Fast Automatic Skinning Transformations

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ETH
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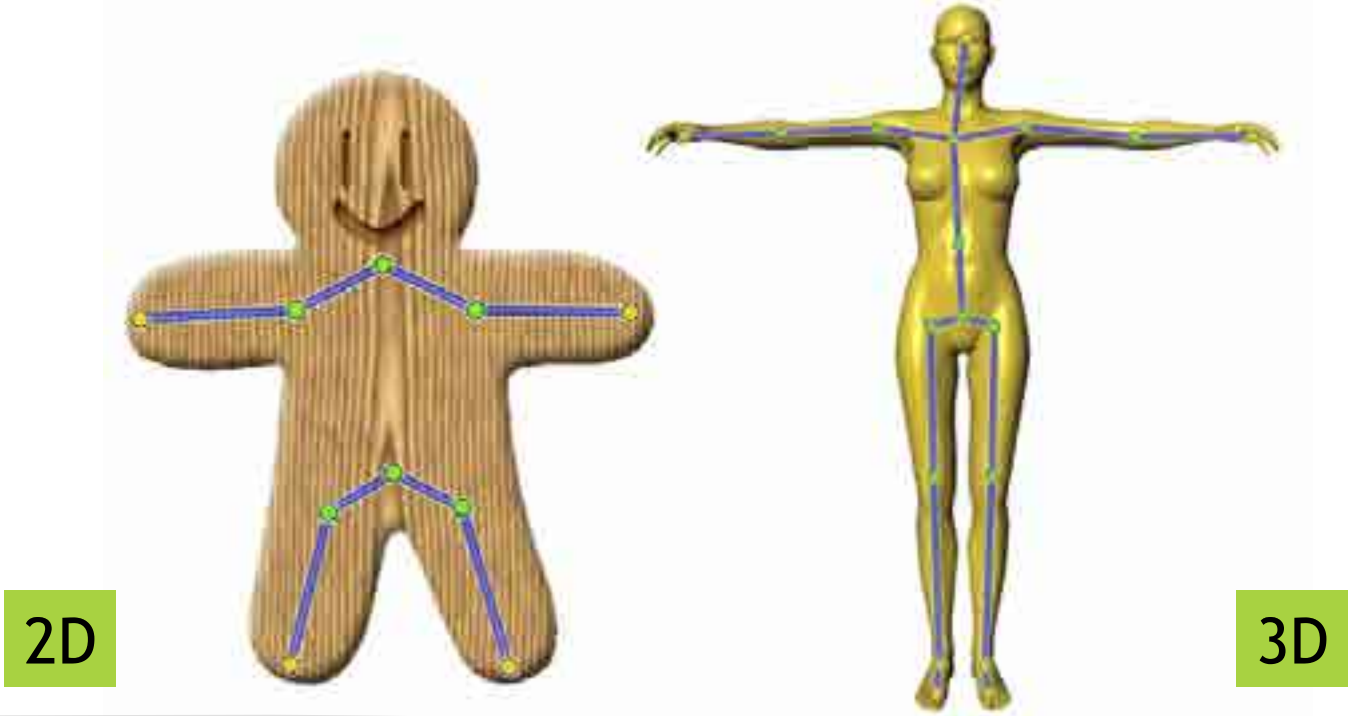
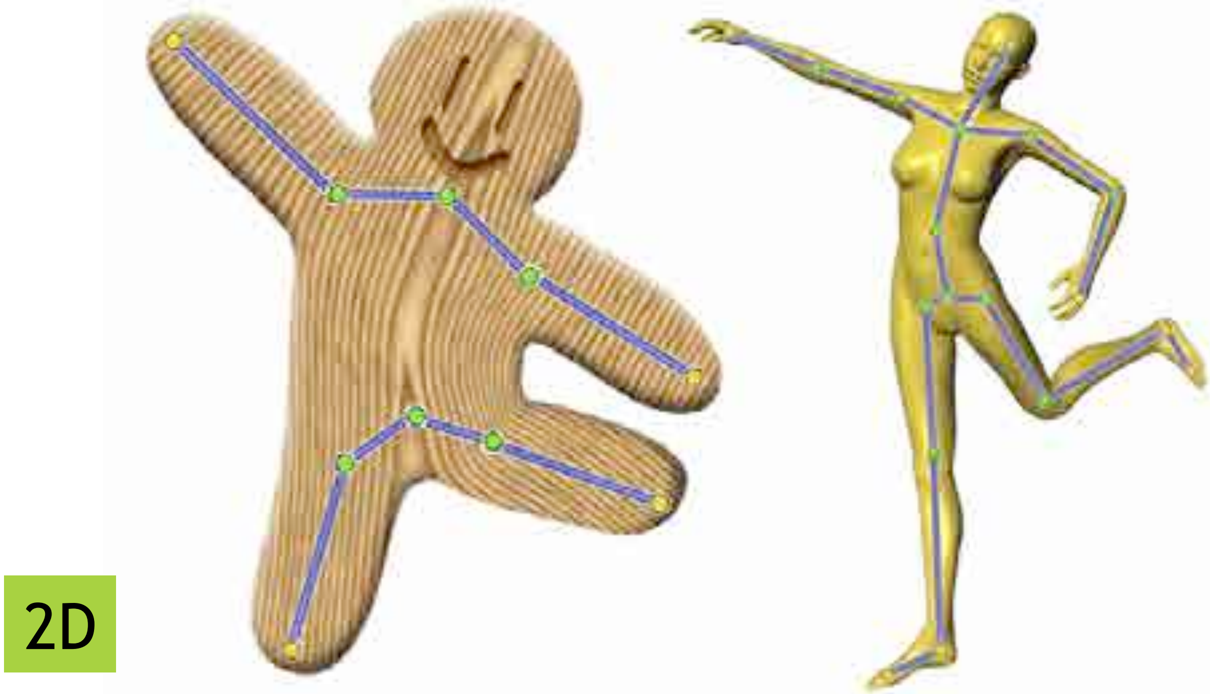
Swiss Federal Institute of Technology Zurich

Real-time performance critical for interactive design and animation

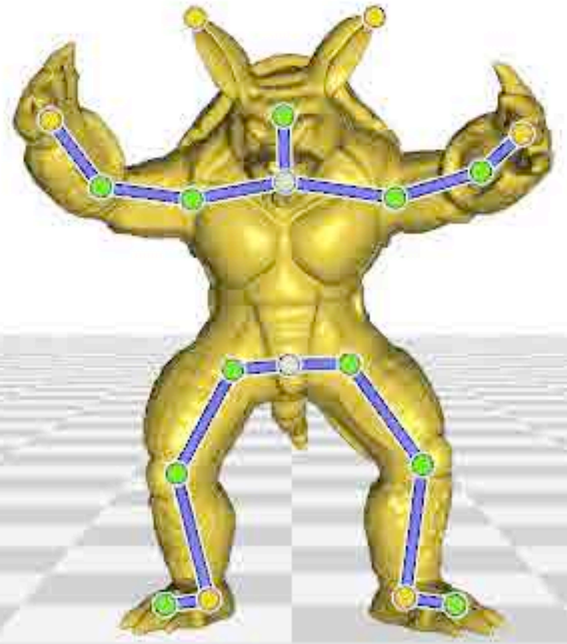


Real-time performance critical for interactive design and animation



We want speeds measured in microseconds

80k triangles
20μs per iteration



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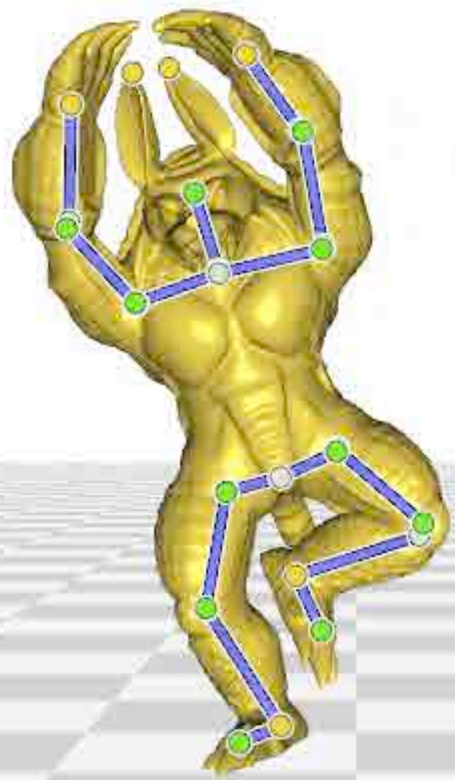
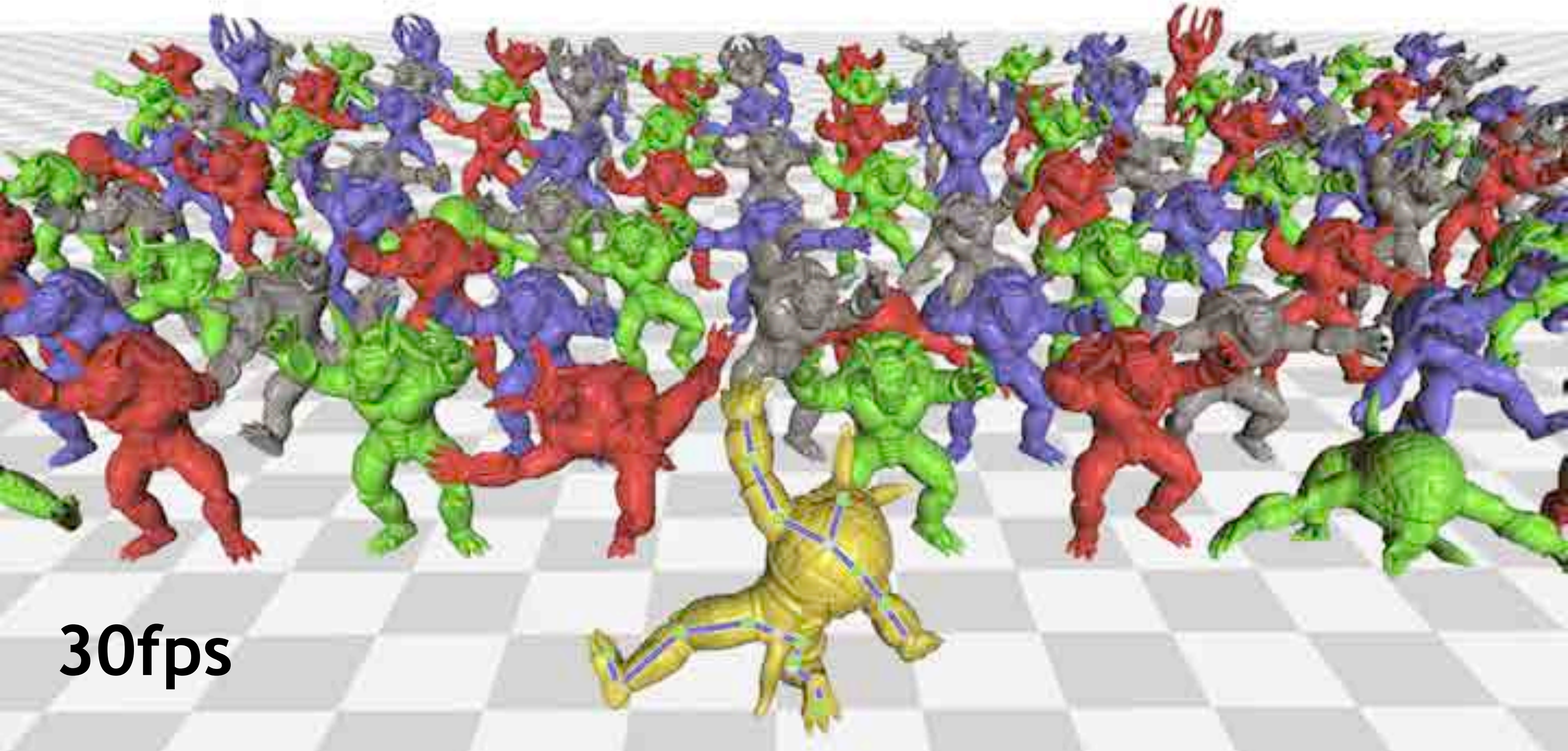
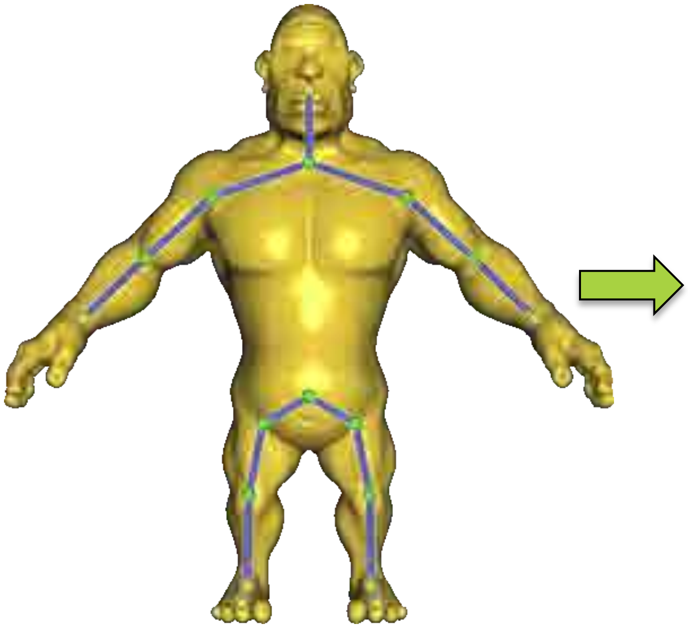


This means speed comparable to rendering

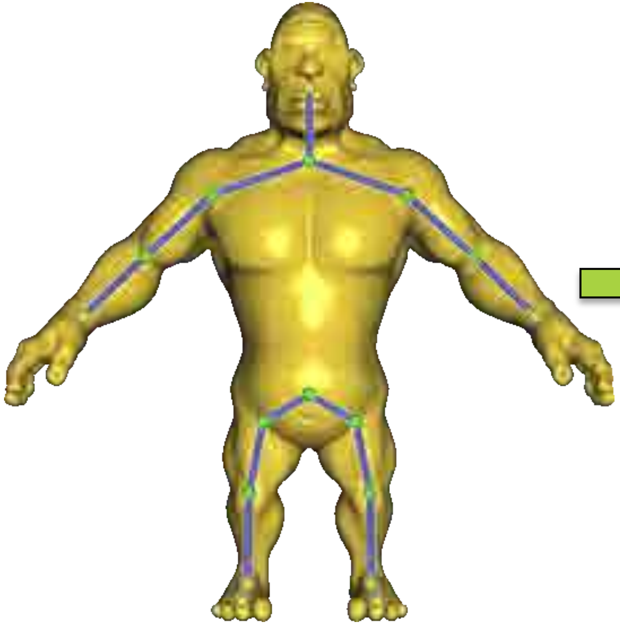


Linear Blend Skinning preferred for real-time performance



place skeleton in shape

Linear Blend Skinning preferred for real-time performance



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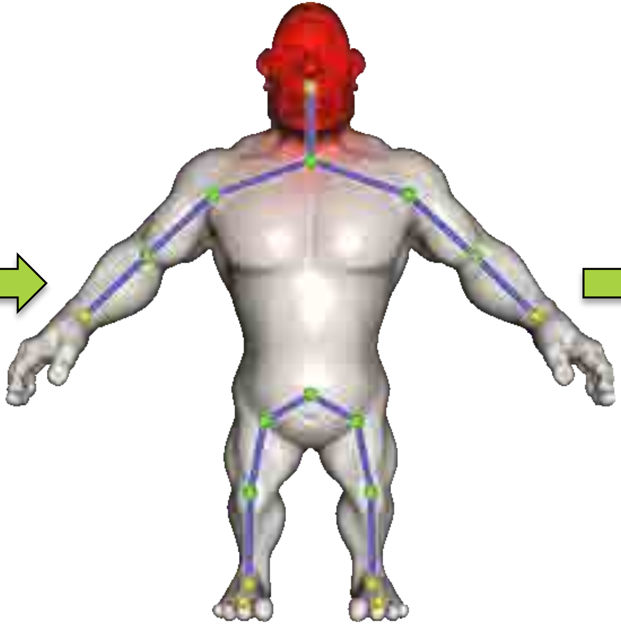
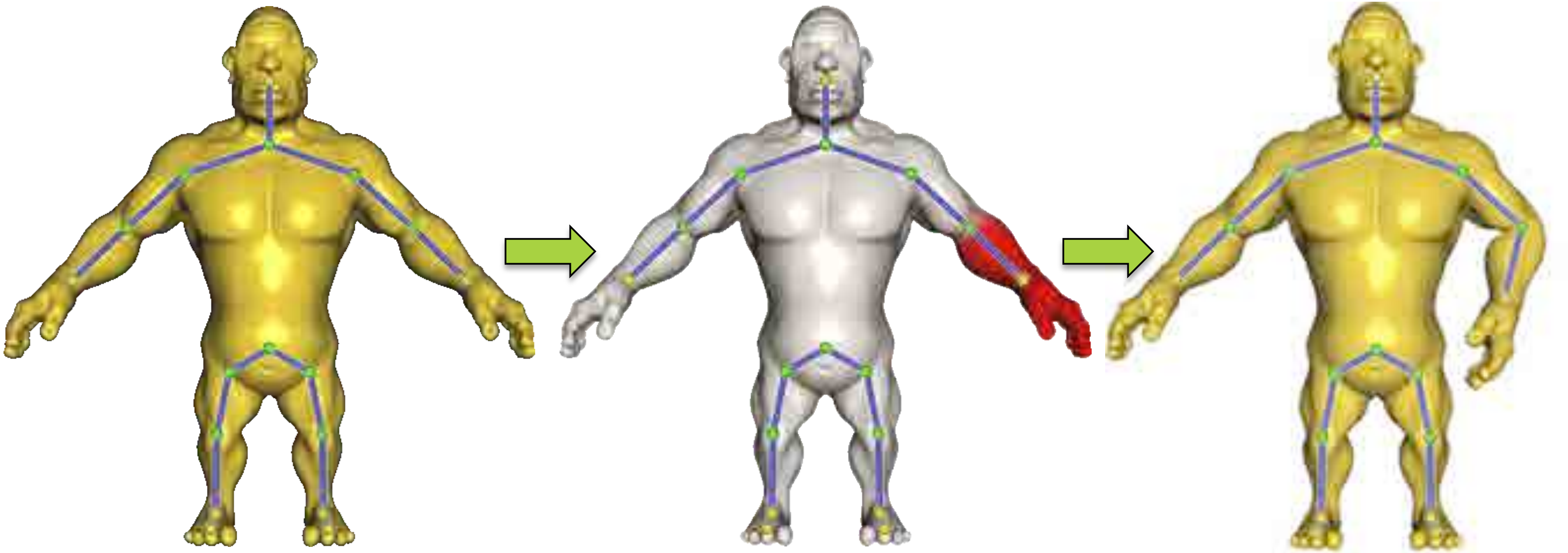


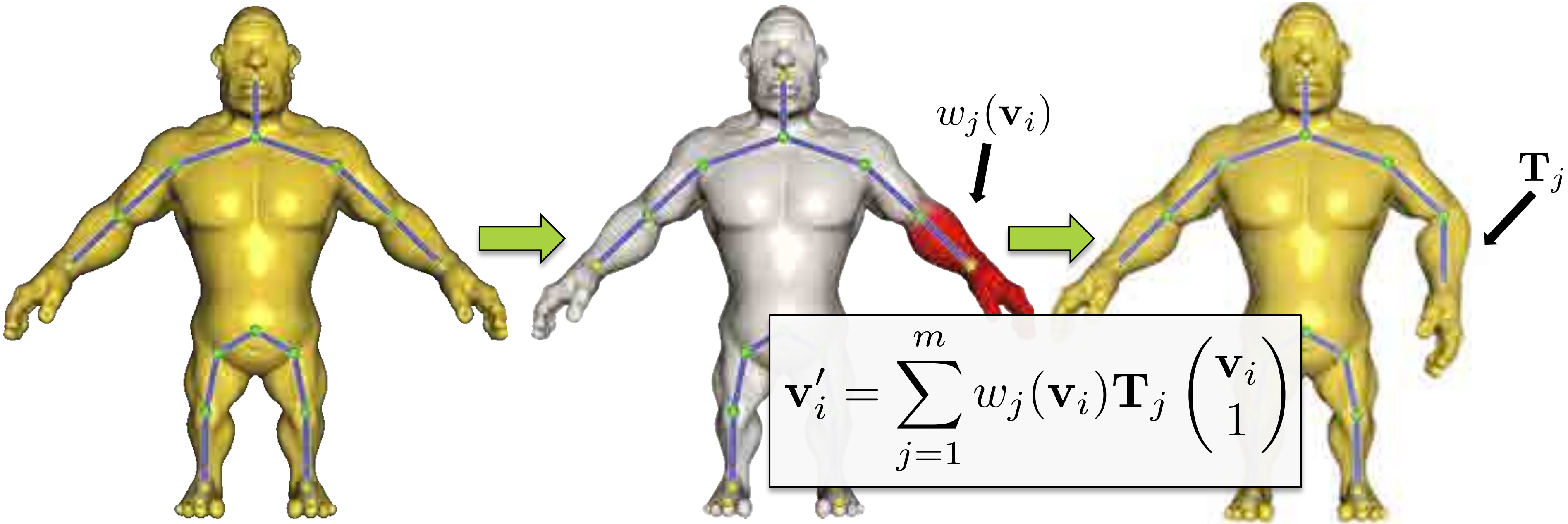
compute/paint weights

Linear Blend Skinning preferred for real-time performance



place skeleton in shape   compute/paint weights   deform bones

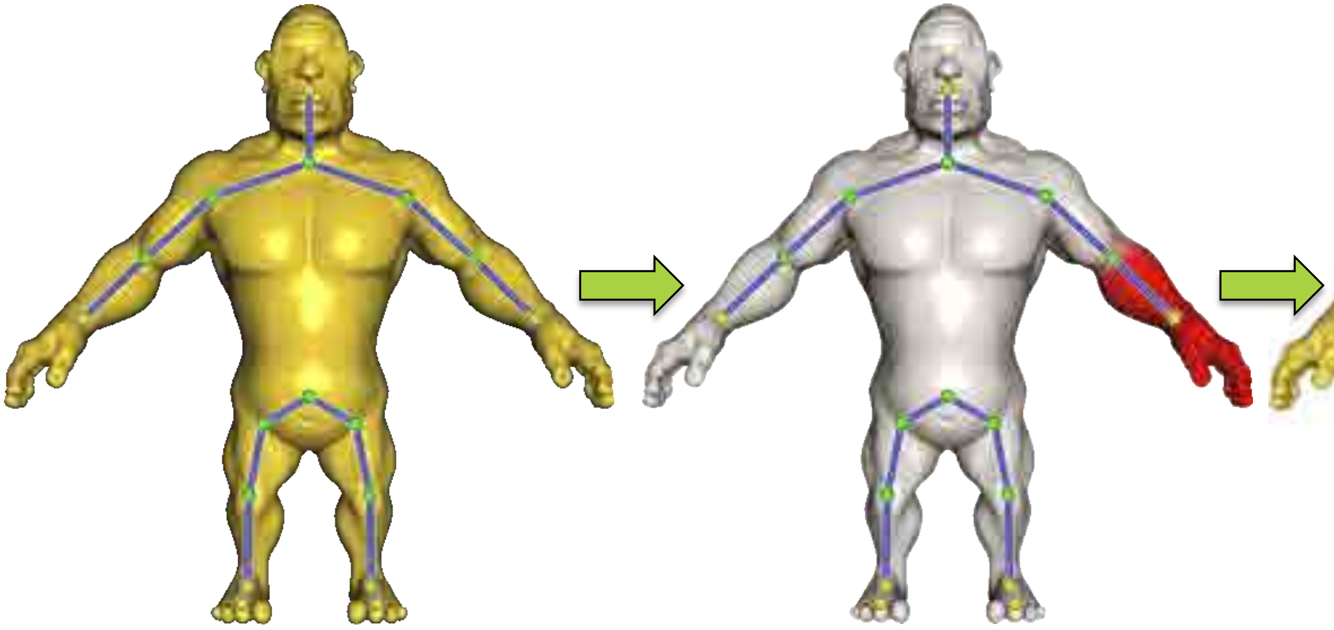
Linear Blend Skinning preferred for real-time performance



$$\mathbf{v}_i' = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix} \mathbf{v}_i \\ 1 \end{pmatrix}$$

place skeleton in shape | compute/paint weights | deform bones

Linear Blend Skinning preferred for real-time performance



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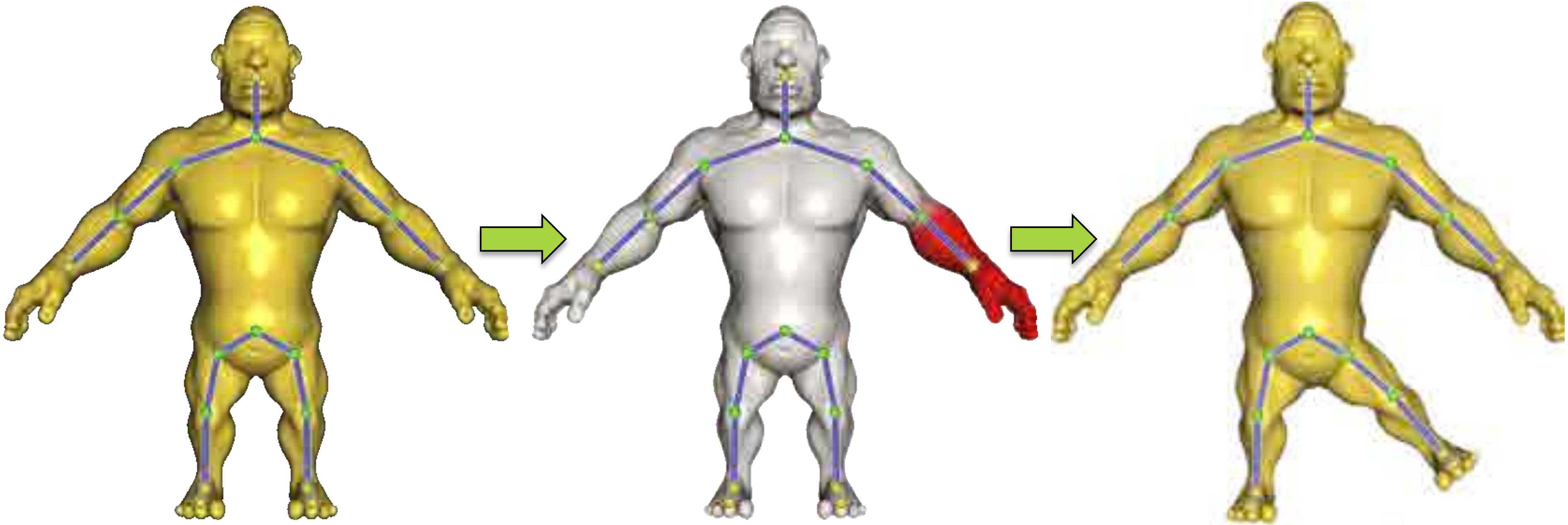

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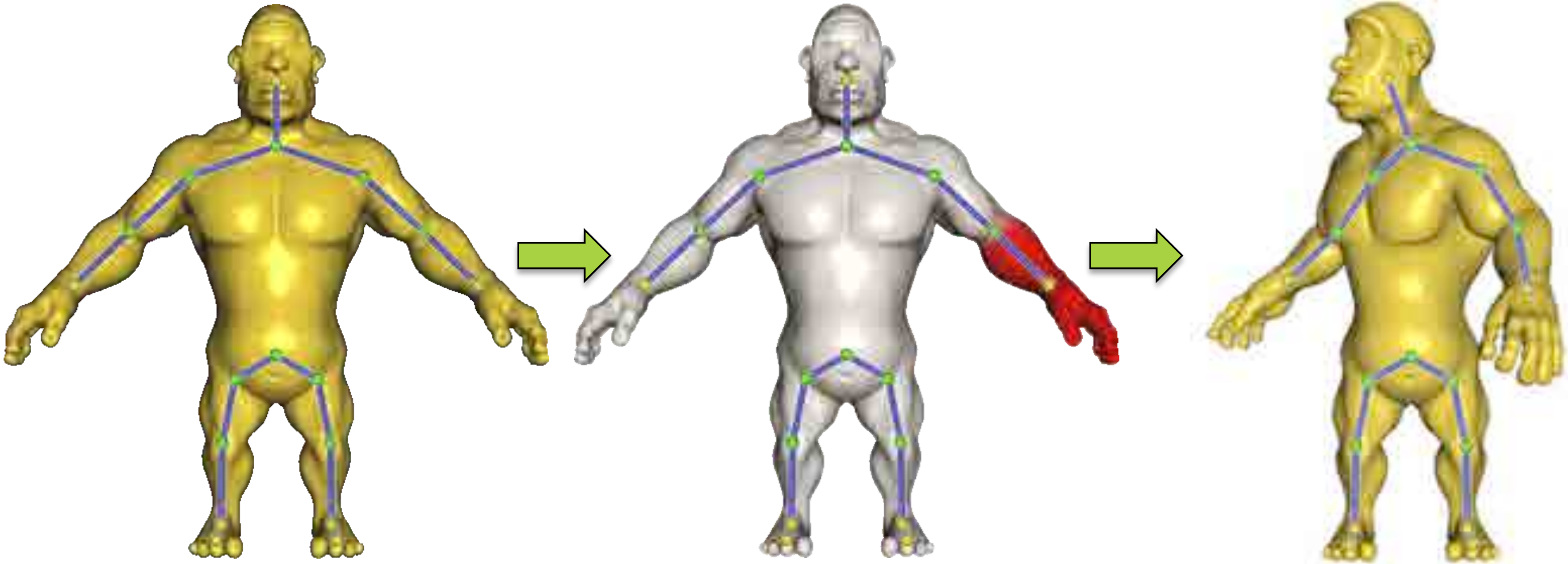

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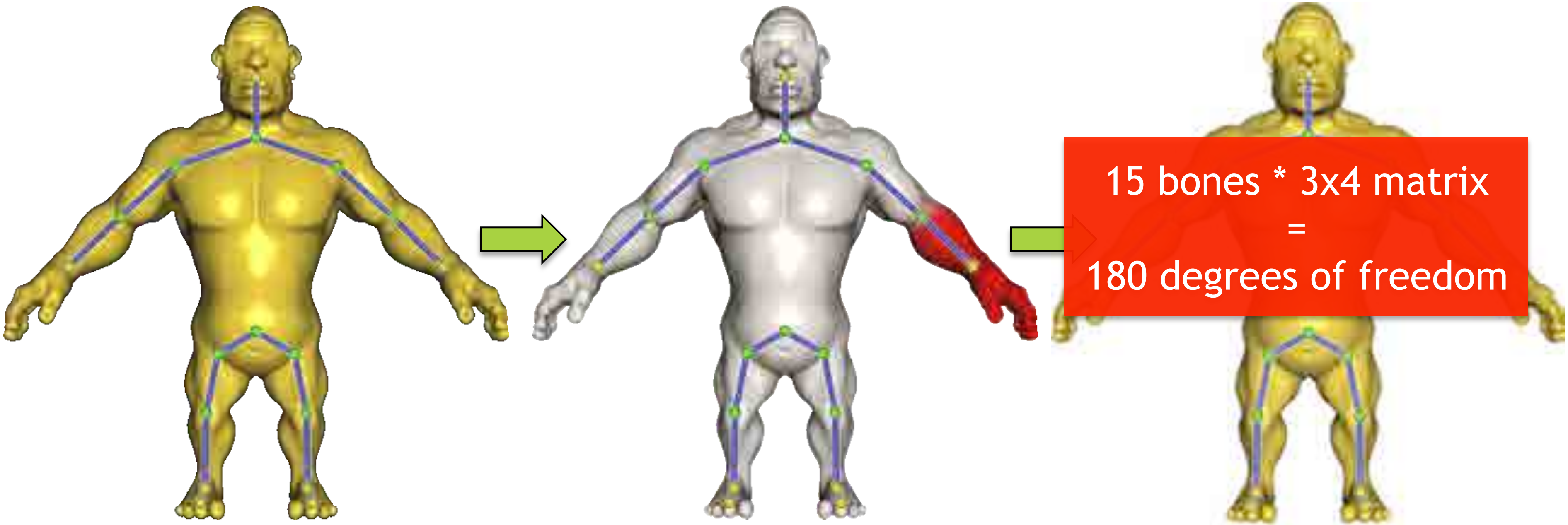
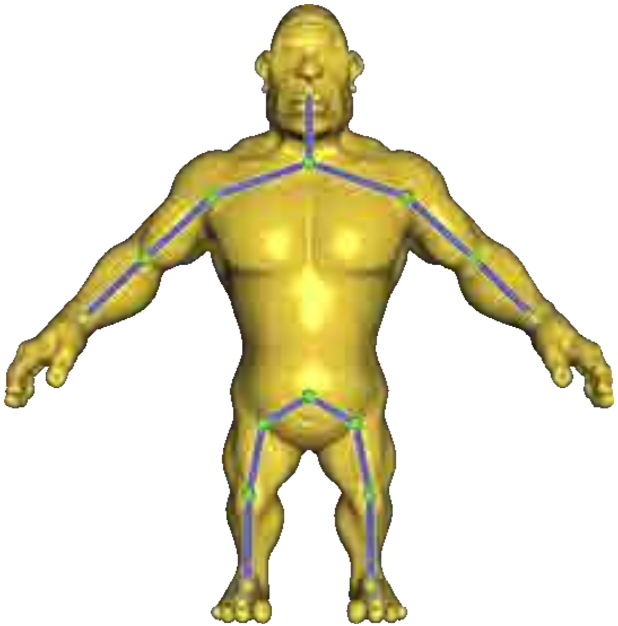


place skeleton in shape | compute/paint weights | deform bones

LBS generalizes to different handle types

$$\mathbf{v}'_i = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix}\mathbf{v}_i \\ 1\end{pmatrix}$$



skeletons

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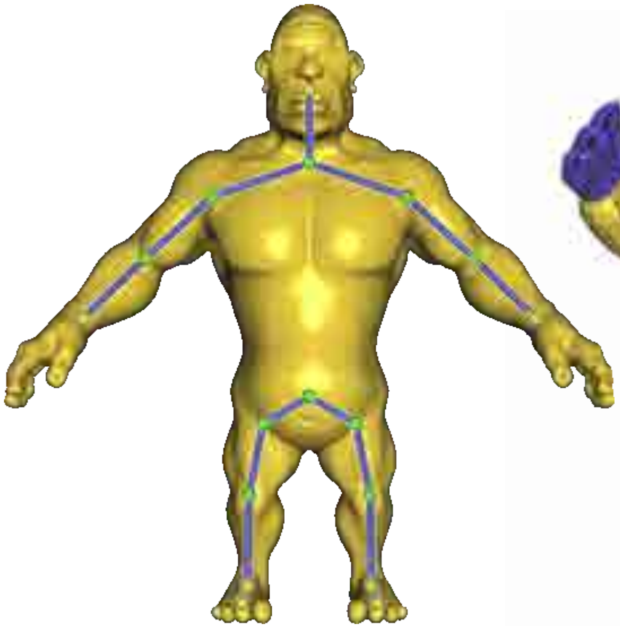
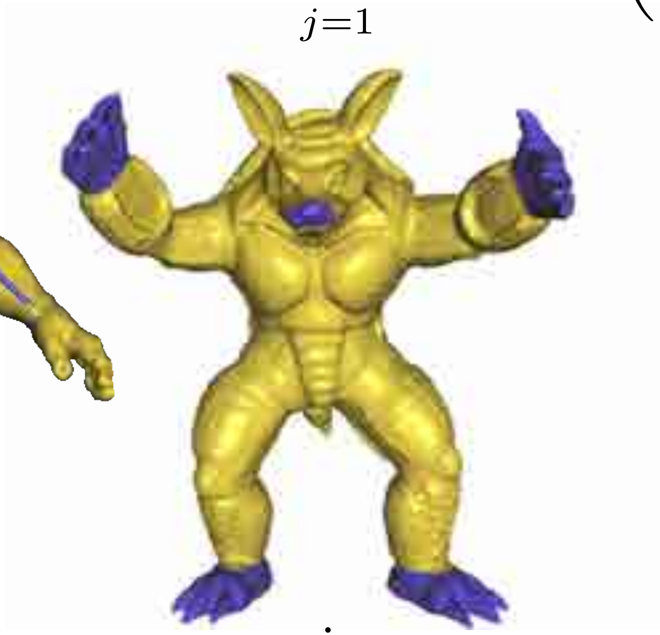


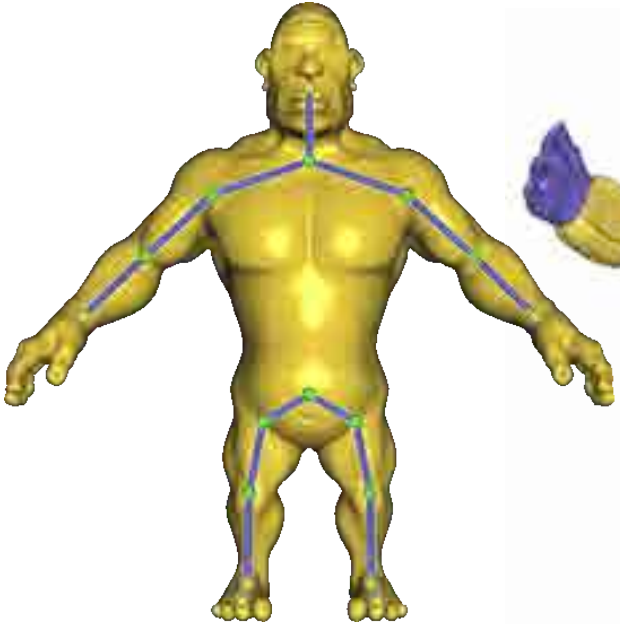
skeletons



regions

LBS generalizes to different handle types

$$\mathbf{v}_i' = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix}\mathbf{v}_i \\ 1\end{pmatrix}$$



skeletons

regions

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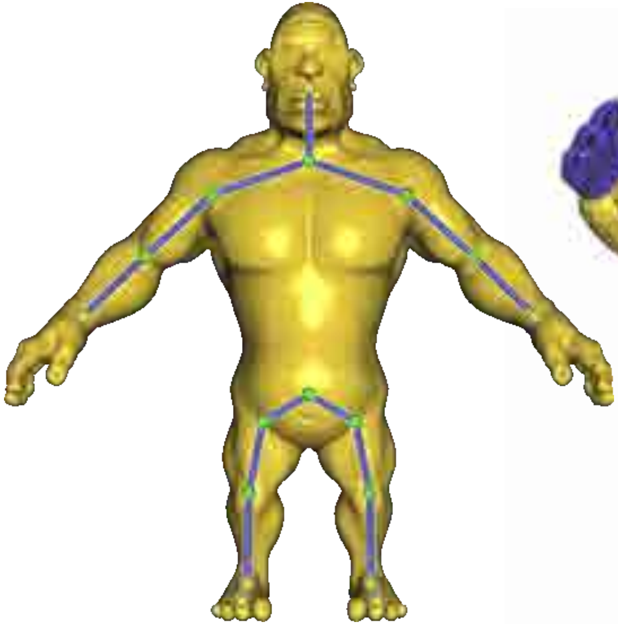


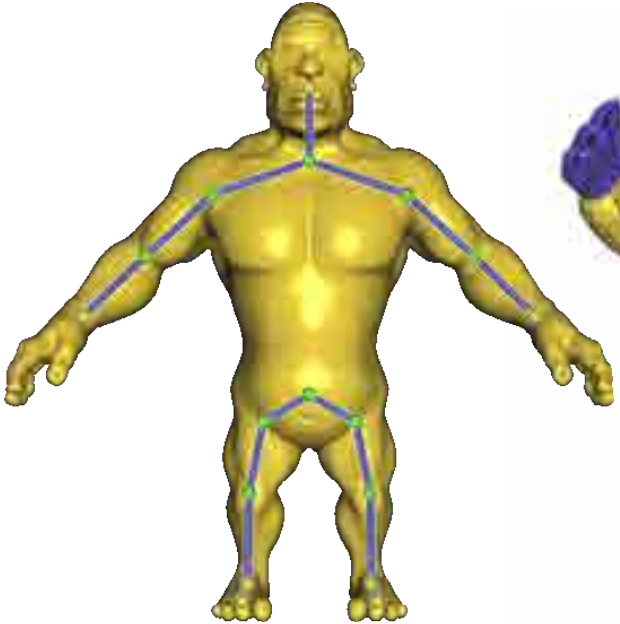
skeletons

regions

points

LBS generalizes to different handle types

$$\mathbf{v}'_i = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix} \mathbf{v}_i \\ 1 \end{pmatrix}$$


skeletons


regions


points

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User specifies subset of parameters, optimize to find remaining ones

Full optimization

$$\underset{\mathbf{V}'}{\arg\min}\ E(\mathbf{V}')$$

Mesh vertex positions

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User specifies subset of parameters, optimize to find remaining ones

Full optimization

$$\underset{\mathbf{V}'}{\arg\min}\; E(\mathbf{V}')$$

Reduced model

$$\mathbf{v}'_i = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix} \mathbf{v}_i \\ 1 \end{pmatrix}$$

Skinning degrees of freedom

User specifies subset of parameters, optimize to find remaining ones

Full optimization

$$\arg\min_{\mathbf{V}'} \; E(\mathbf{V}')$$

Reduced model

$$\mathbf{v}'_i = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix} \mathbf{v}_i \\ 1 \end{pmatrix}$$

Matrix form

$$\mathbf{V}' = \mathbf{M}\mathbf{T}$$

User specifies subset of parameters, optimize to find remaining ones

Full optimization
$$\underset{\mathbf{V}'}{\arg\min}\ E(\mathbf{V}')$$

Reduced model
$$\mathbf{v}'_i = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix}\mathbf{v}_i \\ 1\end{pmatrix}$$

Matrix form
$$\mathbf{V}' = \mathbf{M}\mathbf{T}$$

Reduced optimization
$$\underset{\mathbf{T}}{\arg\min}\ E(\mathbf{M}\mathbf{T})$$

Enforce user constraints as linear equalities

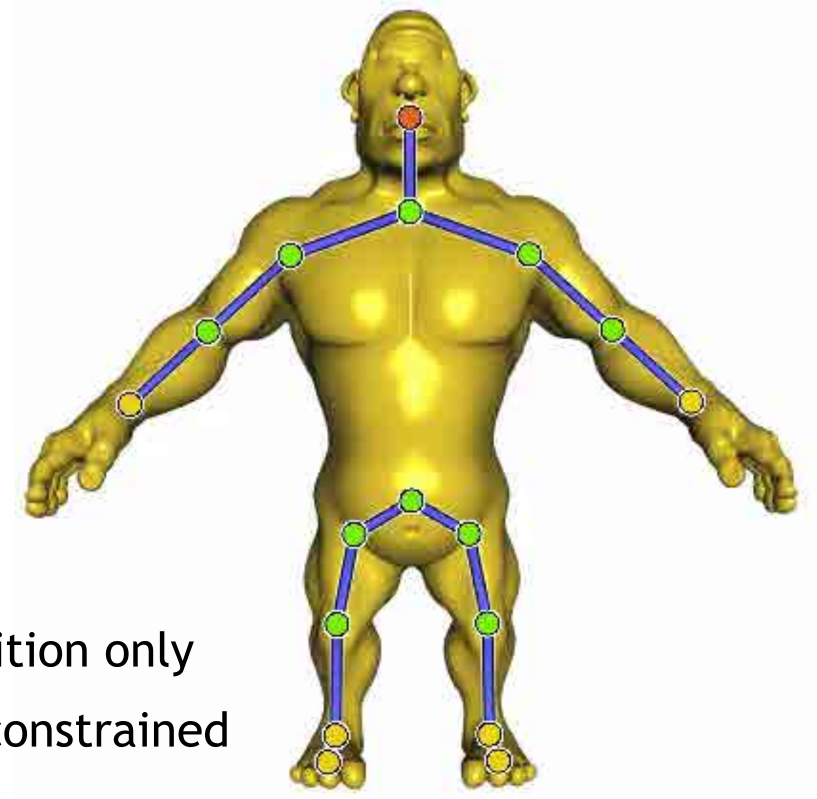
Reduced optimization

$$\arg\min_{\mathbf{T}} \; E(\mathbf{MT})$$

User constraints

$$\underbrace{\begin{bmatrix} \mathbf{I}_{\text{full}} \\ \mathbf{M}_{\text{pos}} \end{bmatrix}}_{\mathbf{M}_{\text{eq}}} \mathbf{T} = \underbrace{\begin{bmatrix} \mathbf{T}_{\text{full}} \\ \mathbf{P}_{\text{pos}} \end{bmatrix}}_{\mathbf{P}_{\text{eq}}}$$

- Full
- Position only
- Unconstrained



Enforce user constraints as linear equalities

Reduced optimization

$$\underset{\mathbf{T}}{\arg\min}\ E(\mathbf{MT})$$

User constraints

$$\underbrace{\begin{bmatrix} \mathbf{I}_{\text{full}} \\ \mathbf{M}_{\text{pos}} \end{bmatrix}}_{\mathbf{M}_{\text{eq}}} \mathbf{T} = \underbrace{\begin{bmatrix} \mathbf{T}_{\text{full}} \\ \mathbf{P}_{\text{pos}} \end{bmatrix}}_{\mathbf{P}_{\text{eq}}}$$

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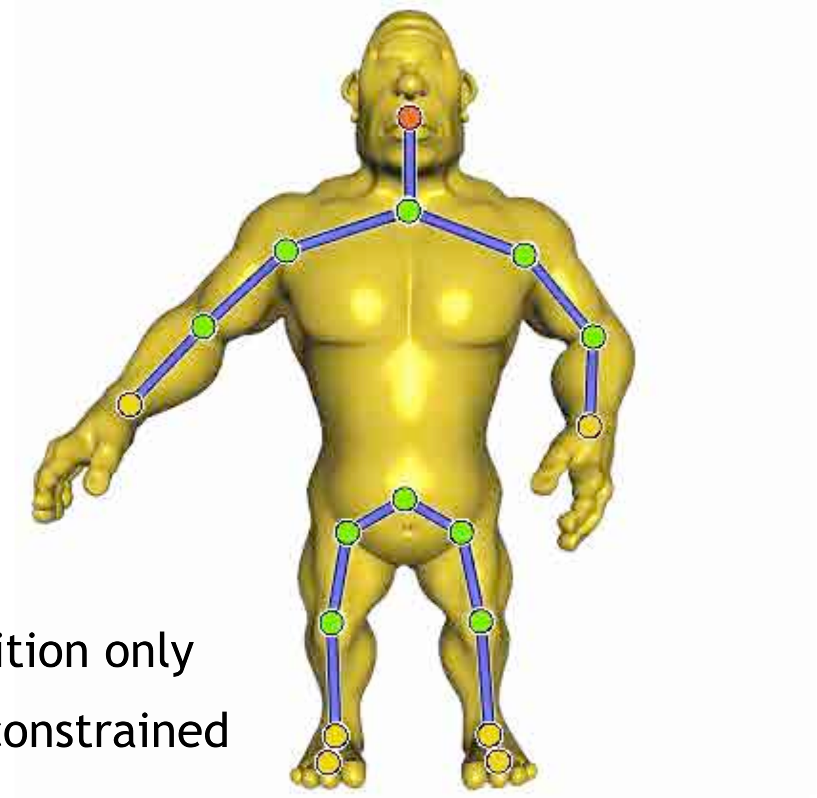

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Enforce user constraints as linear equalities

Reduced optimization

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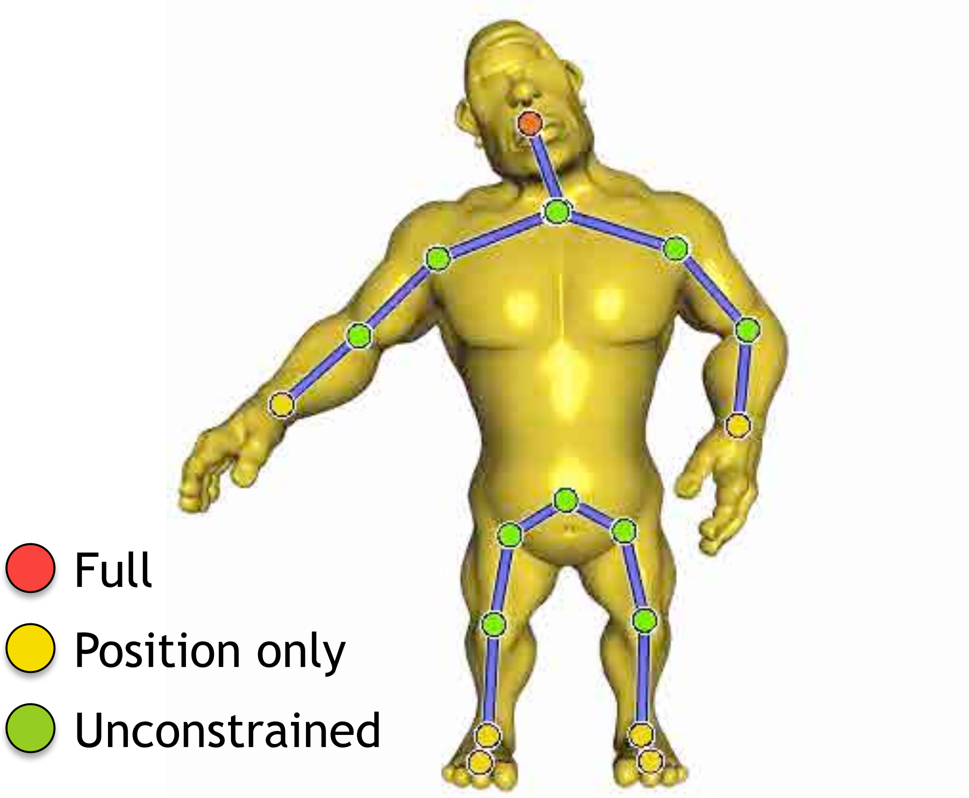

We reduce any *as-rigid-as-possible* energy

Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2} \sum_{k=1}^{r} \sum_{(i,j) \in \mathcal{E}_k} c_{ijk} \| (\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k (\mathbf{v}_i - \mathbf{v}_j) \|^2$$

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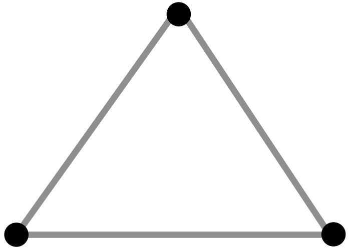
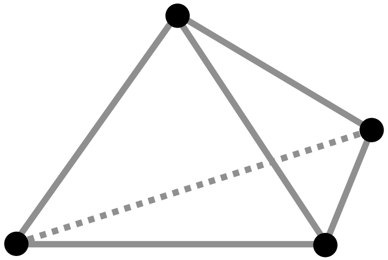
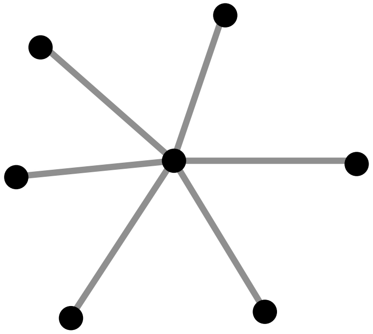


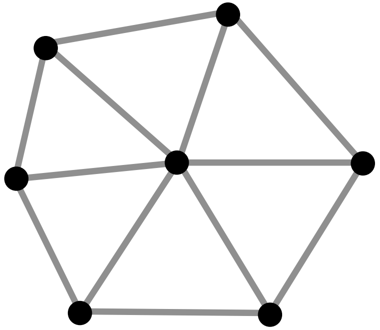
triangles
Liu et al. 08



tetrahedra
Chao et al. 10



"spokes"
Sorkine & Alexa 07



"spokes and rims"
Chao et al. 10

We reduce any *as-rigid-as-possible* energy

Full energies

$$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2} \sum_{k=1}^{r} \sum_{(i,j)\in\mathcal{E}_k} c_{ijk} \| (\mathbf{v}_i' - \mathbf{v}_j') - \mathbf{R}_k (\mathbf{v}_i - \mathbf{v}_j) \|^2$$

Local/Global optimization

Global step: Fix $\mathbf{R}$, minimize with respect to $\mathbf{V}'$

Local step: Fix $\mathbf{V}'$, minimize with respect to $\mathbf{R}$

We reduce any *as-rigid-as-possible* energy

Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2} \sum_{k=1}^{r} \sum_{(i,j)\in\mathcal{E}_k} c_{ijk} \|(\mathbf{v}_i' - \mathbf{v}_j') - \mathbf{R}_k(\mathbf{v}_i - \mathbf{v}_j)\|^2$$

Local/Global optimization

precompute ↓

Global step: large, sparse linear solve $\mathbf{V}' = \mathbf{A}^{-1}\mathbf{b}$

Local step: Fix $\mathbf{V}'$, minimize with respect to $\mathbf{R}$

We reduce any *as-rigid-as-possible* energy

Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2}\sum_{k=1}^{r}\sum_{(i,j)\in\mathcal{E}_k} c_{ijk}\|(\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k(\mathbf{v}_i - \mathbf{v}_j)\|^2$$

Local/Global optimization

Global step: large, sparse linear solve $\mathbf{V}' = \mathbf{A}^{-1}\mathbf{b}$

Local step: 3x3 SVD for each rotation in $\mathbf{R}$

We reduce any *as-rigid-as-possible* energy

Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2} \sum_{k=1}^{r} \sum_{(i,j) \in \mathcal{E}_k} c_{ijk} \| (\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k (\mathbf{v}_i - \mathbf{v}_j) \|^2$$

Local/Global optimization

precompute

Global step: small, dense linear solve $\mathbf{T} = \tilde{\mathbf{A}}^{-1} \tilde{\mathbf{b}}$

Local step: 3x3 SVD for each rotation in $\mathbf{R}$

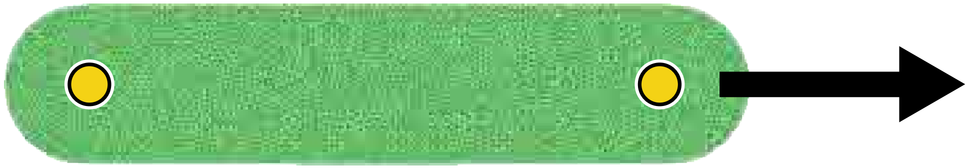
Substitute

$$\mathbf{V}' = \mathbf{M}\mathbf{T}$$

Similar to:

[Huang et al. 06]
[Der et al. 06]
[Au et al. 07]
[Hildebrandt et al. 12]

Direct reduction of elastic energies brings speed up and regularization…



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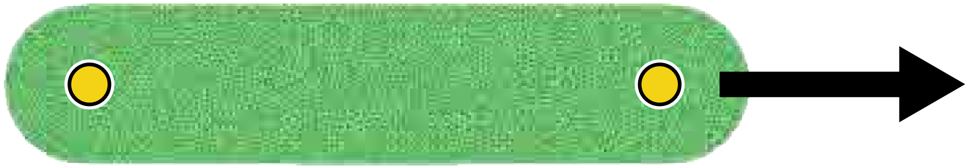
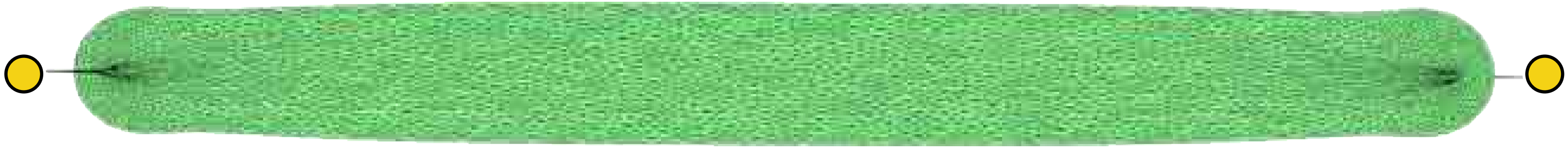


Full ARAP solution



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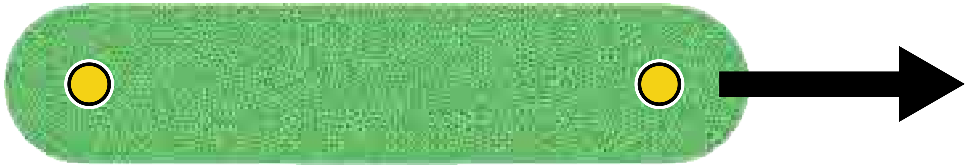

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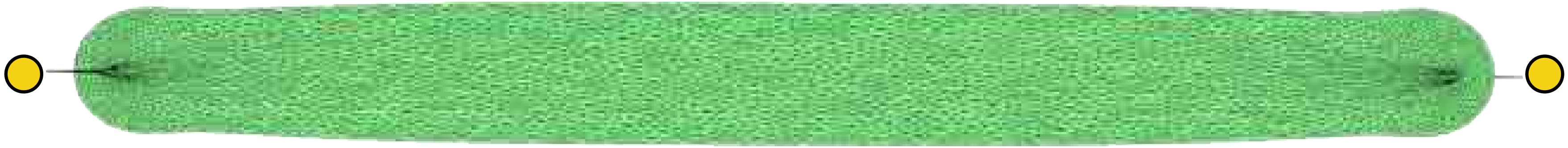
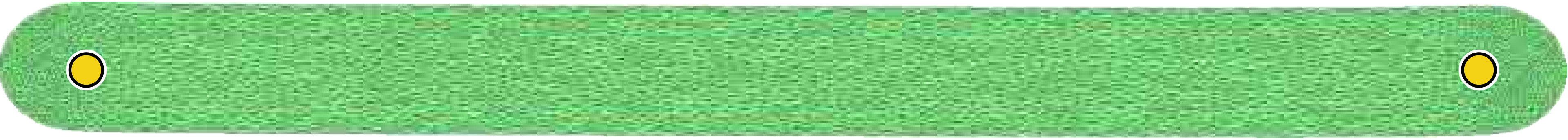


Our smooth subspace solution $\mathbf{V}' = \mathbf{MT}$



We reduce any *as-rigid-as-possible* energy

Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2} \sum_{k=1}^{r} \sum_{(i,j) \in \mathcal{E}_k} c_{ijk} \| (\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k (\mathbf{v}_i - \mathbf{v}_j) \|^2$$

Local/Global optimization

Global step: small, dense linear solve $\mathbf{T} = \tilde{\mathbf{A}}^{-1} \tilde{\mathbf{b}}$

Local step: 3x3 SVD for each rotation in $\mathbf{R}$

But #rotations ~ full mesh discretization

Substitute
$$\mathbf{V}' = \mathbf{MT}$$

We reduce any *as-rigid-as-possible* energy

Full energies $$E(\mathbf{V}',\mathbf{R}) = \frac{1}{2}\sum_{k=1}^{r}\sum_{(i,j)\in\mathcal{E}_k} c_{ijk}\|(\mathbf{v}'_i-\mathbf{v}'_j)-\mathbf{R}_k(\mathbf{v}_i-\mathbf{v}_j)\|^2$$

Local/Global optimization

Global step: small, dense linear solve $\mathbf{T} = \tilde{\mathbf{A}}^{-1}\tilde{\mathbf{b}}$

Local step: 3x3 SVD for each rotation in $\mathbf{R}$

Substitute

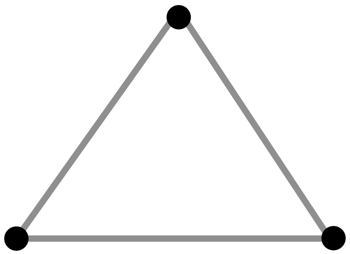
$$\mathbf{V}' = \mathbf{M}\mathbf{T}$$

Cluster

$$\mathcal{E}_k$$

Rotation evaluations may be reduced by clustering in *weight space*

Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2}\sum_{k=1}^{r}\sum_{(i,j)\in\mathcal{E}_k} c_{ijk}\|(\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k(\mathbf{v}_i - \mathbf{v}_j)\|^2$$



triangles
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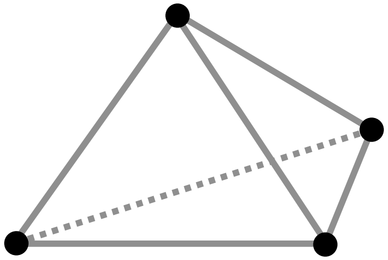

tetrahedra
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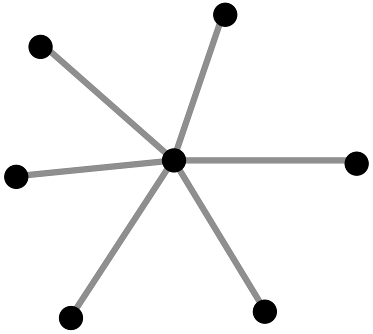
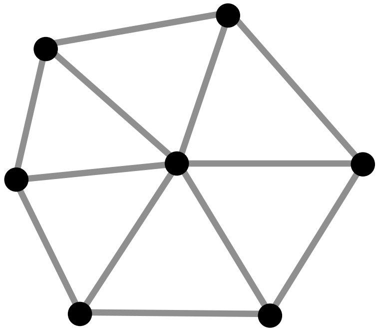


"spokes"
Sorkine & Alexa 07



"spokes and rims"
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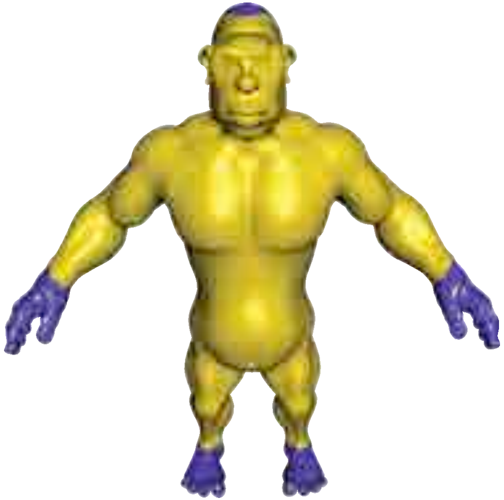
Rotation evaluations may be reduced by k-means clustering in *weight space*

Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2} \sum_{k=1}^{r} \sum_{(i,j) \in \mathcal{E}_k} c_{ijk} \|(\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k(\mathbf{v}_i - \mathbf{v}_j)\|^2$$

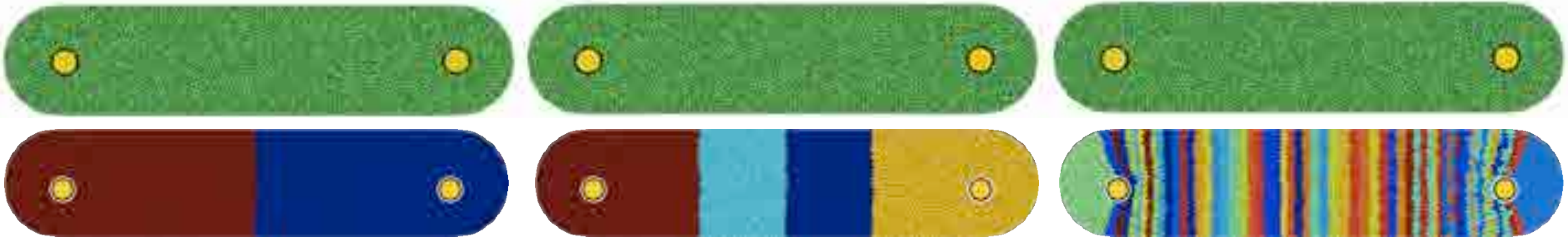


weight space

$$\mathbf{x}_j = \begin{bmatrix} w_1(\mathbf{v}_j) \\ w_2(\mathbf{v}_j) \\ \vdots \\ w_m(\mathbf{v}_j) \end{bmatrix}$$

Rotation evaluations may be reduced by clustering in *weight space*

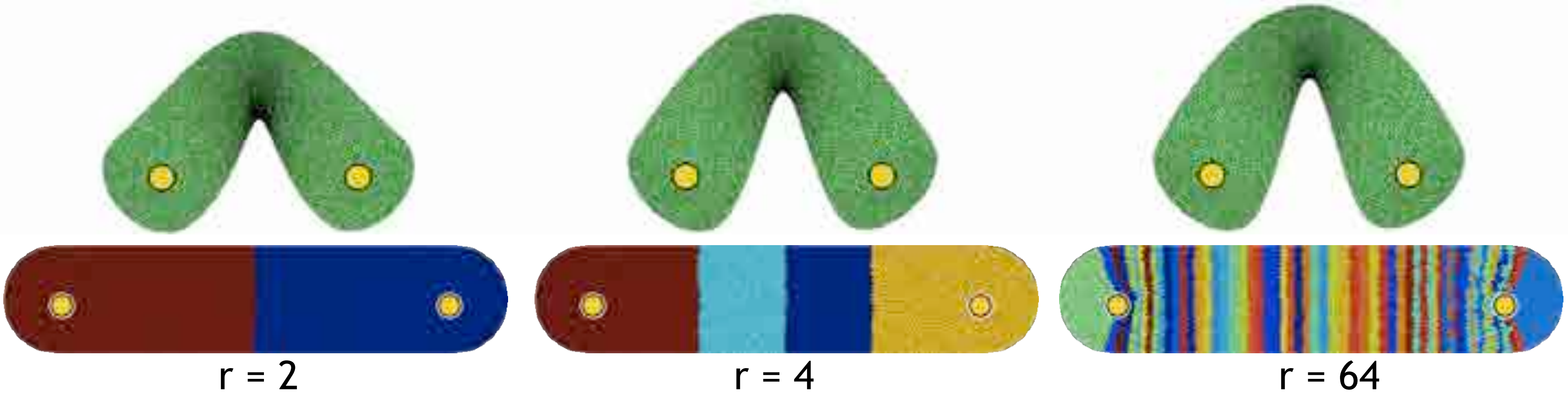
Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2}\sum_{k=1}^{r}\sum_{(i,j)\in\mathcal{E}_k} c_{ijk}\|(\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k(\mathbf{v}_i - \mathbf{v}_j)\|^2$$



r = 2 r = 4 r = 64

Rotation evaluations may be reduced by clustering in *weight space*

Full energies $$E(\mathbf{V}',\mathbf{R}) = \frac{1}{2}\sum_{k=1}^{r}\sum_{(i,j)\in\mathcal{E}_k} c_{ijk}\|(\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k(\mathbf{v}_i - \mathbf{v}_j)\|^2$$



We reduce any *as-rigid-as-possible* energy

Full energies $$E(\mathbf{V}', \mathbf{R}) = \frac{1}{2} \sum_{k=1}^{r} \sum_{(i,j)\in\mathcal{E}_k} c_{ijk} \|(\mathbf{v}'_i - \mathbf{v}'_j) - \mathbf{R}_k(\mathbf{v}_i - \mathbf{v}_j)\|^2$$

Local/Global optimization

Global step: small, dense linear solve $\mathbf{T} = \tilde{\mathbf{A}}^{-1}\tilde{\mathbf{b}}$

Local step: 3x3 SVD for each rotation in $\mathbf{R}$

#rotations ~ #T,
independent of full mesh resolution

Substitute

$$\mathbf{V}' = \mathbf{M}\mathbf{T}$$

Cluster

$$\mathcal{E}_k$$

Real-time automatic degrees of freedom

Real-time automatic degrees of freedom

With more and more user constraints we fall back to standard skinning

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With more and more user constraints we fall back to standard skinning

Extra weights would expand subspace…

$$\mathbf{v}_i' = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix} \mathbf{v}_i \\ 1 \end{pmatrix}$$

$$\mathbf{V}' = \mathbf{M}\mathbf{T}$$

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$$\mathbf{v}_i' = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix}\mathbf{v}_i\\1\end{pmatrix} + \sum_{k=1}^{m_{\text{extra}}} w_k(\mathbf{v}_i)\mathbf{T}_k \begin{pmatrix}\mathbf{v}_i\\1\end{pmatrix}$$

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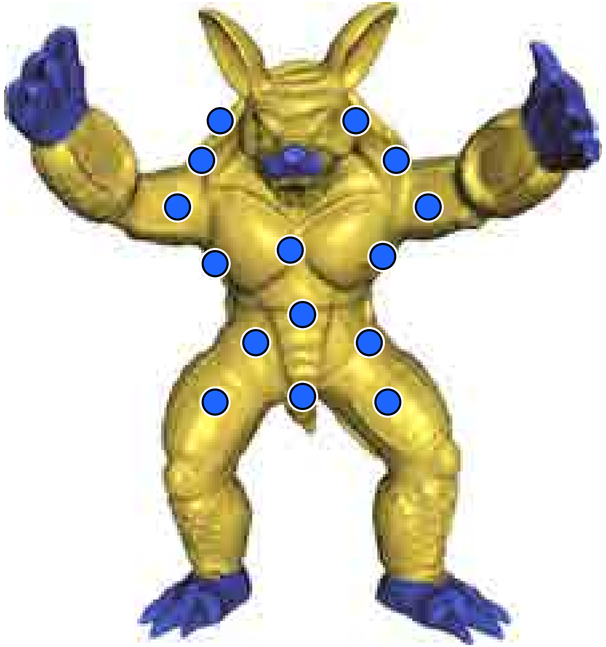
$$\mathbf{v}'_i = \sum_{j=1}^{m} w_j(\mathbf{v}_i)\mathbf{T}_j \begin{pmatrix}\mathbf{v}_i \\ 1\end{pmatrix} + \sum_{k=1}^{m_{\text{extra}}} w_k(\mathbf{v}_i)\mathbf{T}_k \begin{pmatrix}\mathbf{v}_i \\ 1\end{pmatrix}$$

$$\mathbf{V}' = \mathbf{M}\mathbf{T} + \mathbf{M}_{\text{extra}}\mathbf{T}_{\text{extra}}$$

Overlapping b-spline “bumps” in weight space



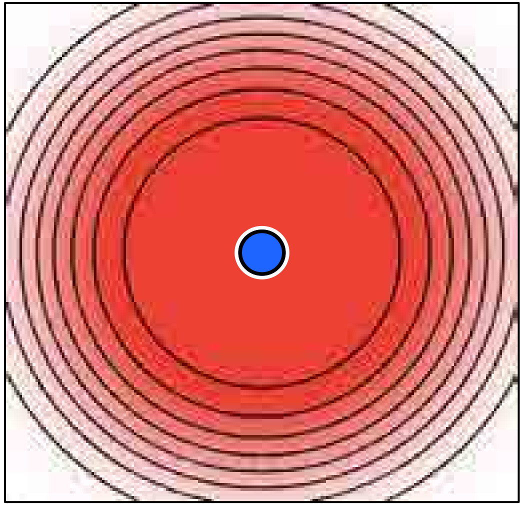
farthest point sampling

weight space

$$\mathbf{x}_j = \begin{bmatrix} w_1(\mathbf{v}_j) \\ w_2(\mathbf{v}_j) \\ \vdots \\ w_m(\mathbf{v}_j) \end{bmatrix}$$

Overlapping b-spline "bumps" in weight space

in weight space



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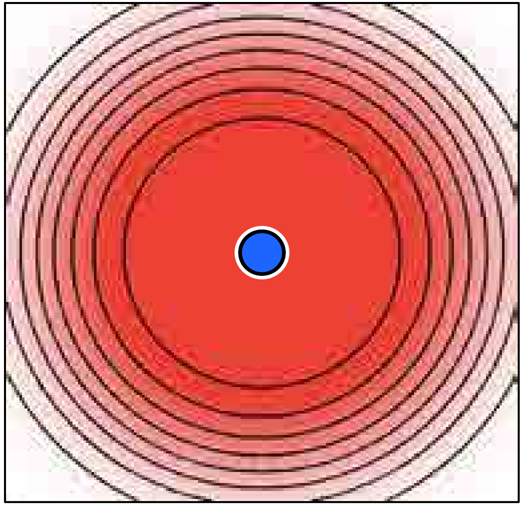
b-spline basis parameterized by distance in weight space

Overlapping b-spline “bumps” in weight space

in weight space



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b-spline basis parameterized by distance in weight space

Extra weights expand deformation subspace


no extra weights


15 extra weights

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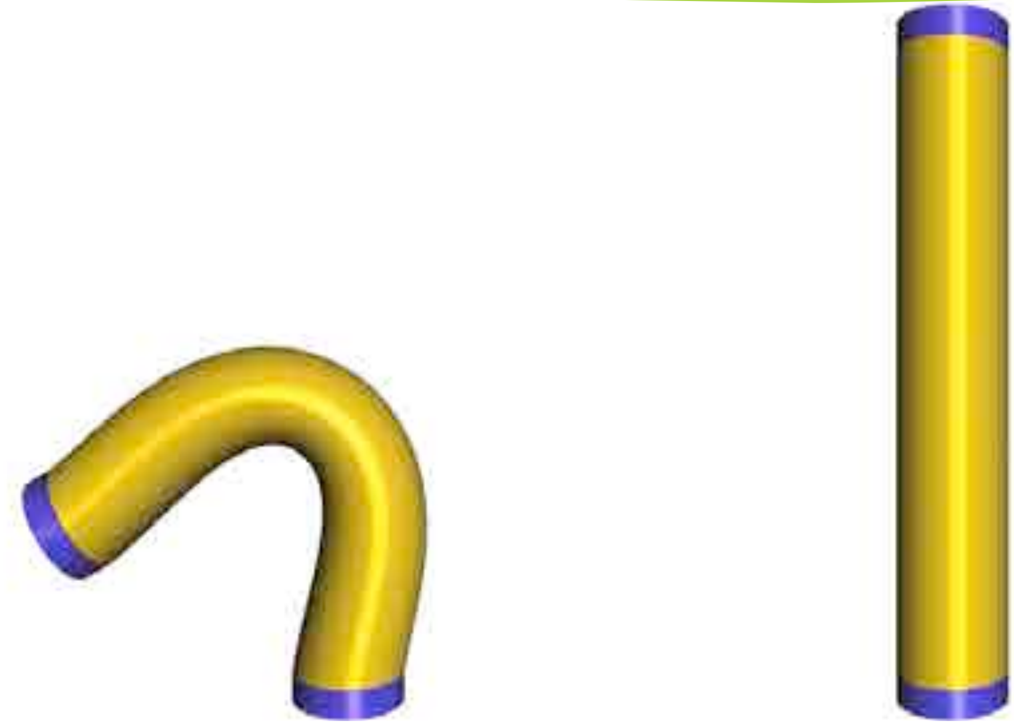
15 extra weights

Subspace now rich enough for fast variational modeling



Full non-linear optimization
[Botsch et al. 2006]

Our reduced method

Subspace now rich enough for fast variational modeling

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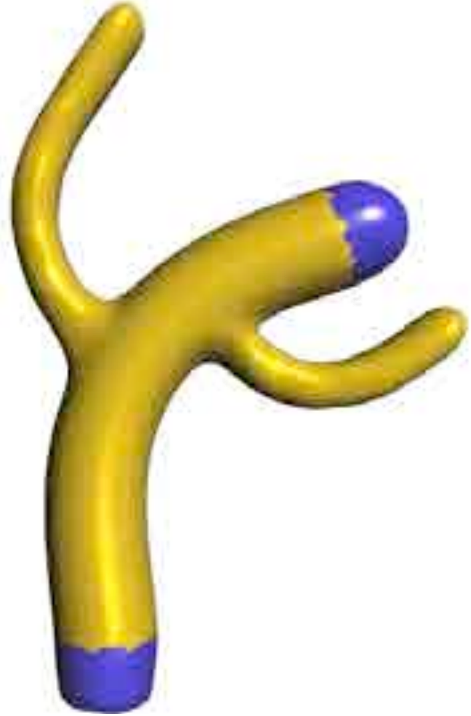

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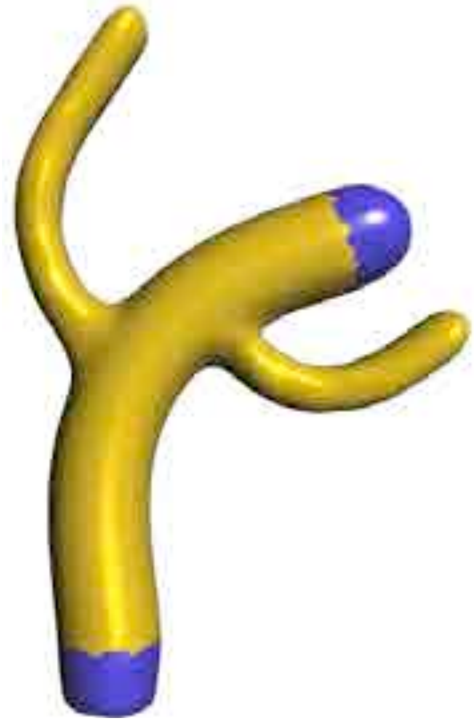

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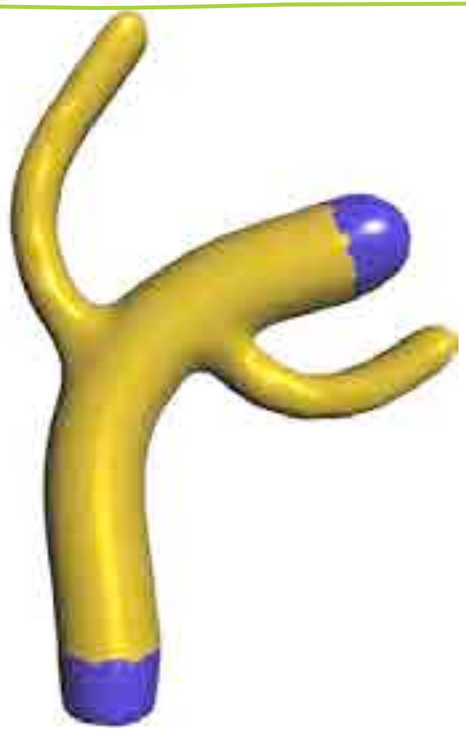

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Our reduced method

Final algorithm is simple and FAST

Precomputation per shape+rig	*For a 50K triangle mesh:*
- Compute any additional weights	*12 seconds*
- Construct, prefactor system matrices	*2.7 seconds*

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Precomputation per shape+rig *For a 50K triangle mesh:*

- Compute any additional weights *12 seconds*
- Construct, prefactor system matrices *2.7 seconds*

Precomputation when switching constraint type

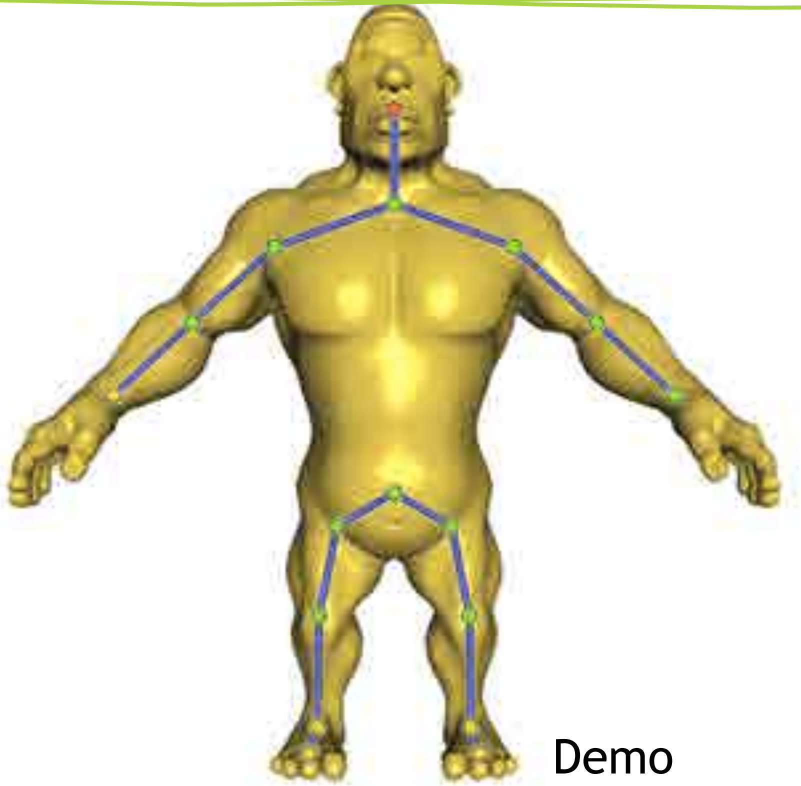
- *Re*-factor global step system *6 milliseconds*

~30 iterations *22 microseconds*

global: #weights by #weights linear solve

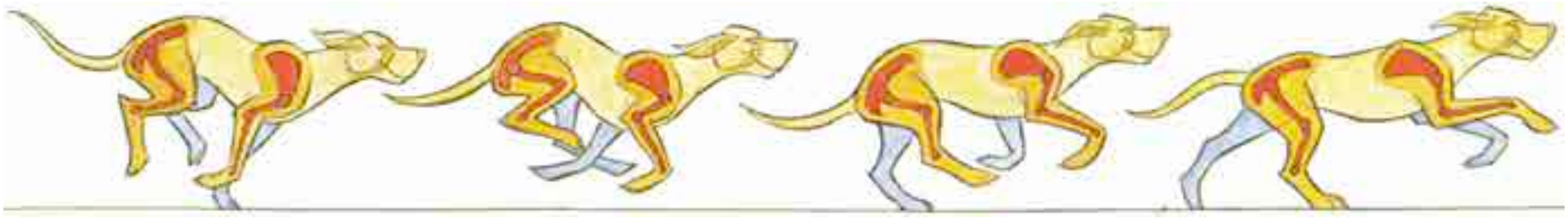
local: #rotations SVDs [McAdams et al. 2011]

Lightning FAST automatic skinning transformations



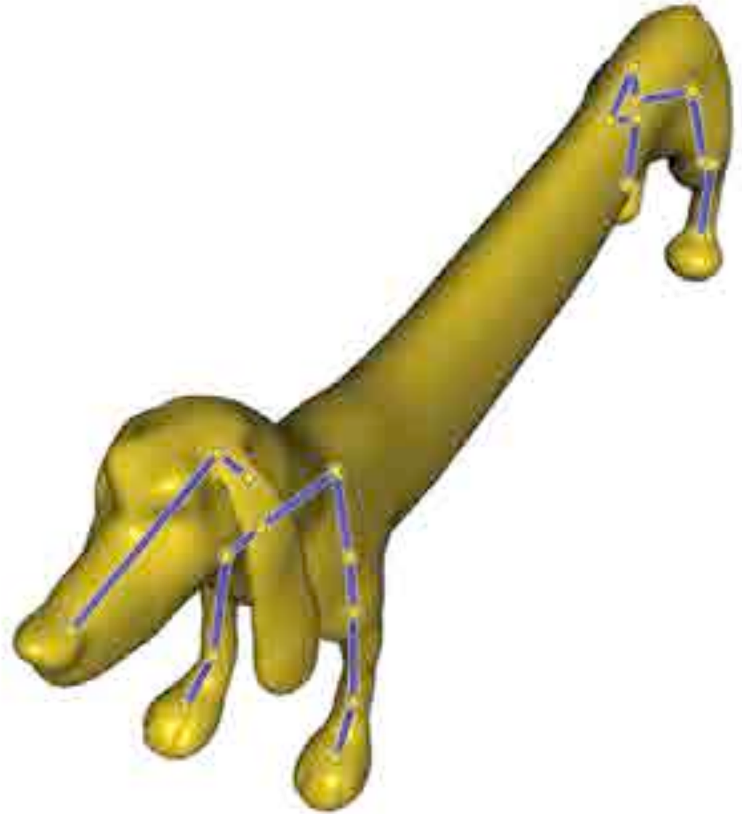
Demo

Extra weights and disjoint skeletons make flexible control easy



From *Cartoon Animation by Preston Blair*

Extra weights and disjoint skeletons make flexible control easy



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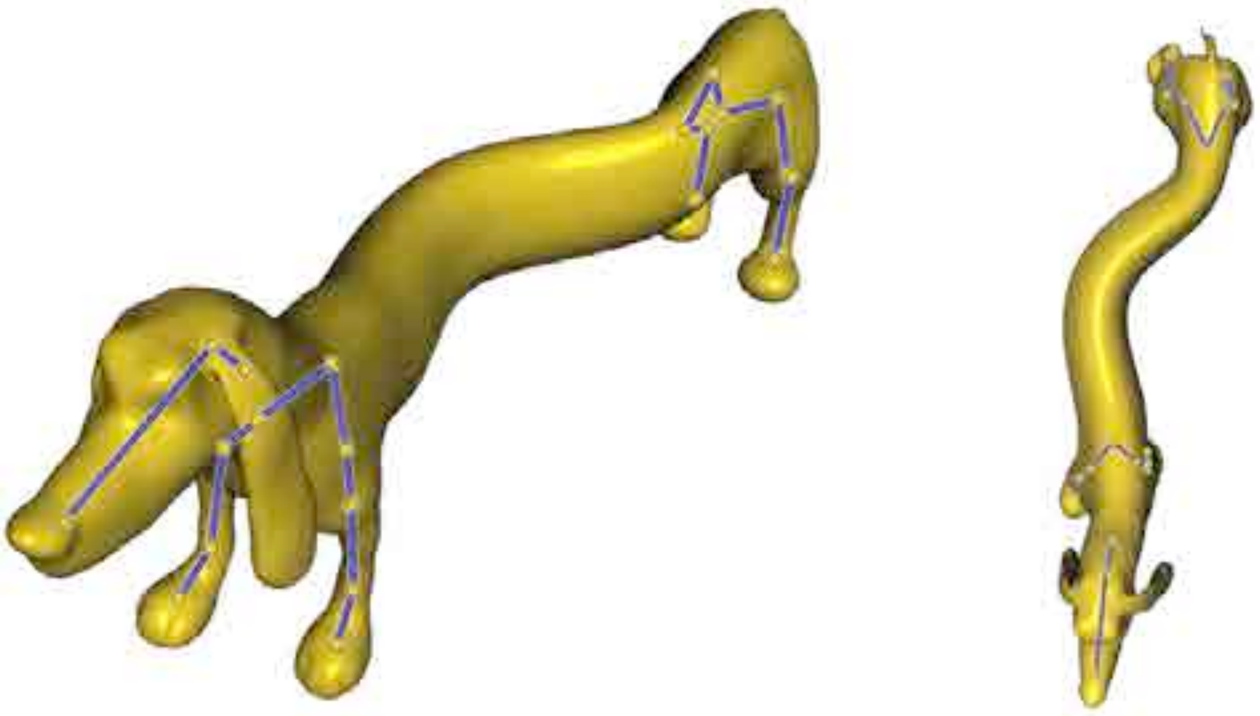

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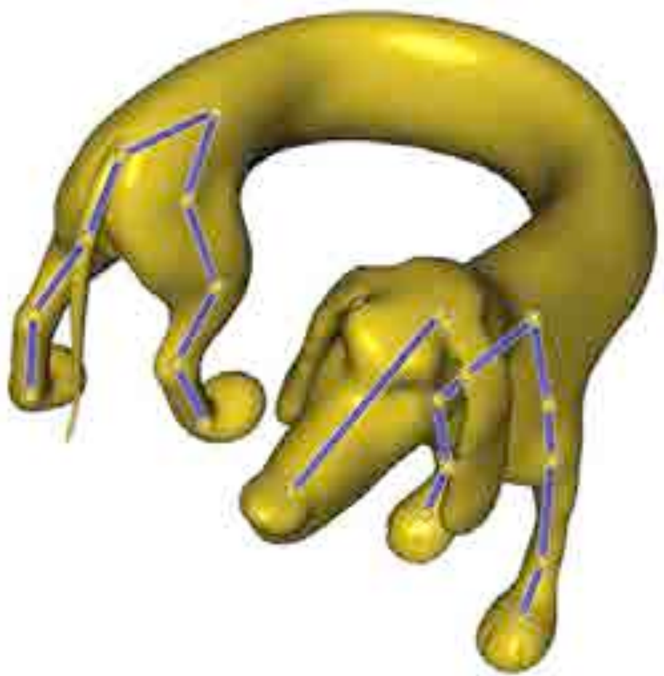

Our reduction preserves nature of different energies, at no extra cost

Surface ARAP

$$\mathbf{V}'_{\text{surf}} = \mathbf{M}_{\text{surf}} T$$

Volumetric ARAP

$$\mathbf{V}'_{\text{vol}} = \mathbf{M}_{\text{vol}} T$$

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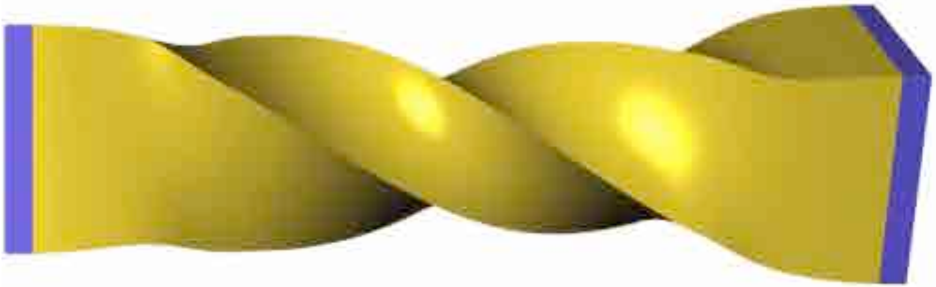
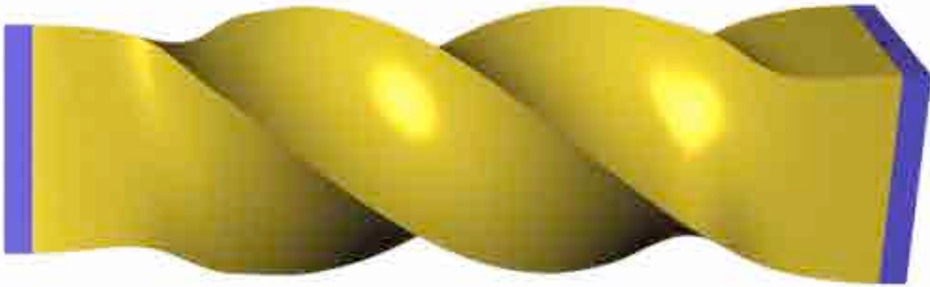
Surface ARAP



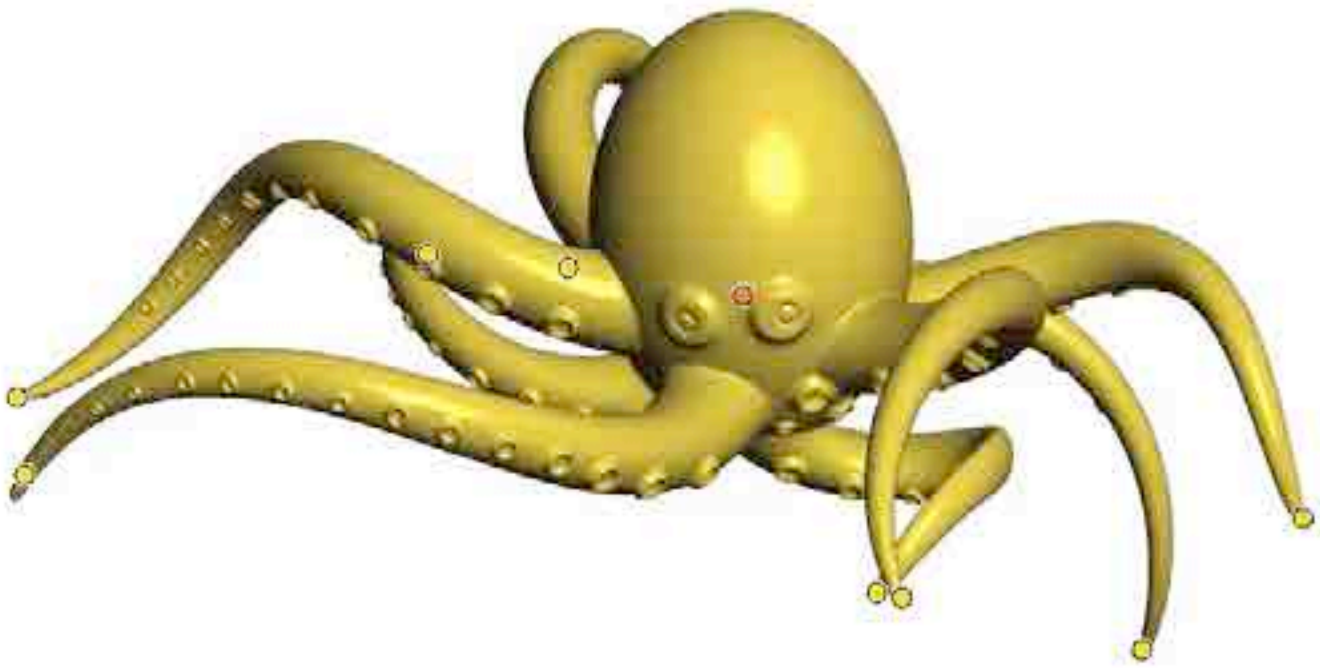
$$\mathbf{V}'_{\text{surf}} = \mathbf{M}_{\text{surf}} T$$

Volumetric ARAP

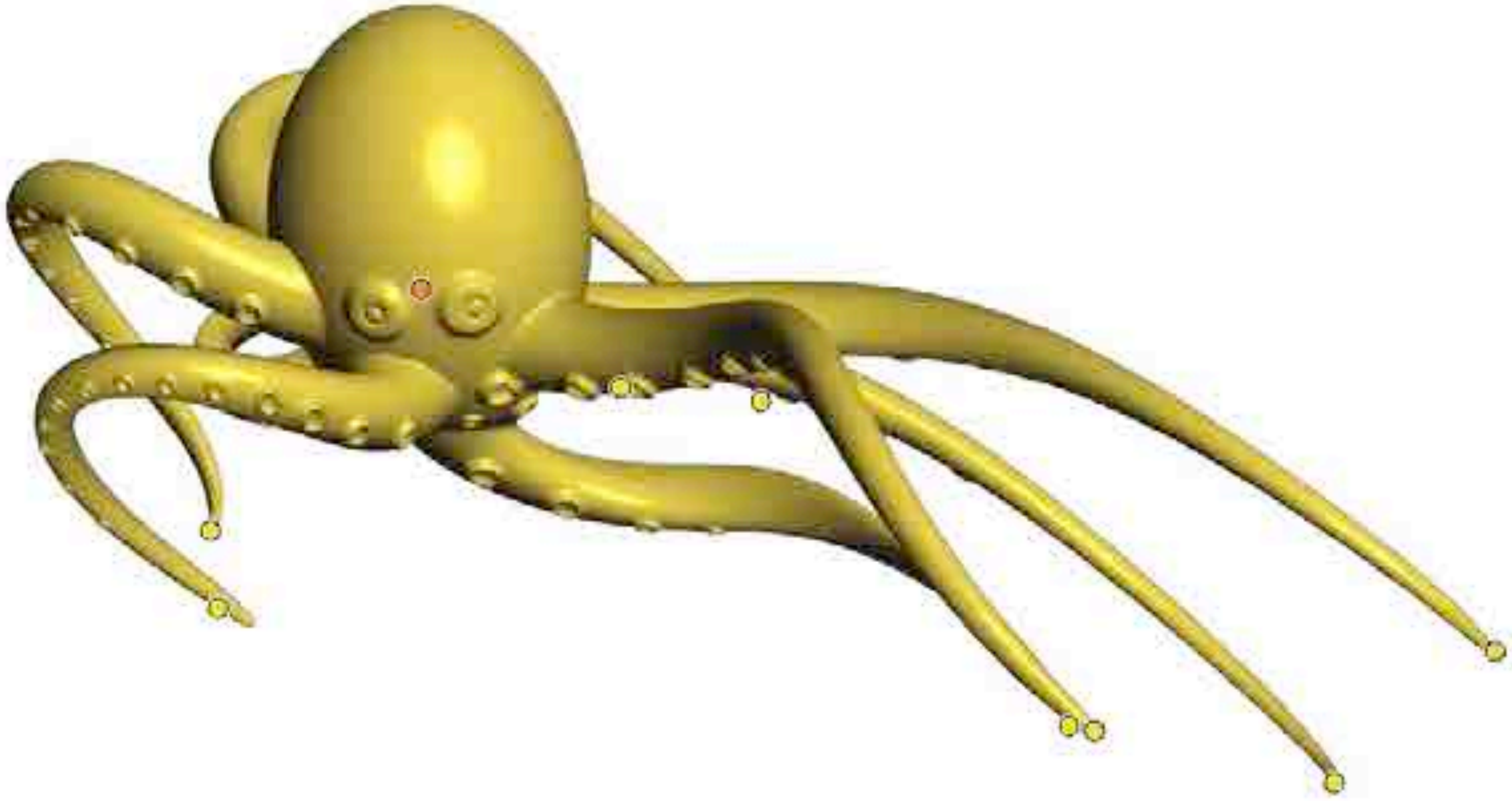


$$\mathbf{V}'_{\text{vol}} = \mathbf{M}_{\text{vol}} T$$

Simple drag-only interface for point handles



Simple drag-only interface for point handles



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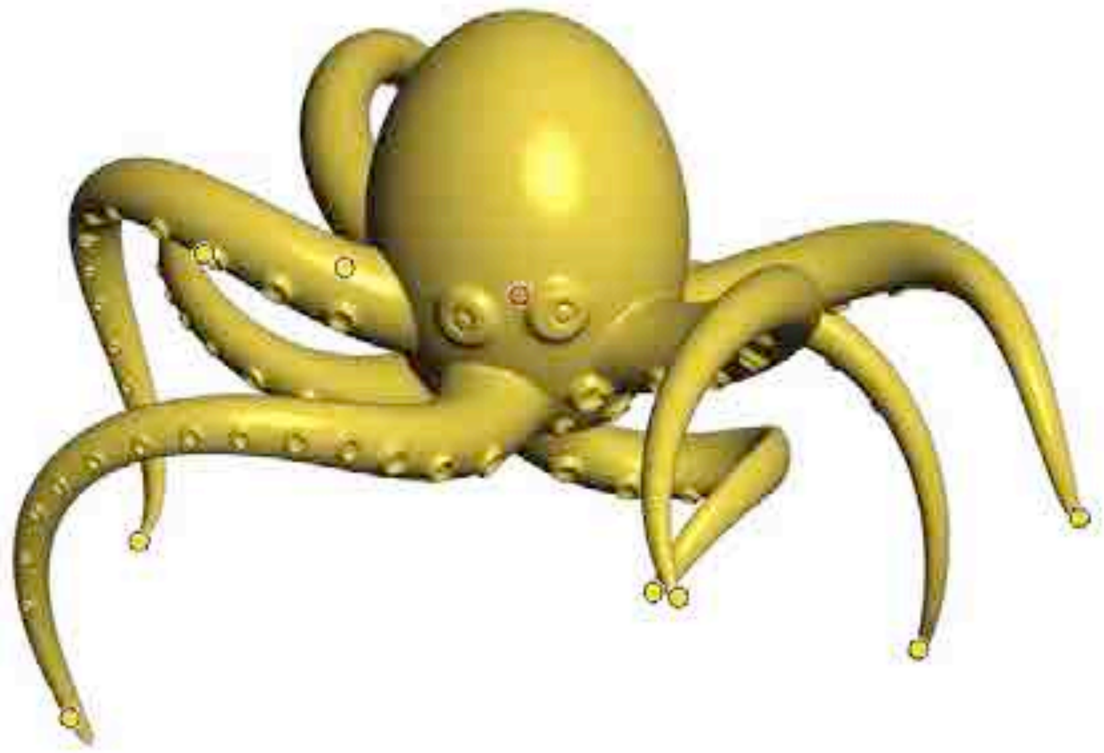

Skinning rig enables FAST deformation

- Substitute $\mathbf{V}' = \mathbf{M}\mathbf{T}$ to reduce DOFs

Skinning rig enables FAST deformation

- Substitute $\mathbf{V}' = \mathbf{MT}$ to reduce DOFs
- Cluster rotations to reduce energy eval.

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Skinning rig enables FAST deformation

- Substitute $\mathbf{V}' = \mathbf{M}\mathbf{T}$ to reduce DOFs
- Cluster rotations to reduce energy eval.
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Each innovation takes advantage of input skinning rig

Future work and discussion

- Alternative additional weights: sparsity?
- Joint limits, balance, etc.

Acknowledgements

We are grateful to Peter Schröder, Emily Whiting, and Maurizio Nitti.

We thank Eftychios Sifakis for his open source fast 3×3 SVD code.

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Fast Automatic Skinning Transformations

http://igl.ethz.ch/projects/fast

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