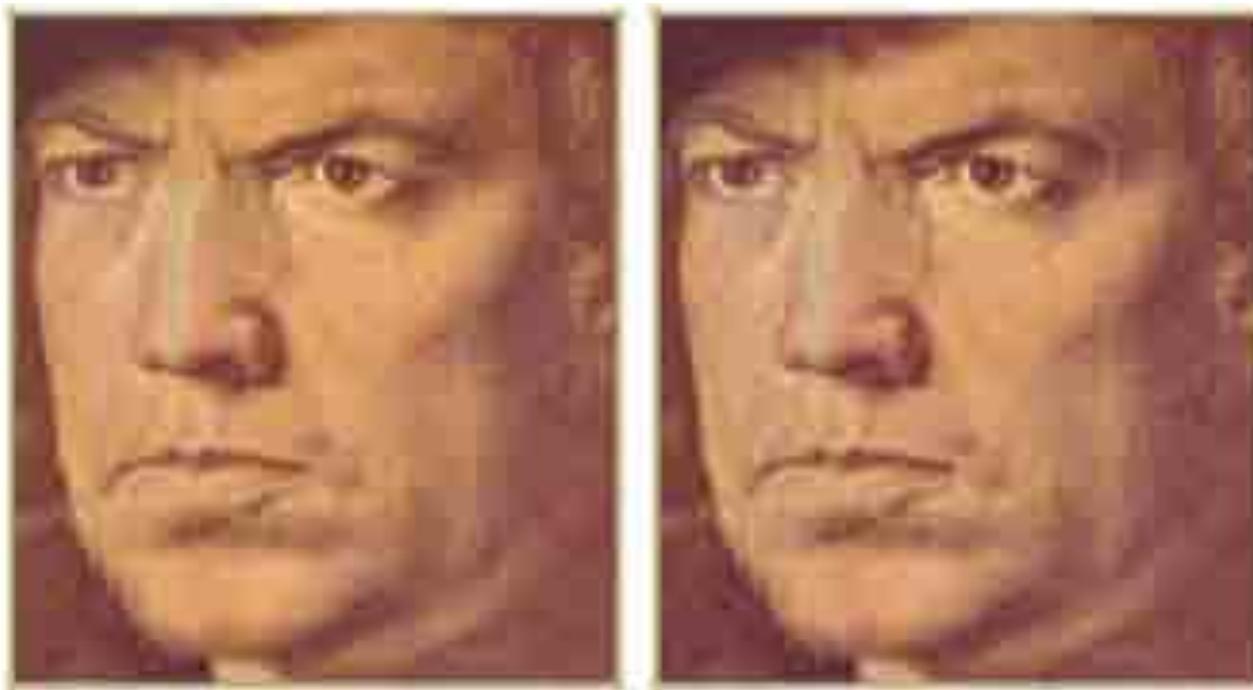


Sparse quadratic programming with constant inequality constraints

2D → less than second per handle

3D → tens of seconds per handle

Variational formulation allows additional, problem-specific constraints



Rotations at point handles may be computed automatically based on translations



without pseudo-edges



with pseudo-edges

Open cages allow arbitrary line constraints



Some tasks are more easily accomplished
by controlling both points and lines



Weights in 3D also retain nice properties, ...

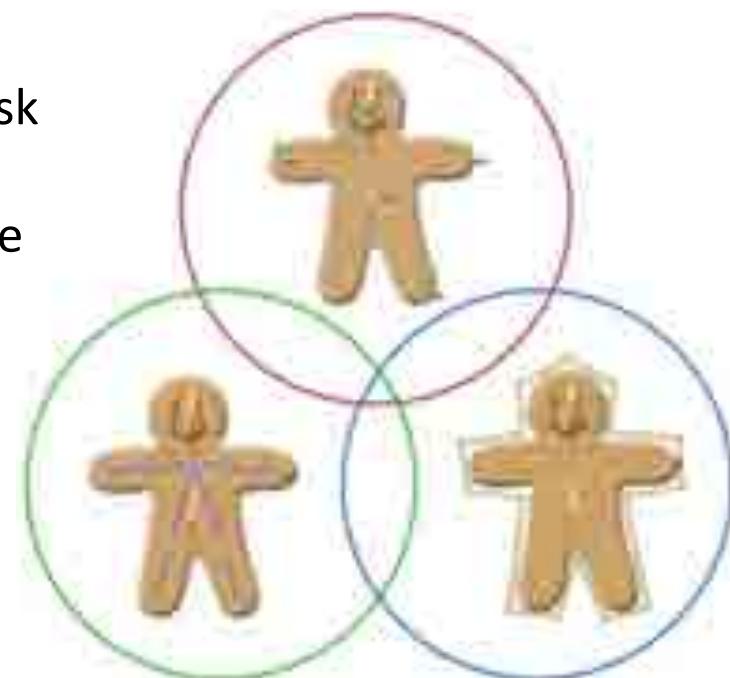


... and unify handle types



In summary, BBW unify popular handle types for intuitive, real-time deformations

- Point, skeleton and cage handles have applications dependent on setting and task
- Bounded biharmonic weights unify handle types
- BBW are:
 - smooth, even at handles
 - local, shape-aware and sparse
 - always between 0 and 1



Future work for bounded biharmonic weights

- Missing degrees of freedom point handles
- Sparsity heuristics, improve precomputation time and storage
- Other uses of bounded biharmonic functions

We gratefully acknowledge...

Jaakko Lehtinen, Bob Sumner and Denis Zorin for illuminating discussions

Annie Ytterberg for her narration of the accompanying video

Yang Song for help implementing the rigidity brush and 2D remeshing

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#48

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Bounded Biharmonic Weights for Real-Time Deformation

MATLAB code: <http://igl.ethz.ch/projects/bbw/>

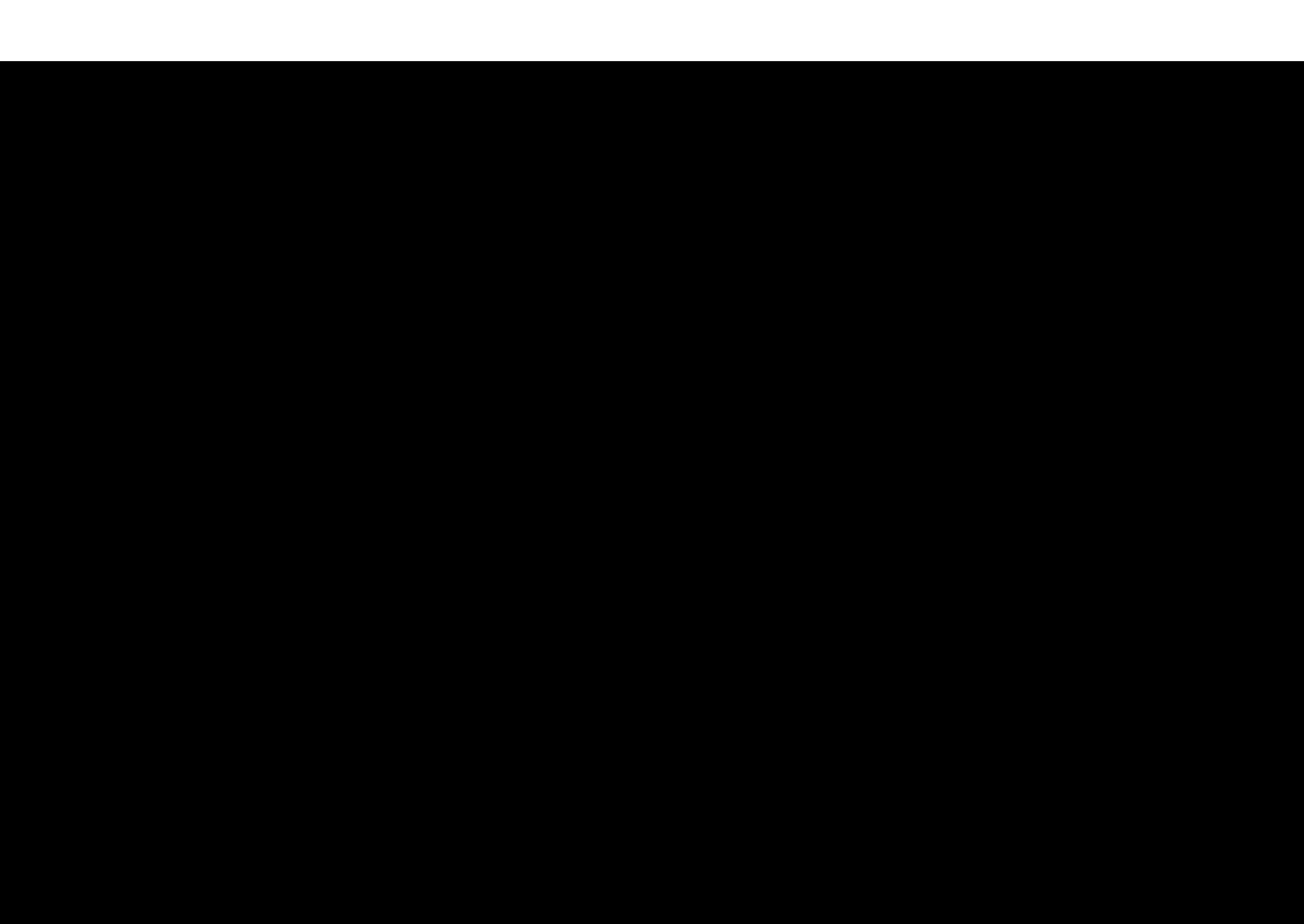
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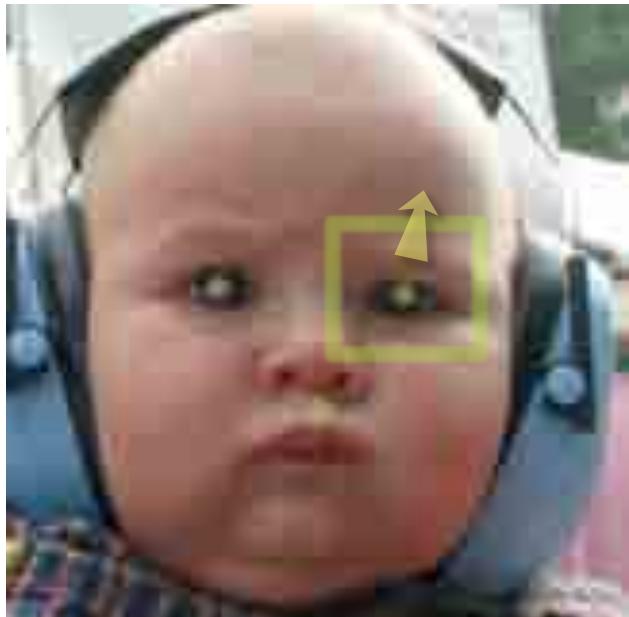
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Our method



Extension of Harmonic Coordinates
[Joshi et al. 2005]

Sparsity helps maintain local influence

Smoothed extension of
Harmonic Coordinates
[Joshi et al. 2005]

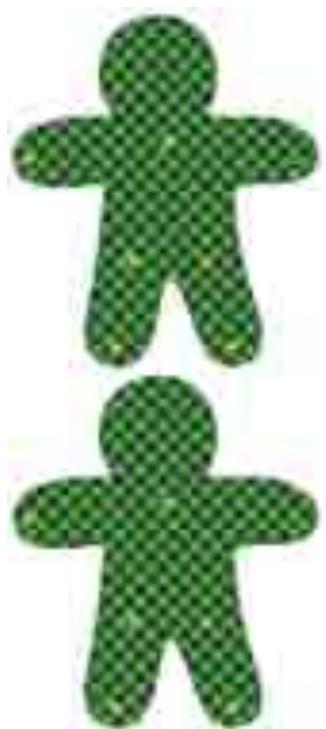


Our method

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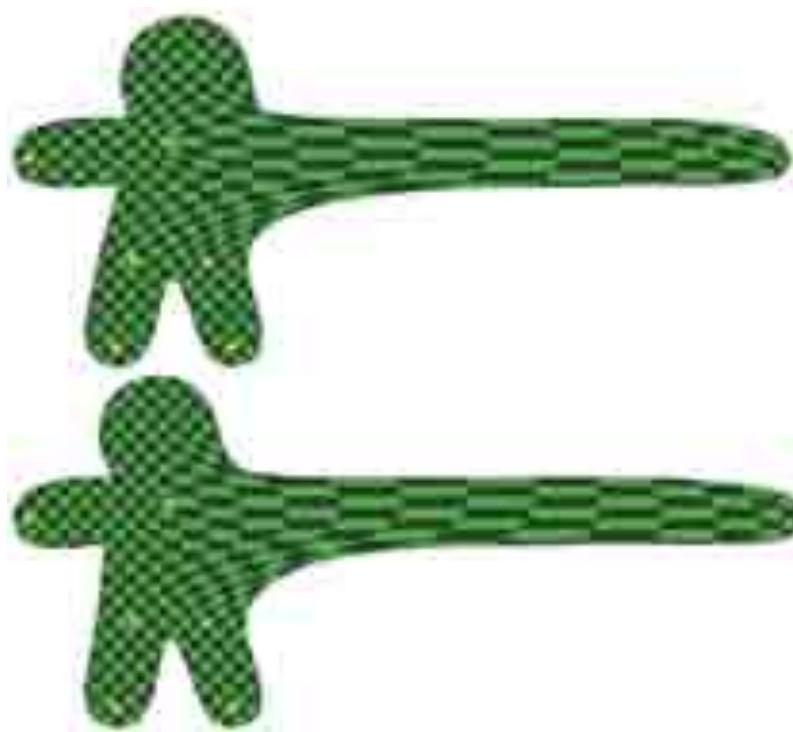
Our method



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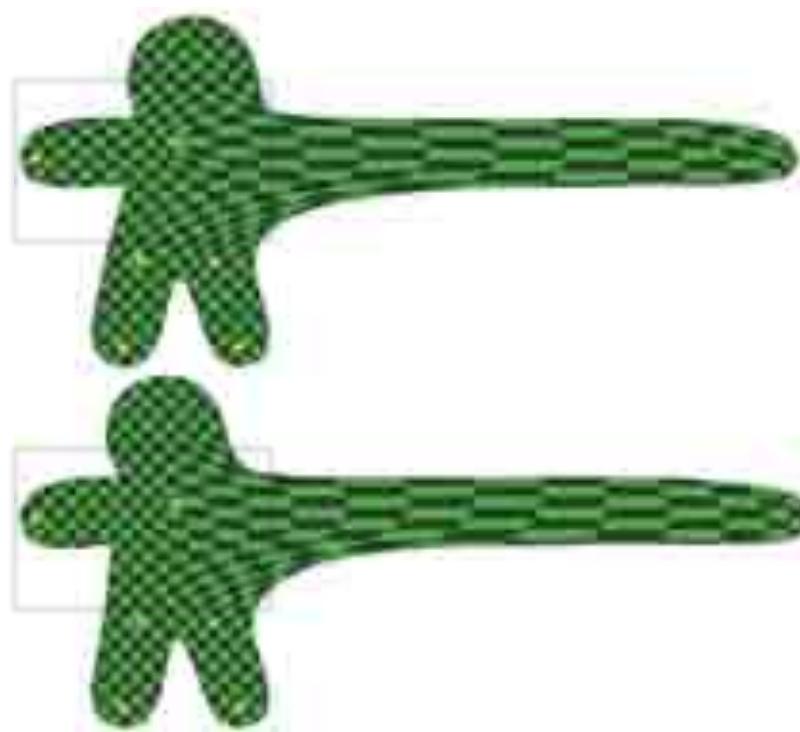
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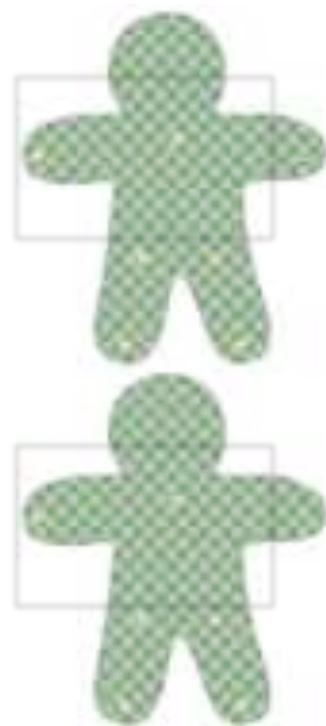
Smoothed extension of
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Our method



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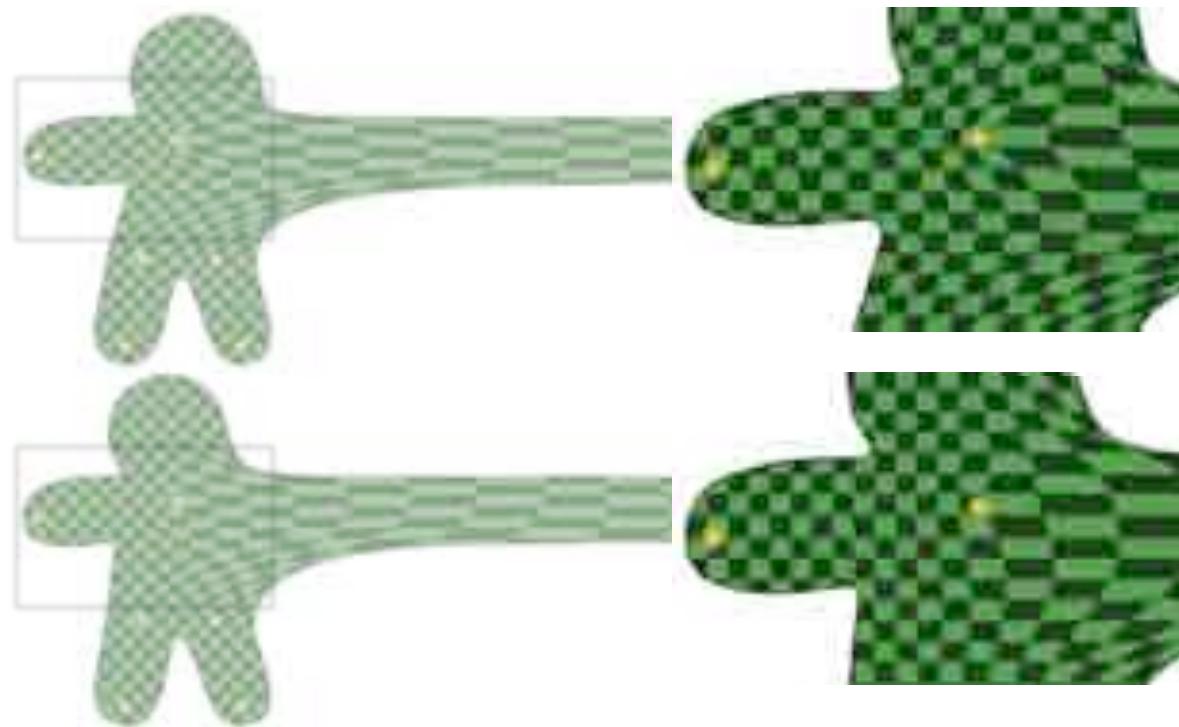
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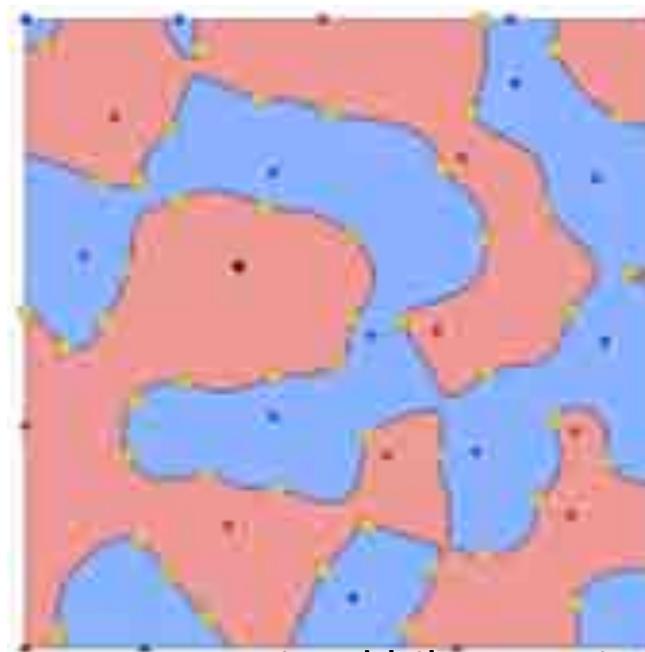
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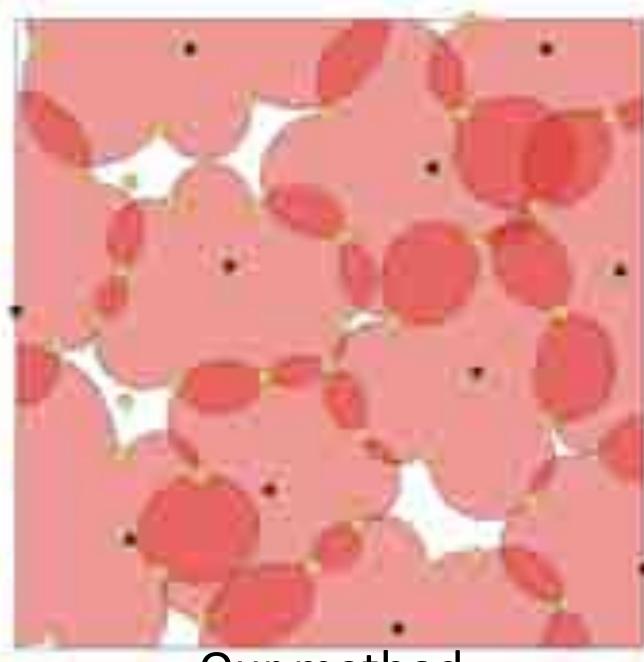


Boundedness also helps maintain local influence

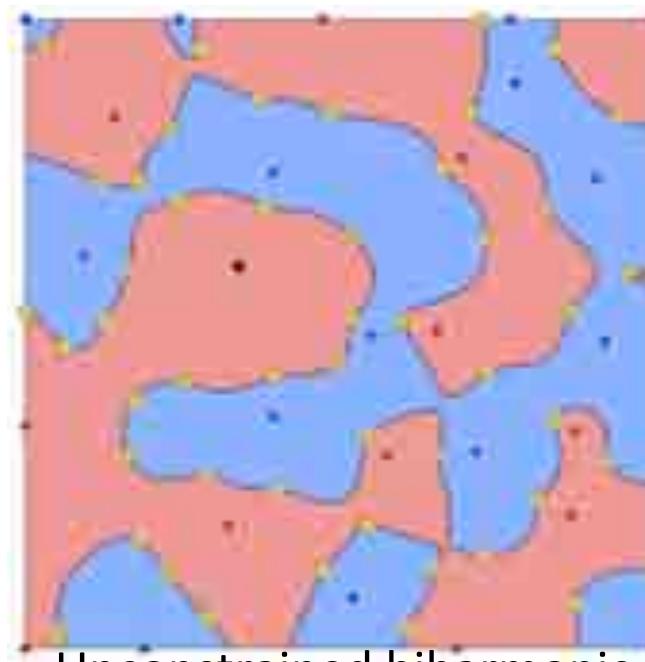


Unconstrained biharmonic
[Botsch and Kobbelt 2004]

Boundedness also helps maintain local influence

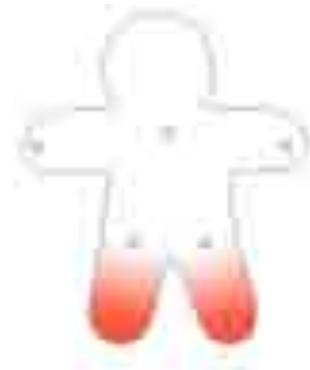


Our method



Unconstrained biharmonic
[Botsch and Kobbelt 2004]

Spurious local maxima also cause unintuitive response



Our method

Extension of unconstrained biharmonic
[Botsch and Kobbelt 2004]



Weights propagate transformations at handles to shape in real-time



Translate

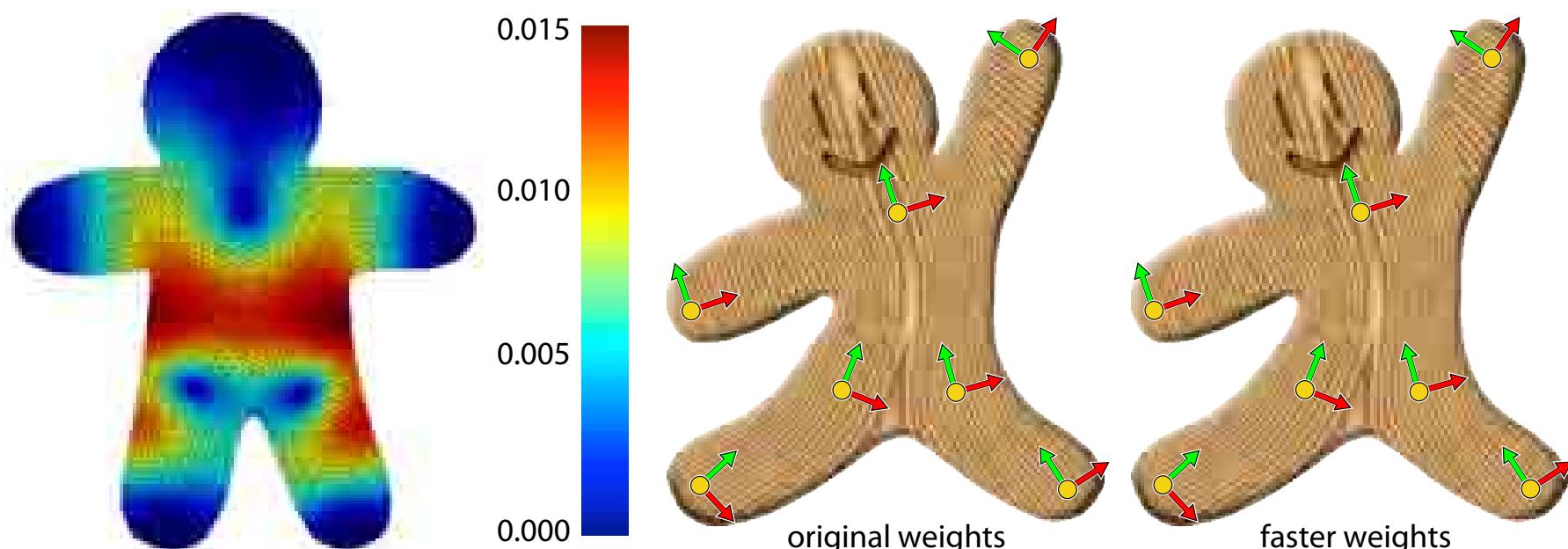


Rotate

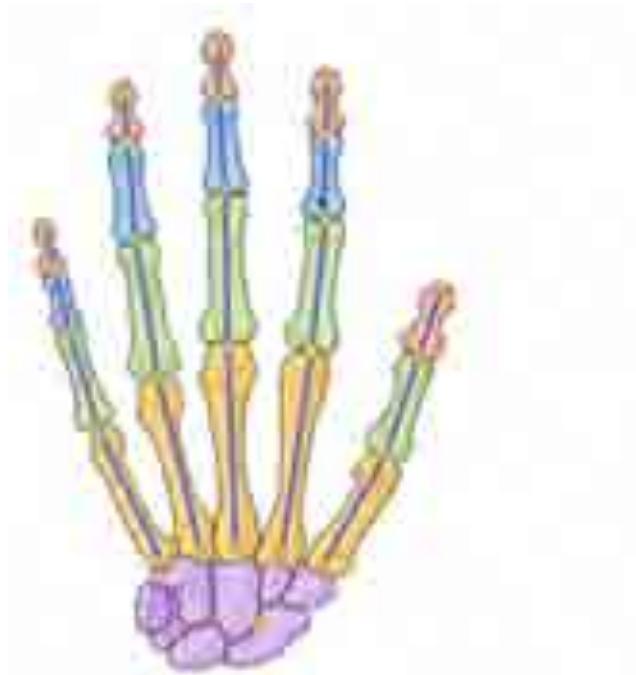


Scale

Dropping partition of unity as explicit constraint does not effect quality



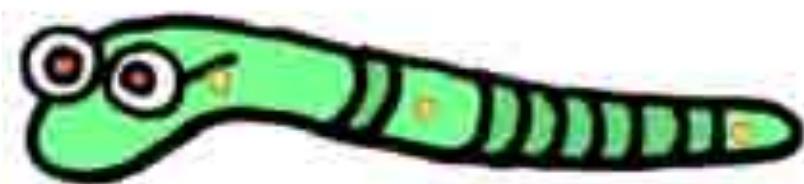
Weights may also define an intuitive,
shape-aware depth ordering in 2D



Rotations at point handles may be computed automatically based on translations



Alternative skinning methods may also take advantage of bounded biharmonic weights



Linear blend skinning



Dual quaternion skinning