# Robust Inside-Outside Segmentation using Generalized Winding Numbers


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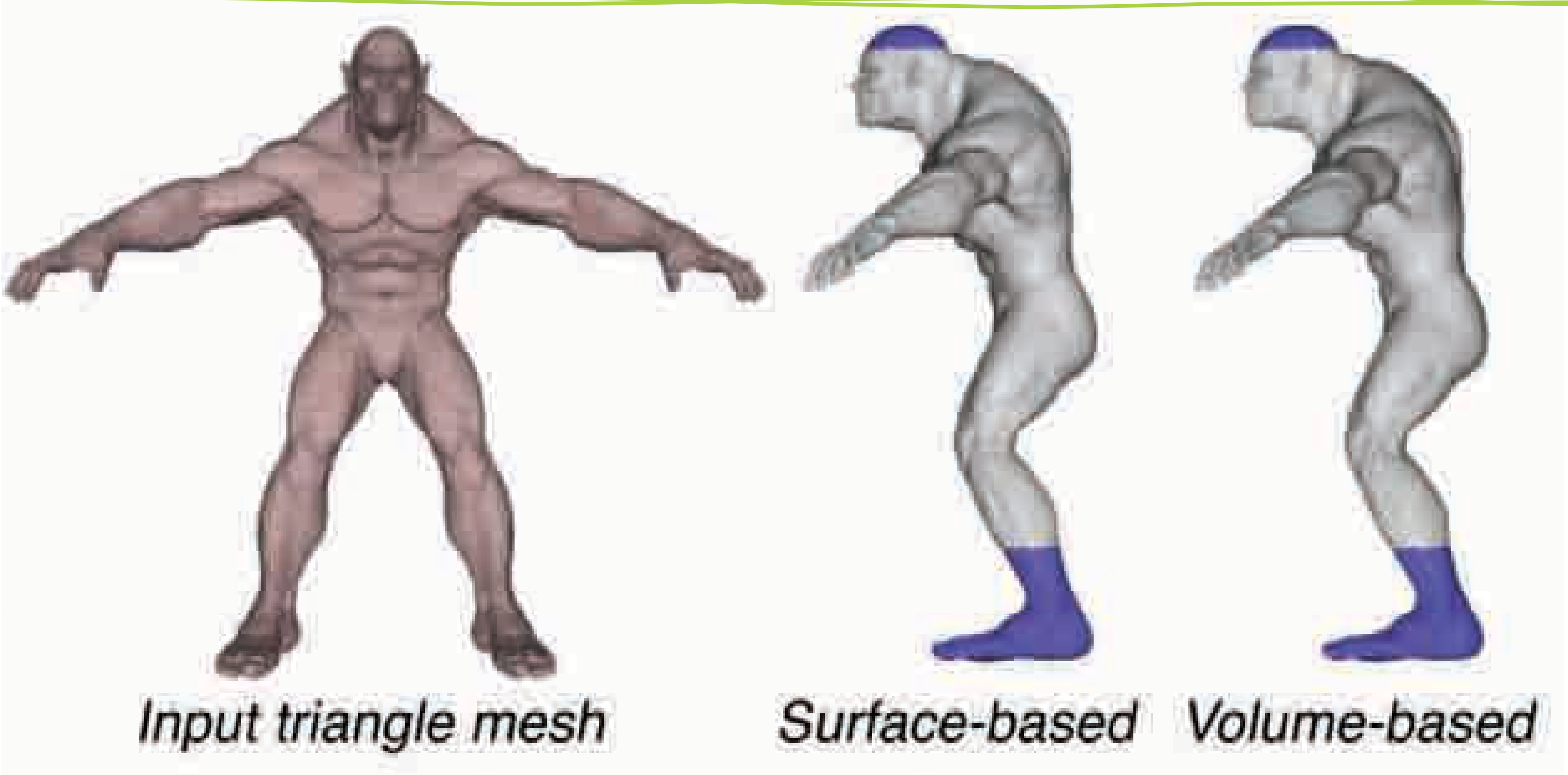
October 9, 2013

# Processing solid shapes requires volumetric representation



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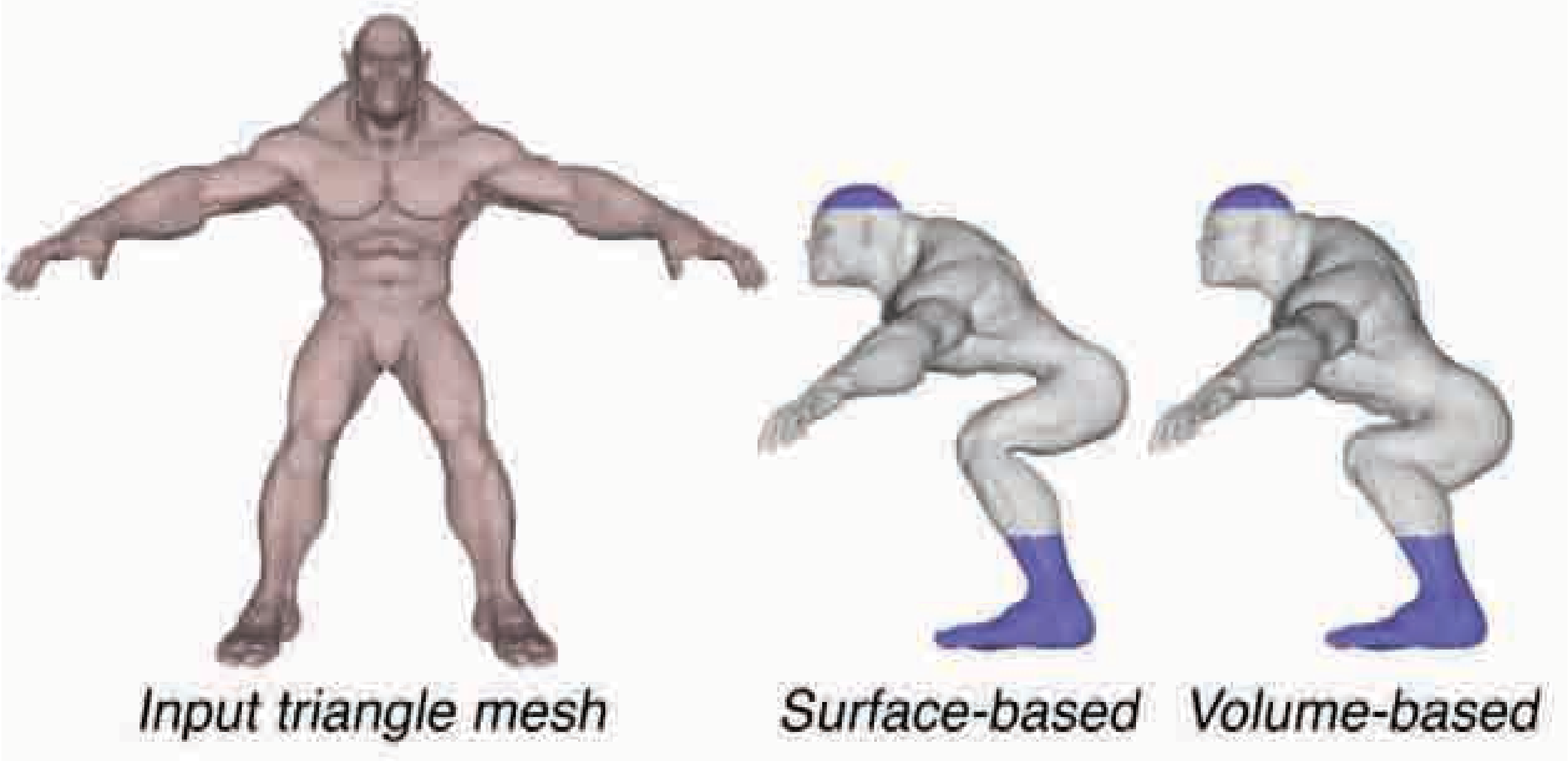
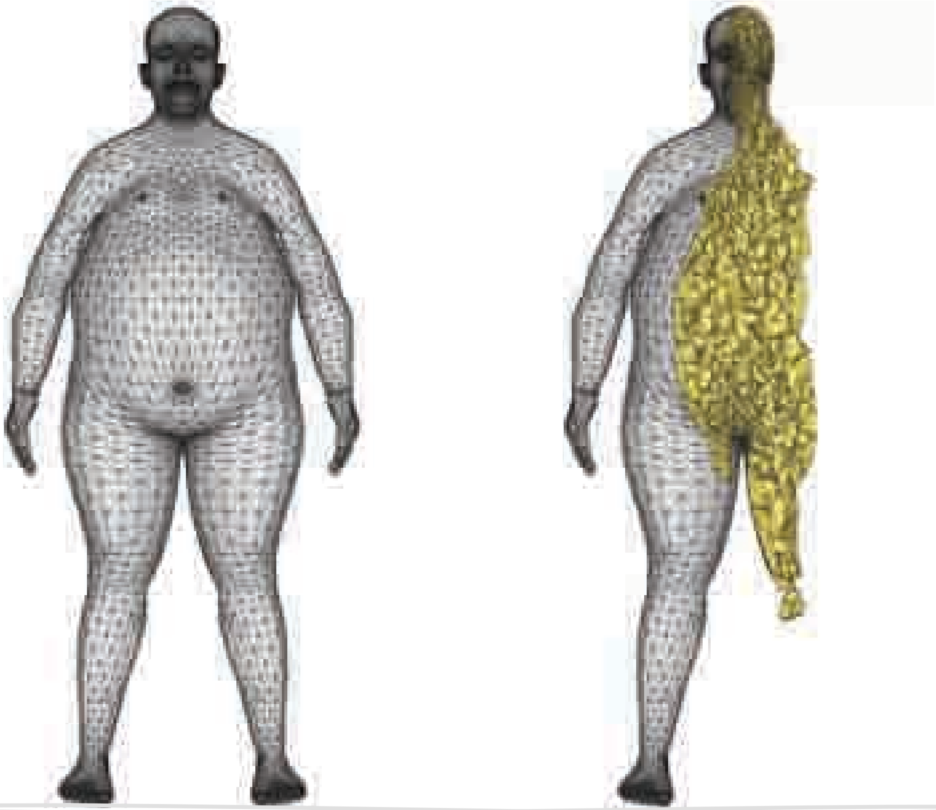


# Explicit representations are essential

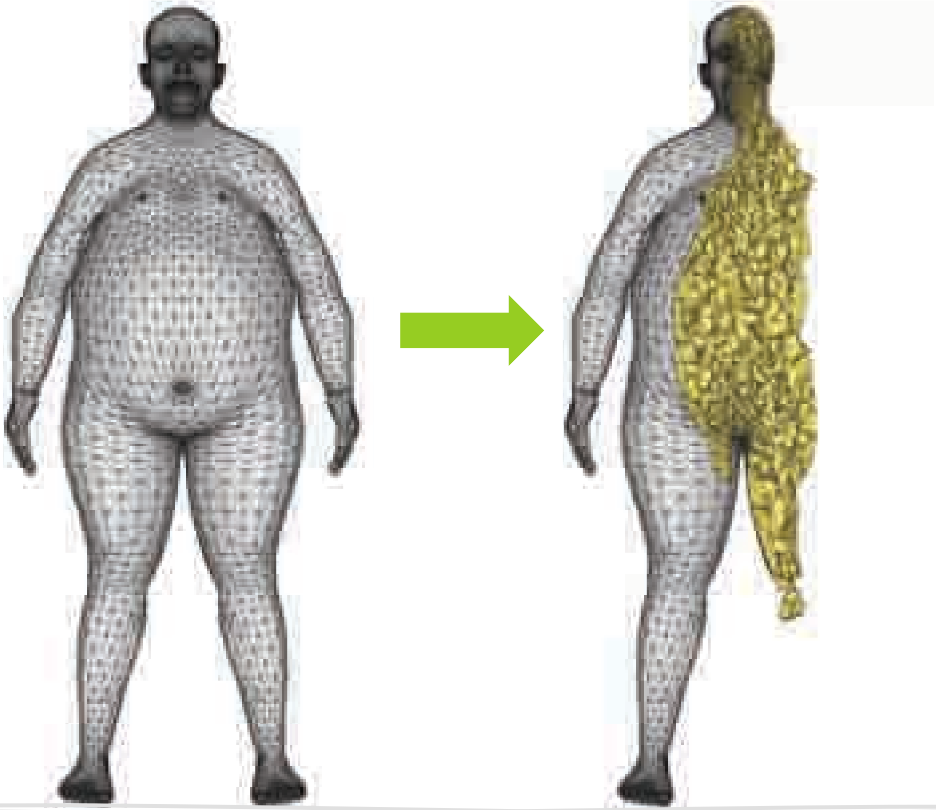
triangle mesh



tetrahedral mesh

# Explicit representations are essential

triangle mesh
*watertight*



tetrahedral mesh
*made by TetGen*

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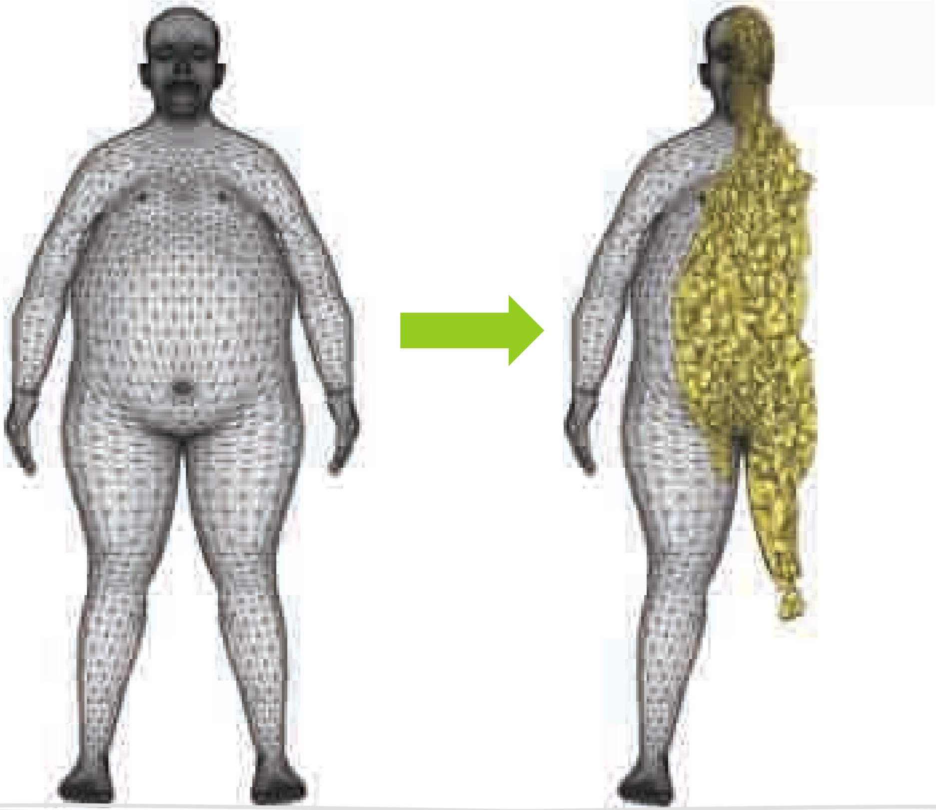
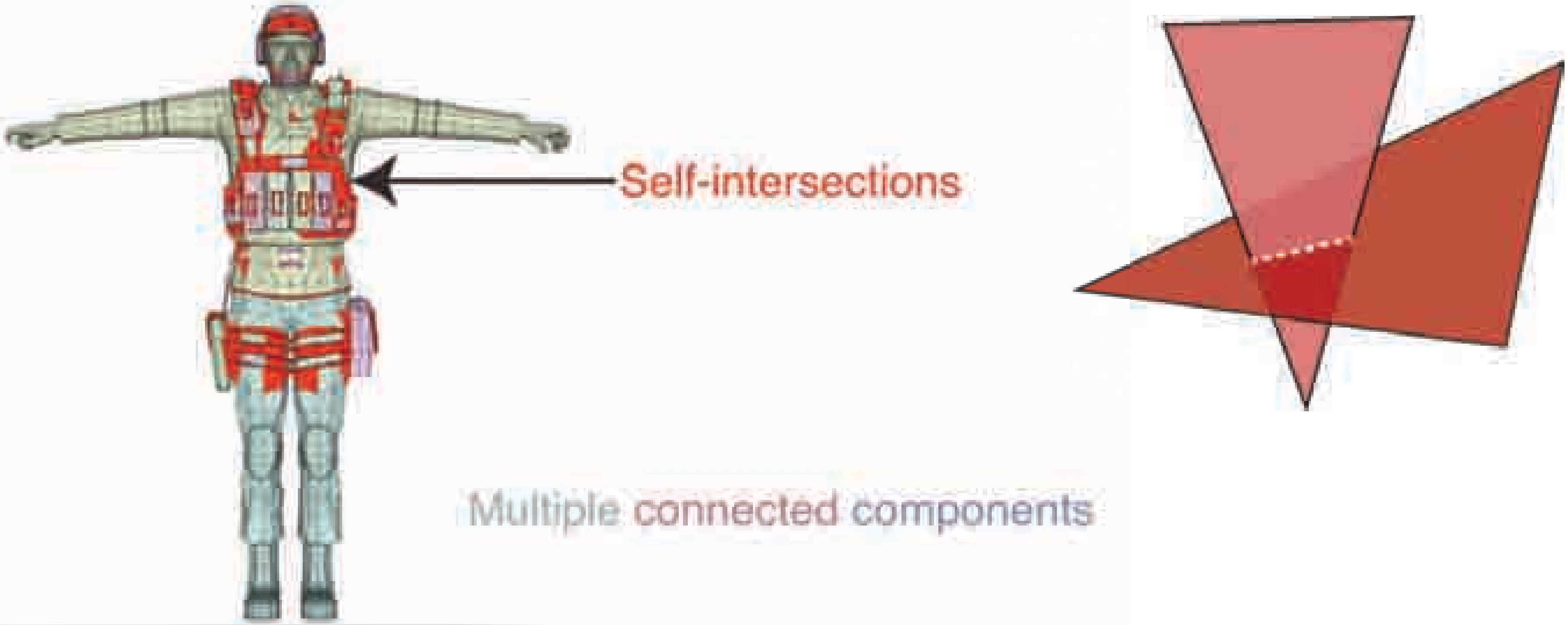
triangle mesh
*watertight*



tetrahedral mesh
*made by TETGEN*

quality elements
varying density
conform to input

# Apparent surface descriptions of solids are *unmeshable* with current tools



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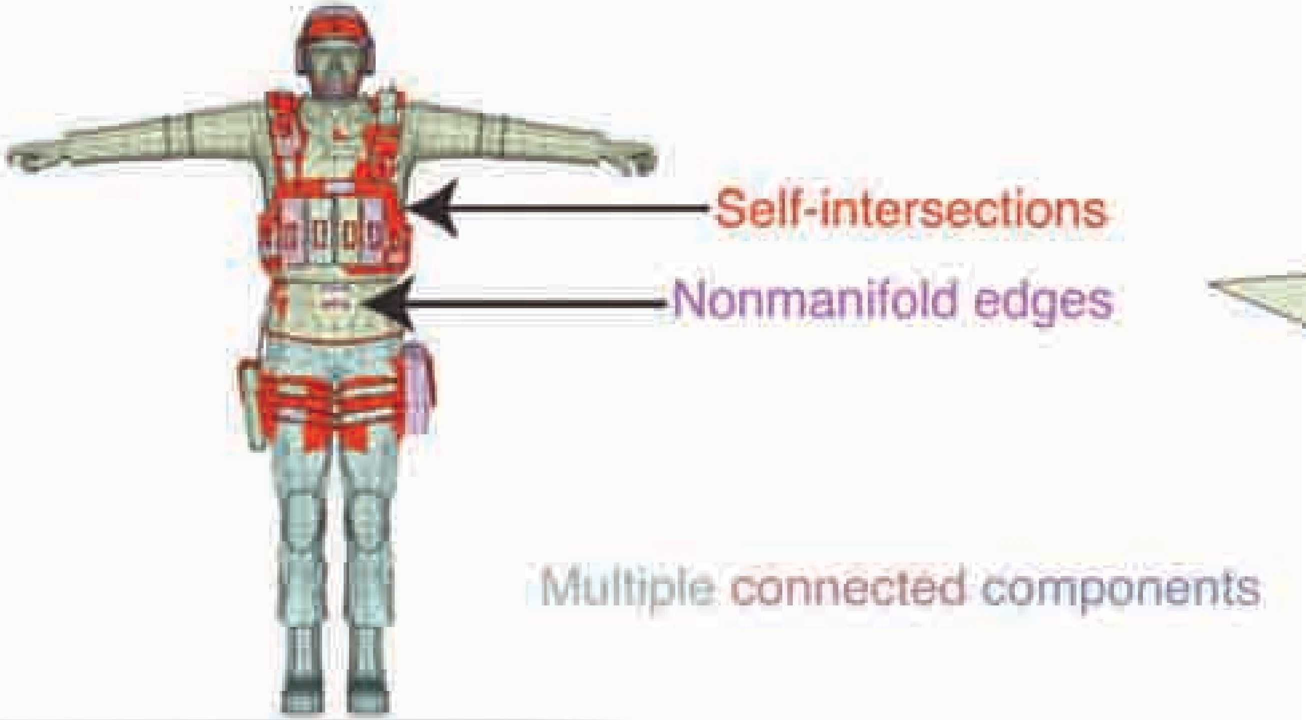


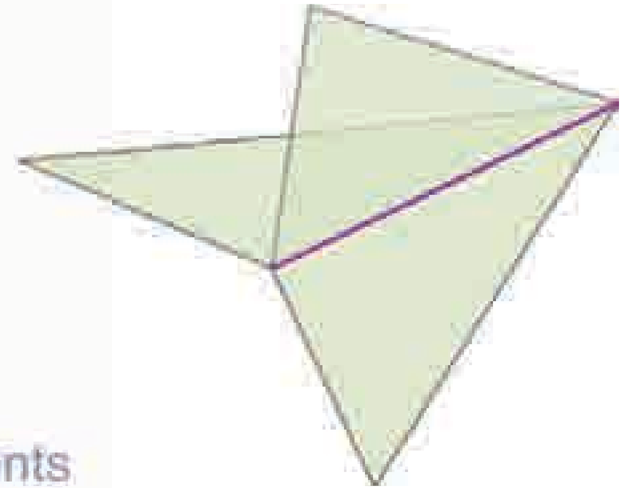

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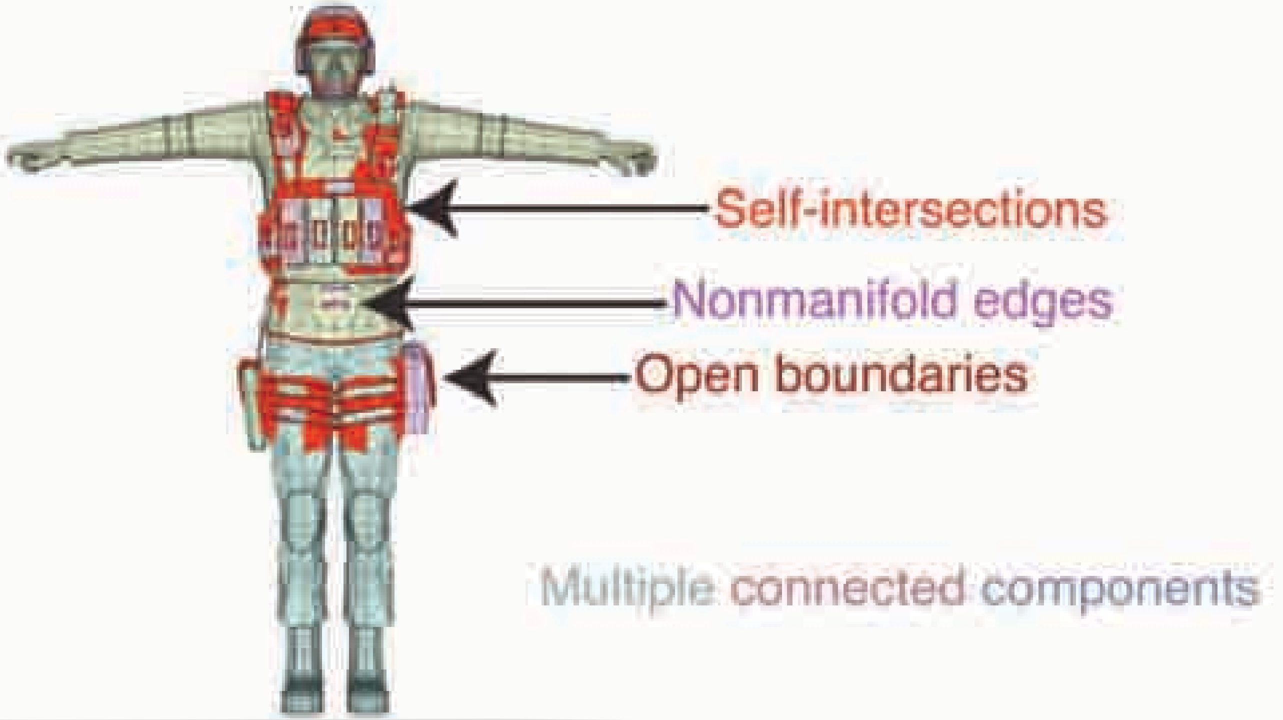
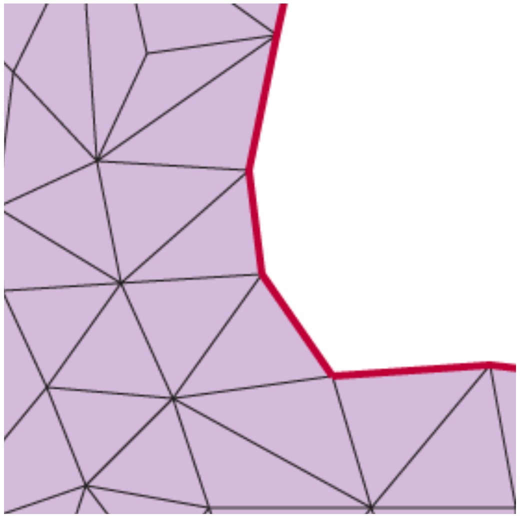
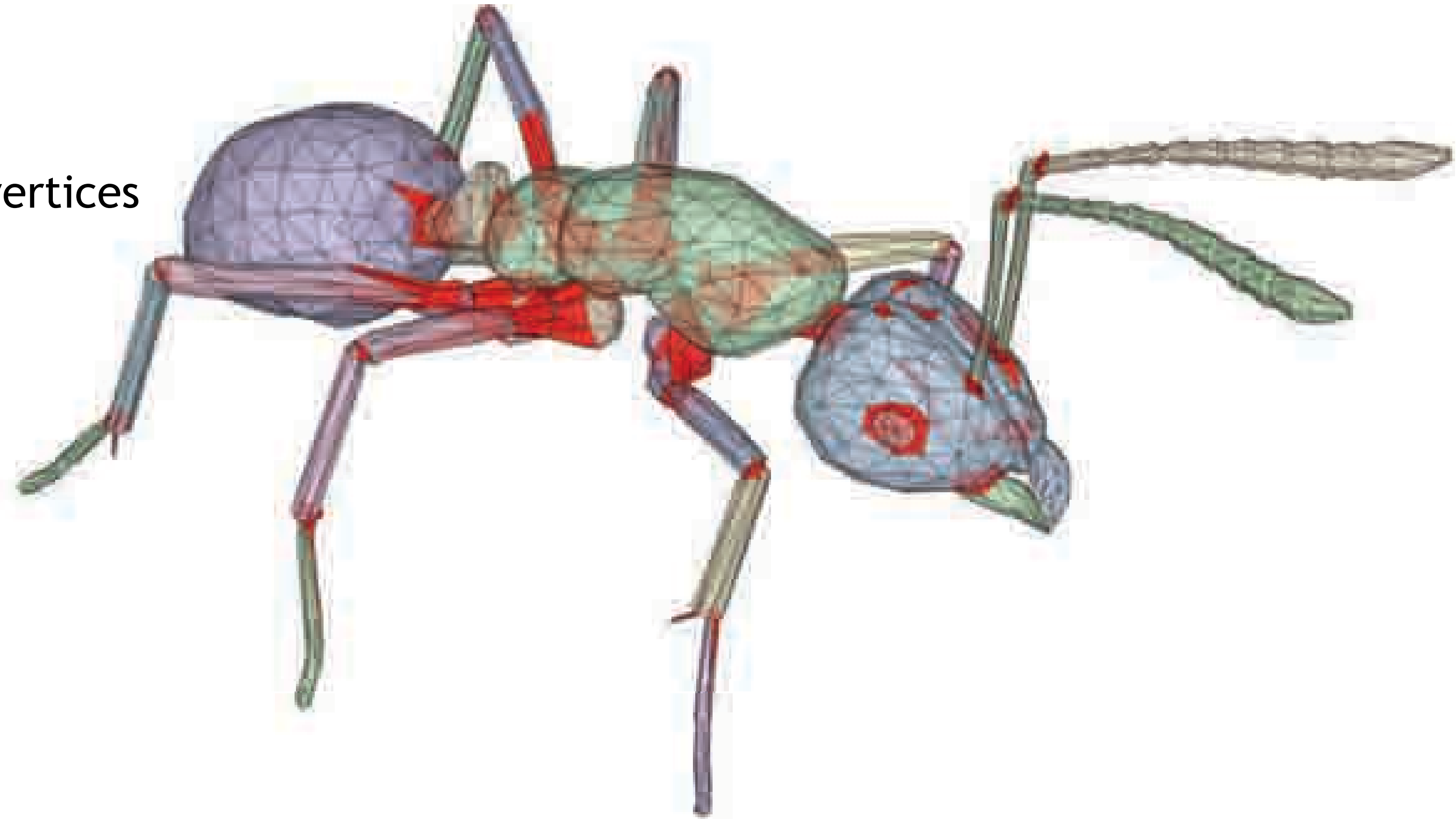


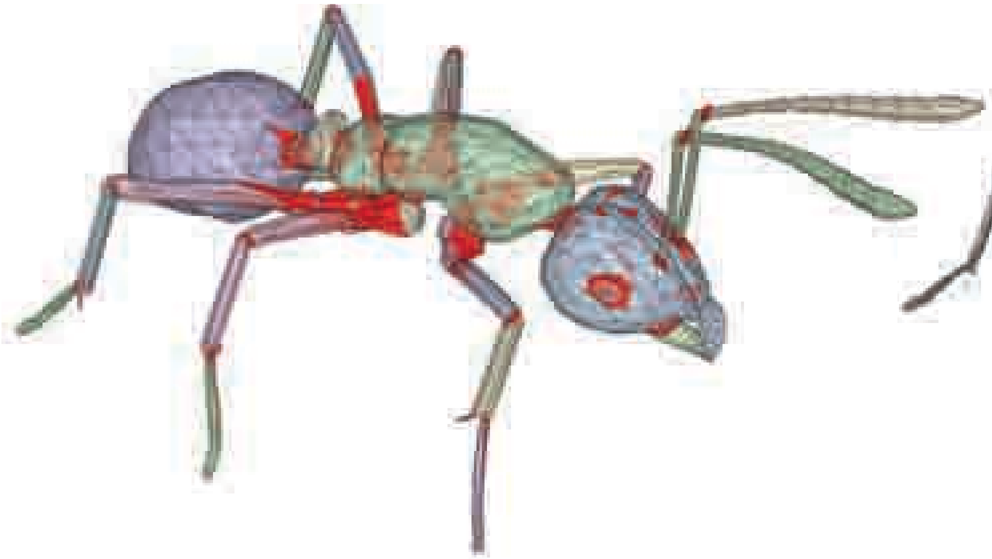



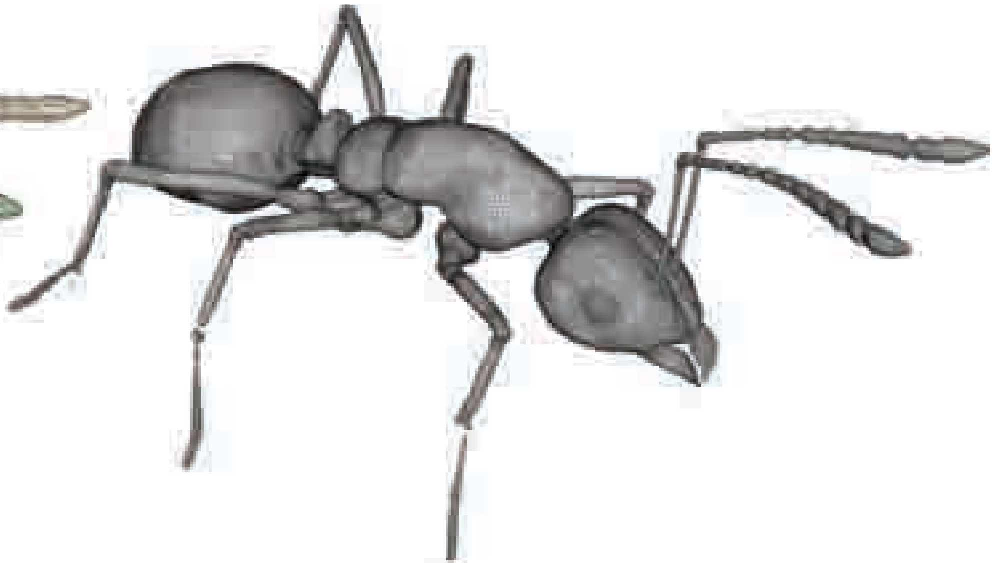
# Meshes are often output of human creativity

only 4000 vertices



# Treating as scanned objects is inappropriate





only 4000 vertices

[Kazhdan et al. 2006]
over 130000 vertices!

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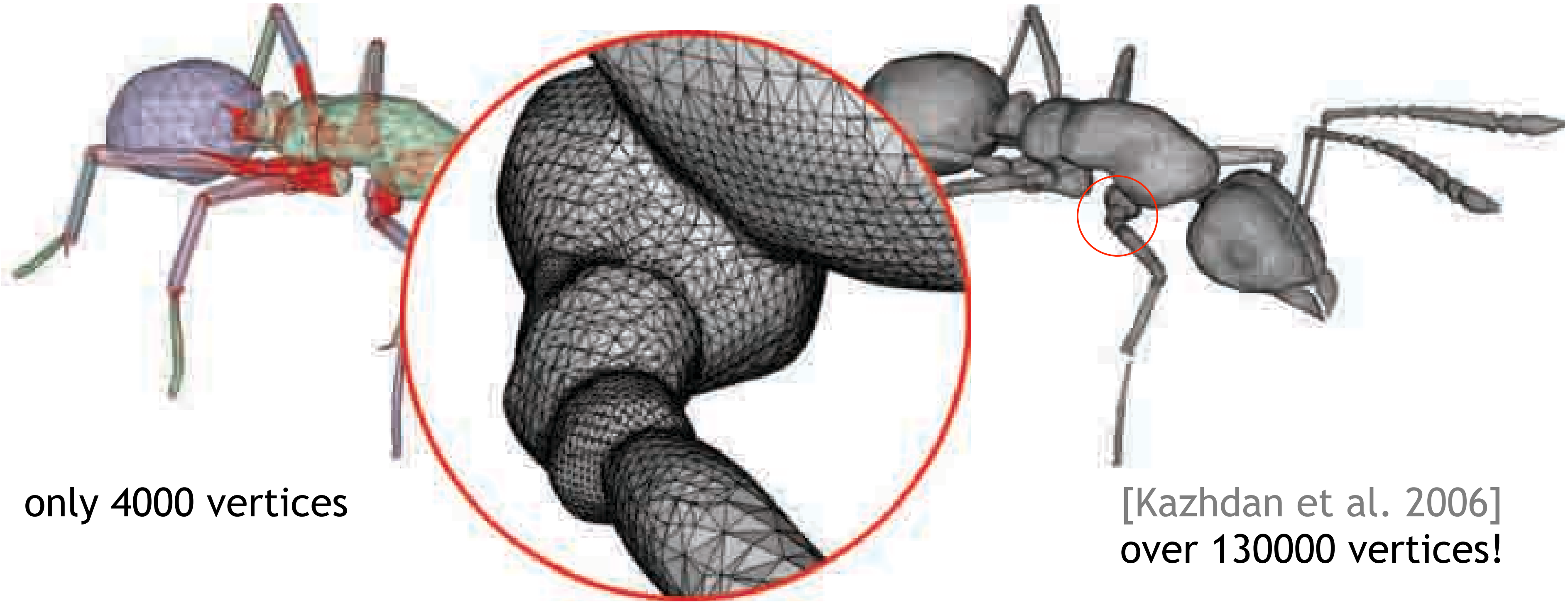

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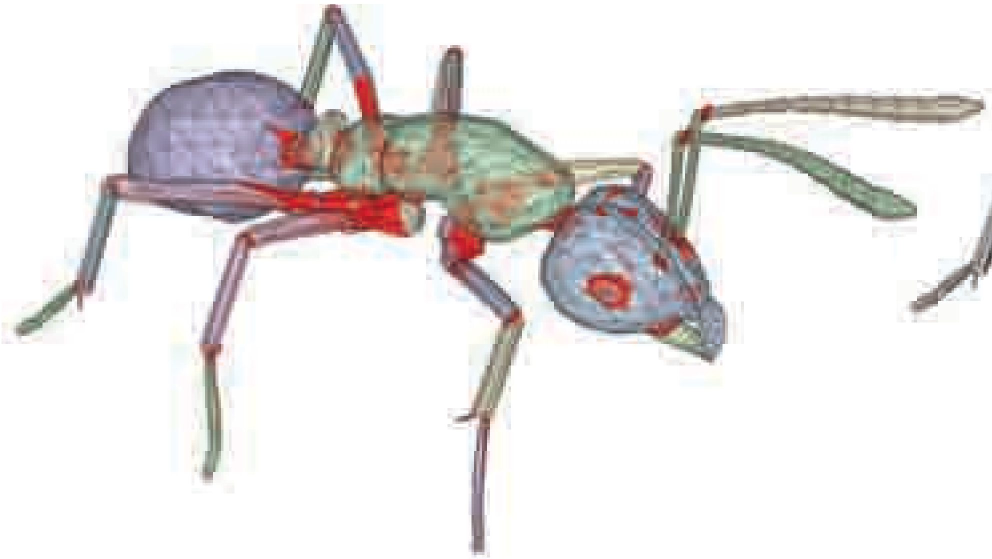
[Kazhdan et al. 2006]
over 130000 vertices!

# Volume mesh should conform to input



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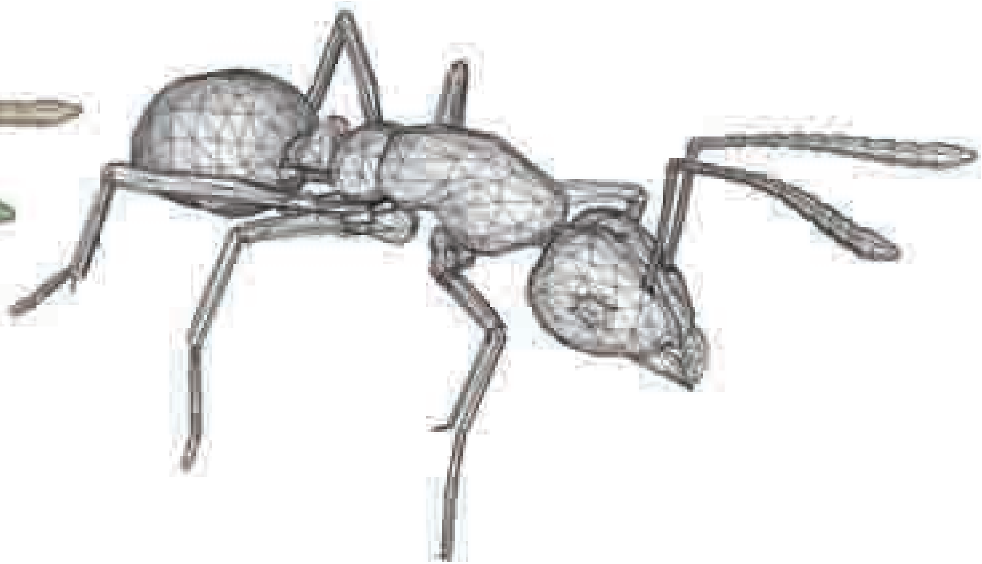

our output tet mesh
only 4500 vertices



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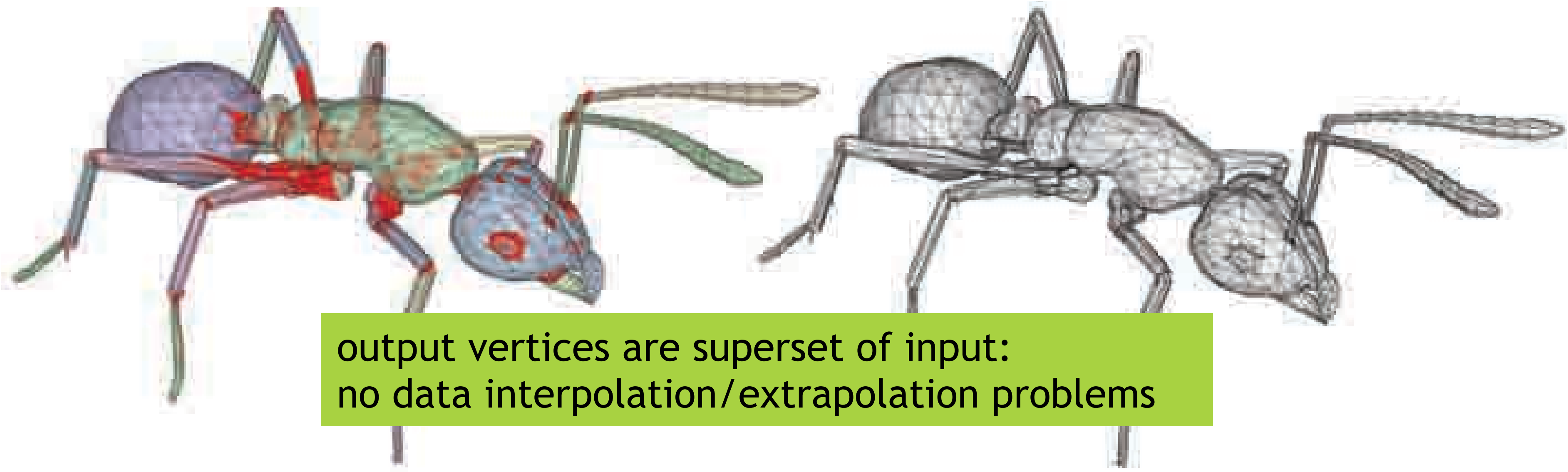
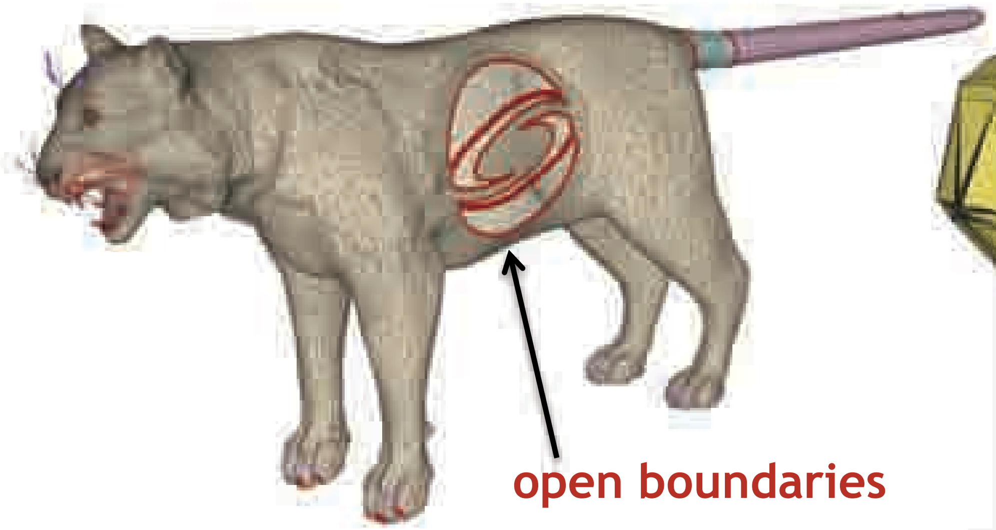


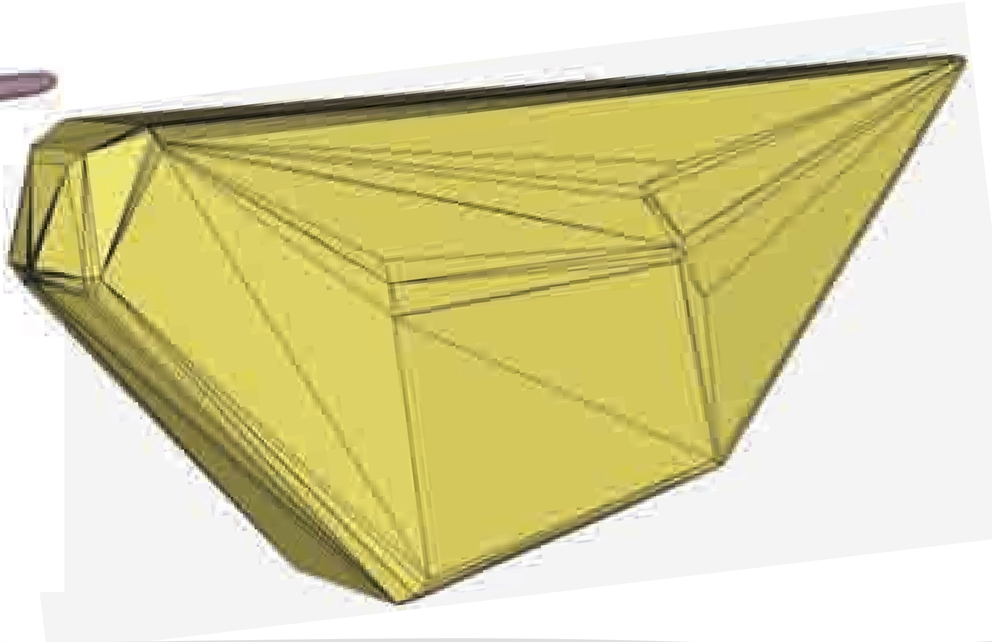
output vertices are superset of input:
no data interpolation/extrapolation problems

only 4000 vertices

our output tet mesh
only 4500 vertices

# Can mesh the entire convex hull, but what's inside? What's outside?



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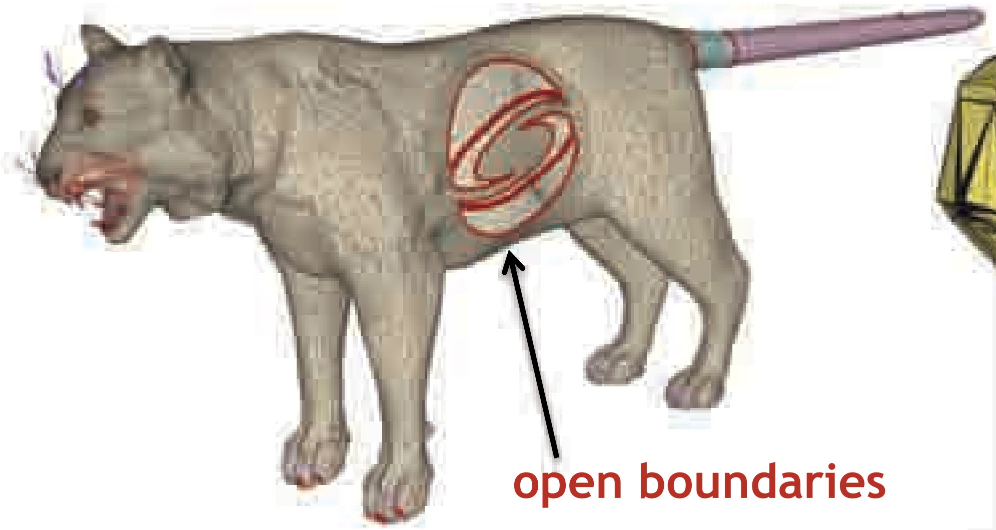


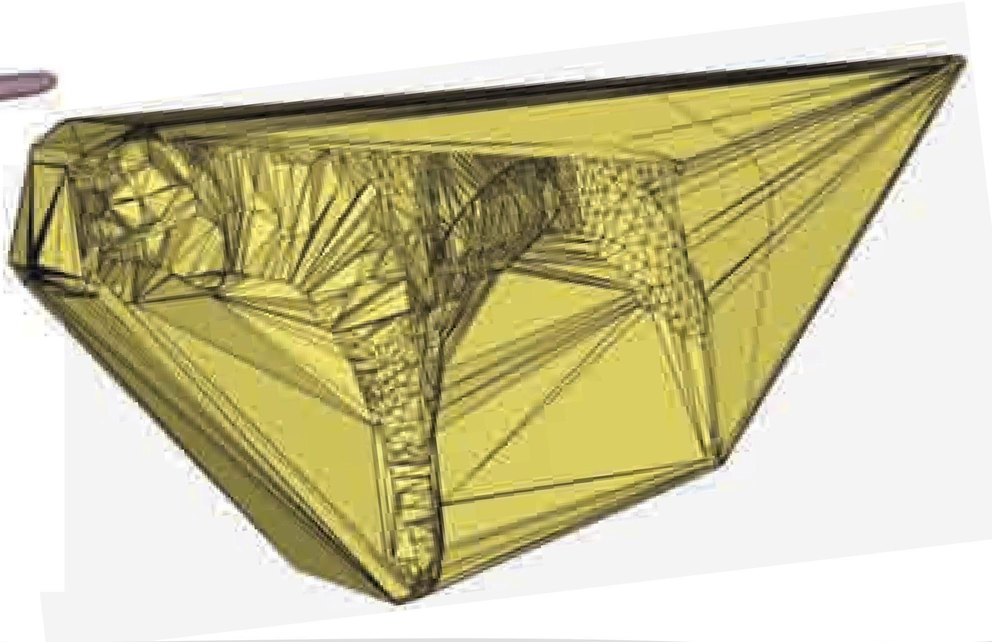

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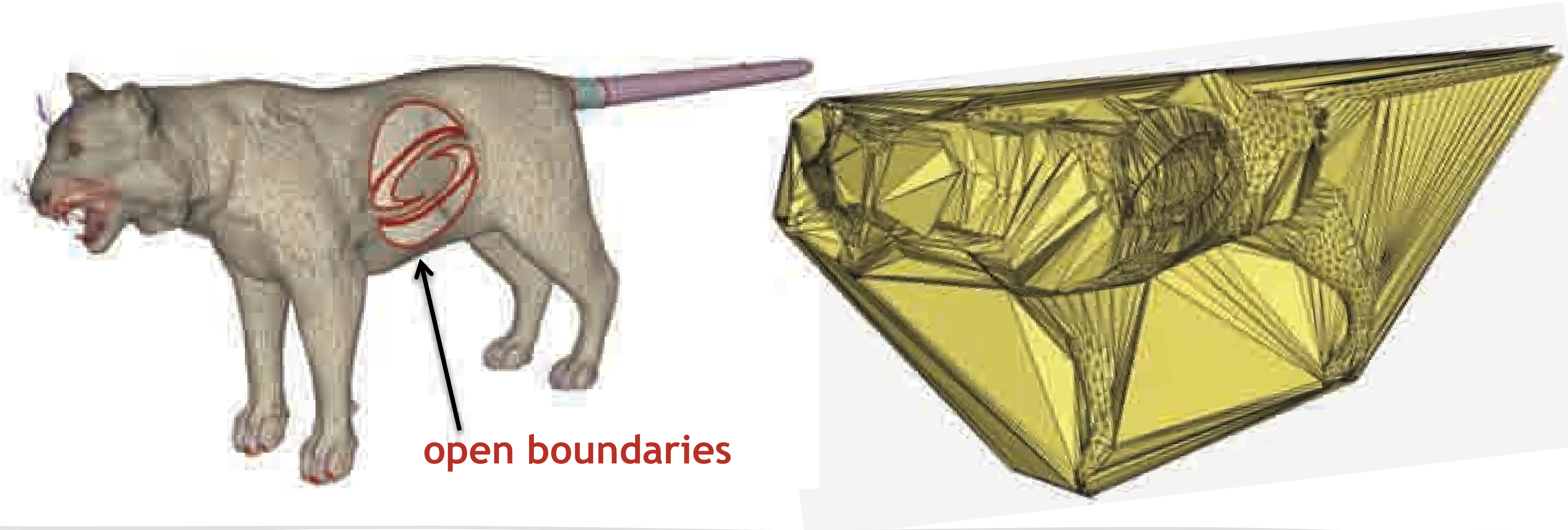
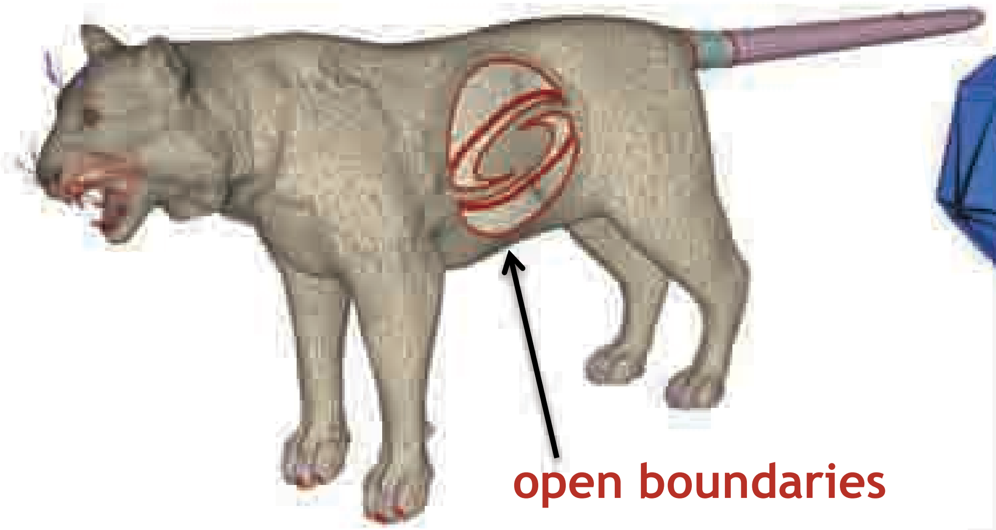
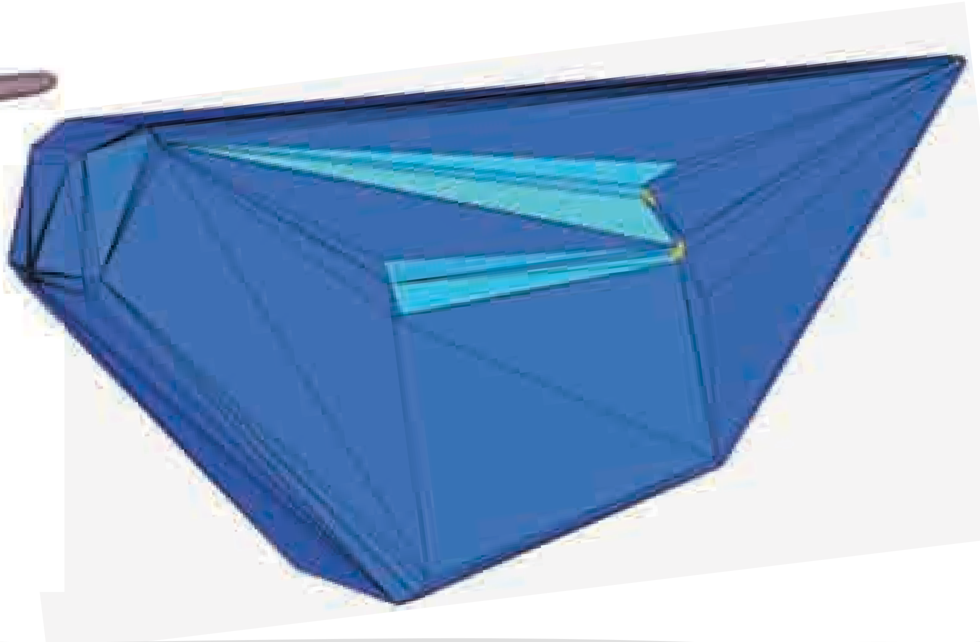


# Generalized function indicates *insideness*

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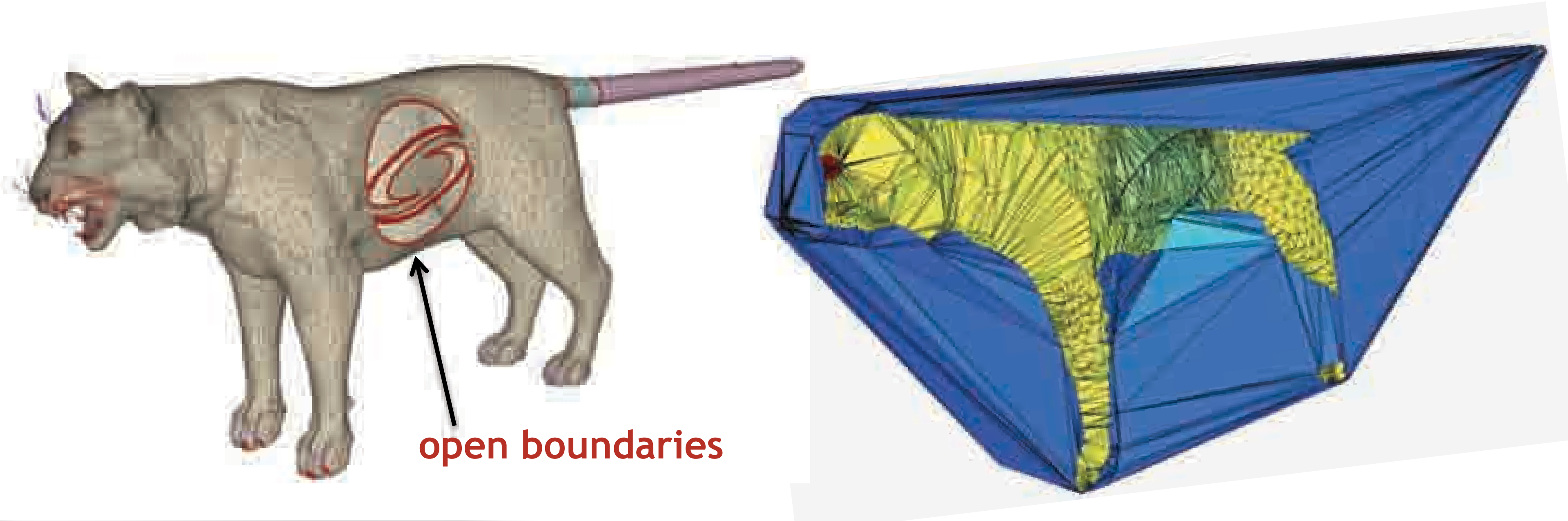

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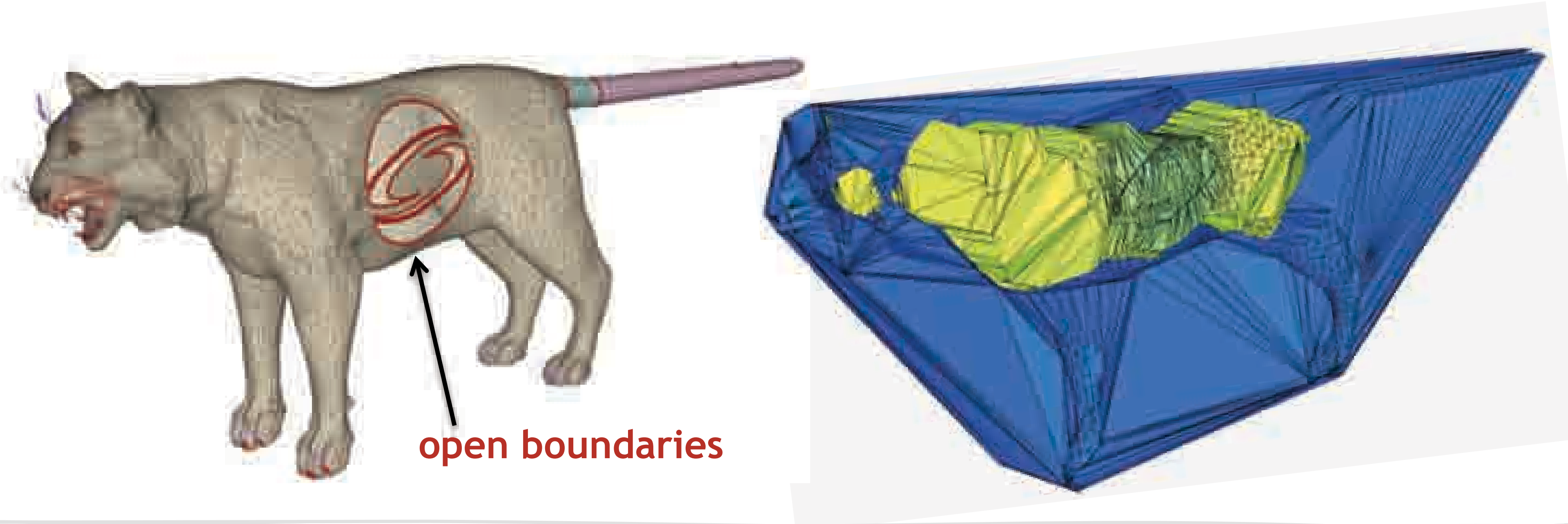

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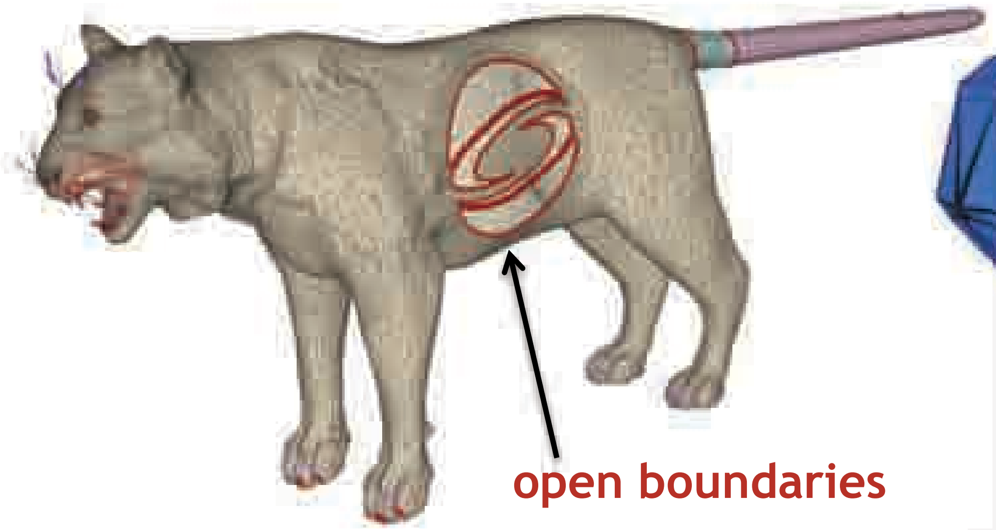


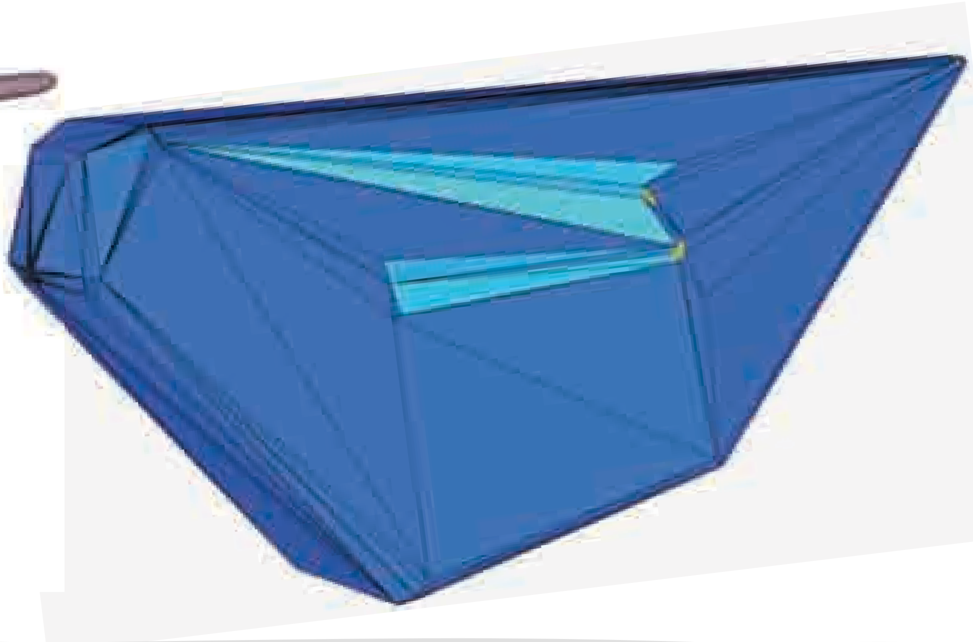

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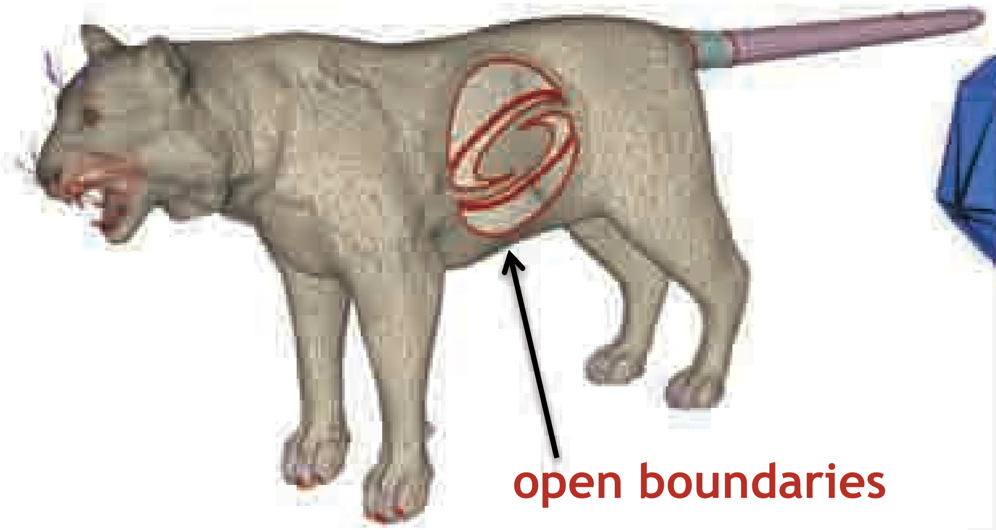


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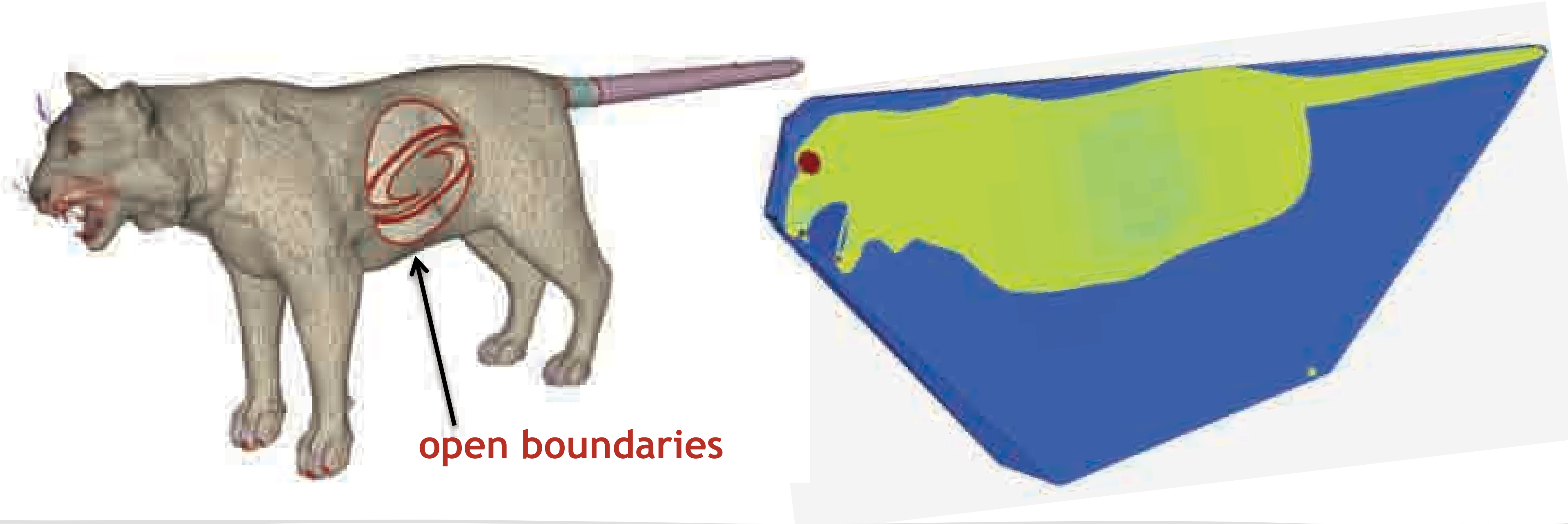
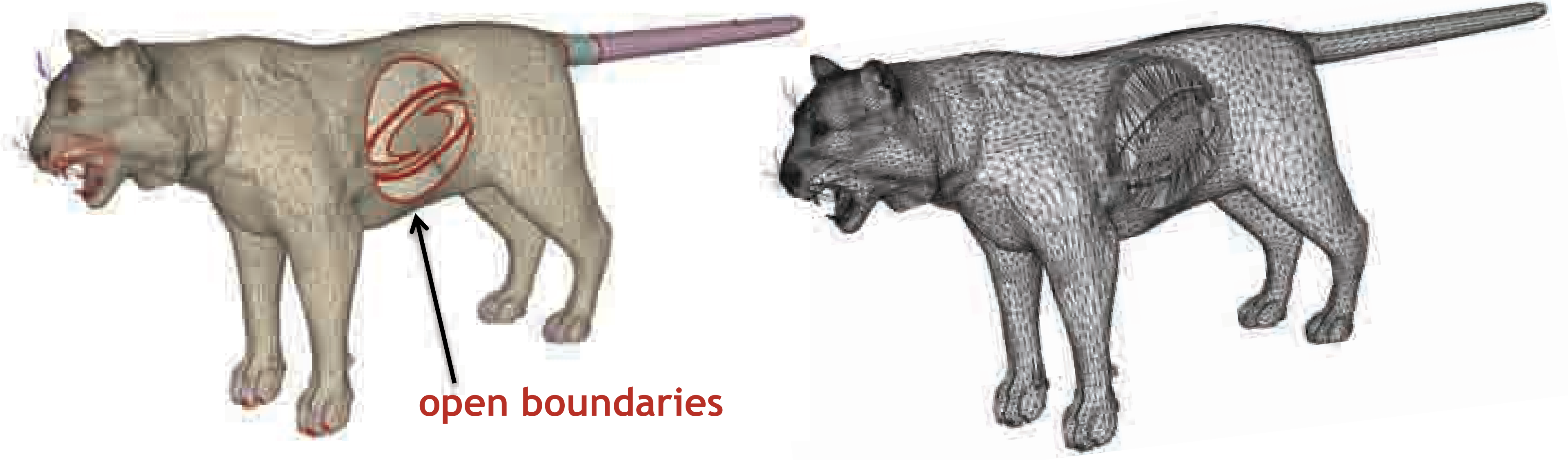


# Function guides a crisp segmentation



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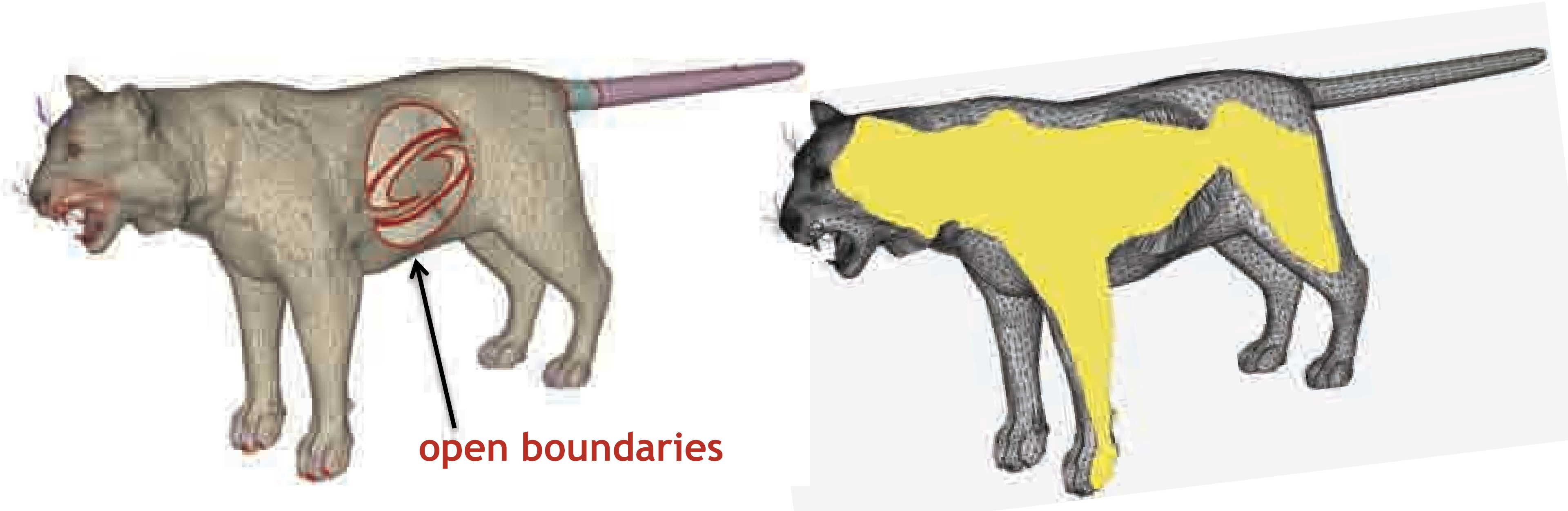

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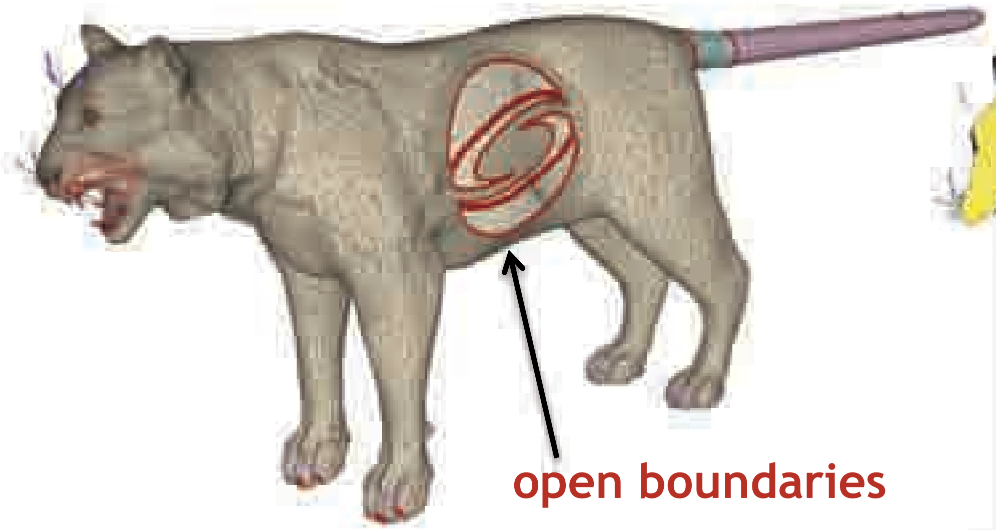
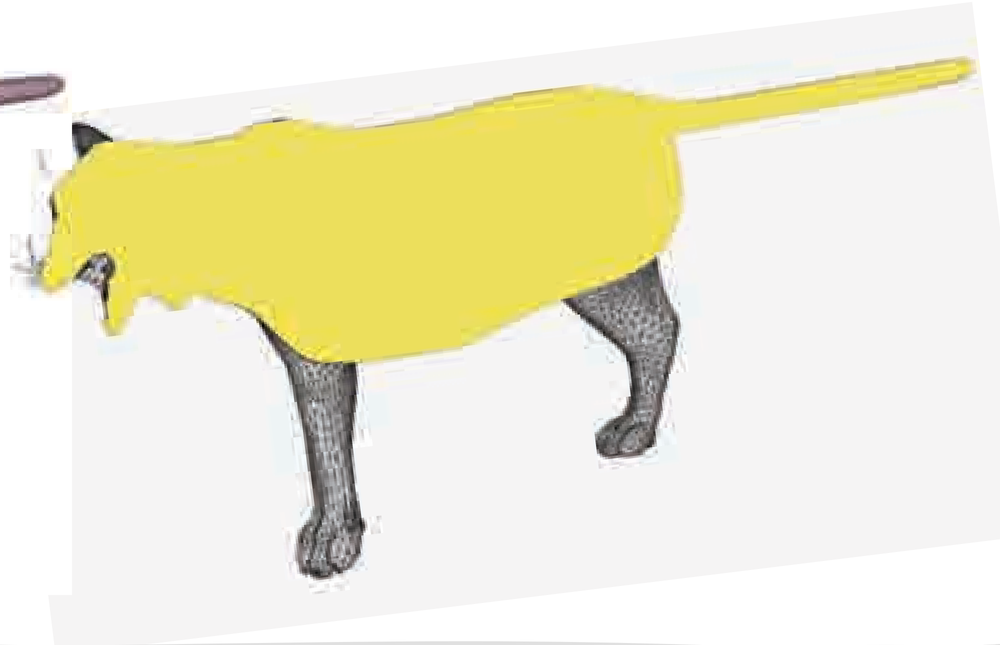
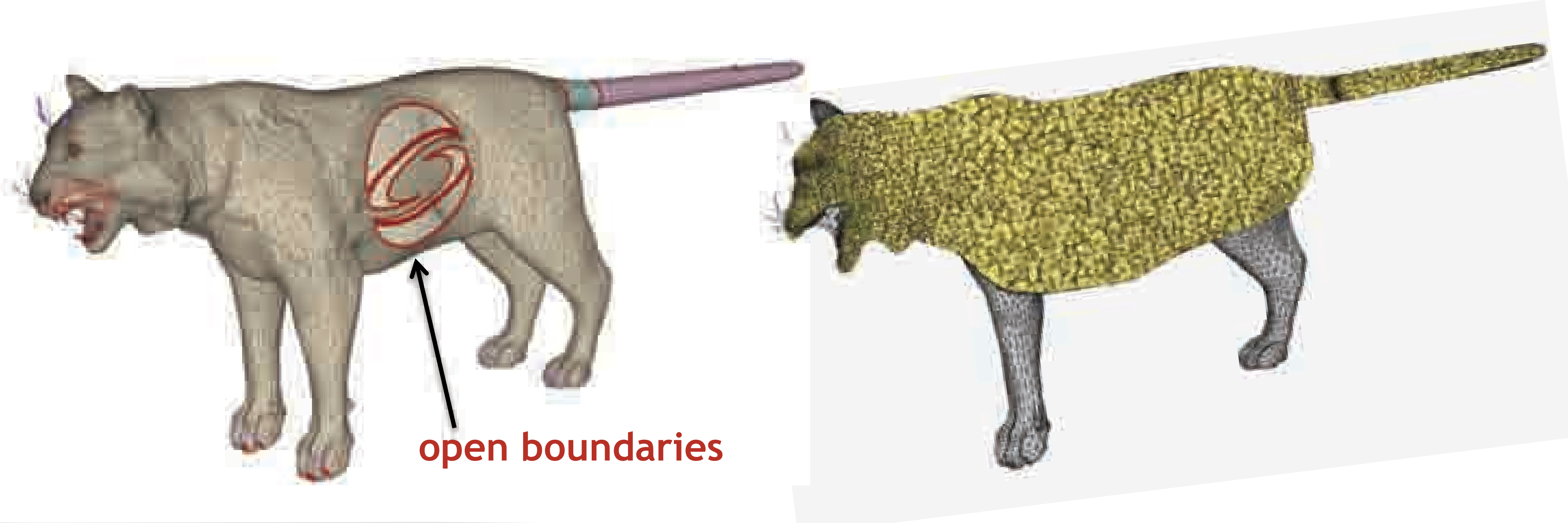


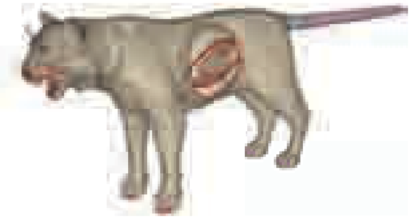



# Output is minimal, ripe for post-processing

Refined mesh using TETGEN, STELLAR, etc.



# Idea: mesh entire convex hull, segment inside tets from outside ones



input triangles

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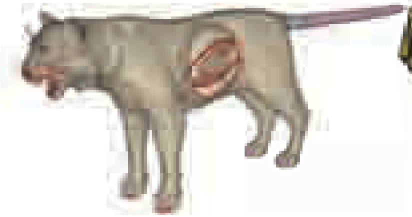

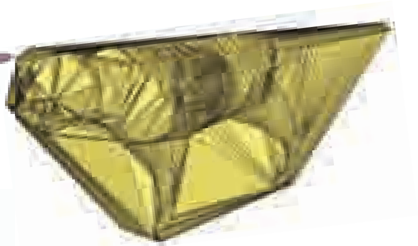

input triangles → CDT →

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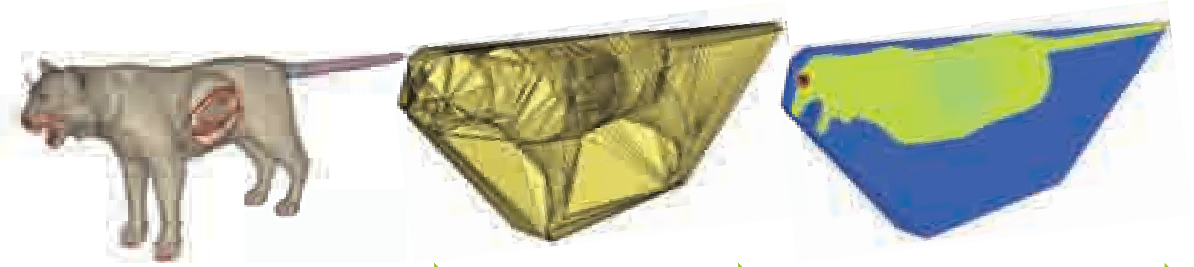

input triangles → CDT → measure of insideness →

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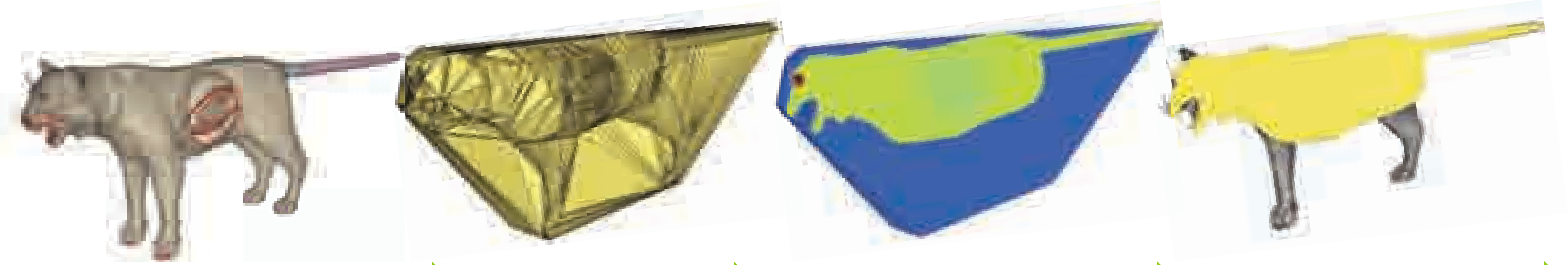

input triangles → CDT → measure of insideness → segment CDT →

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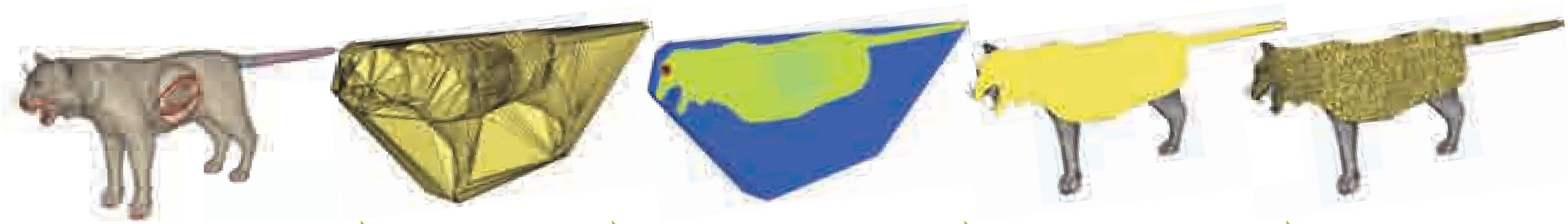
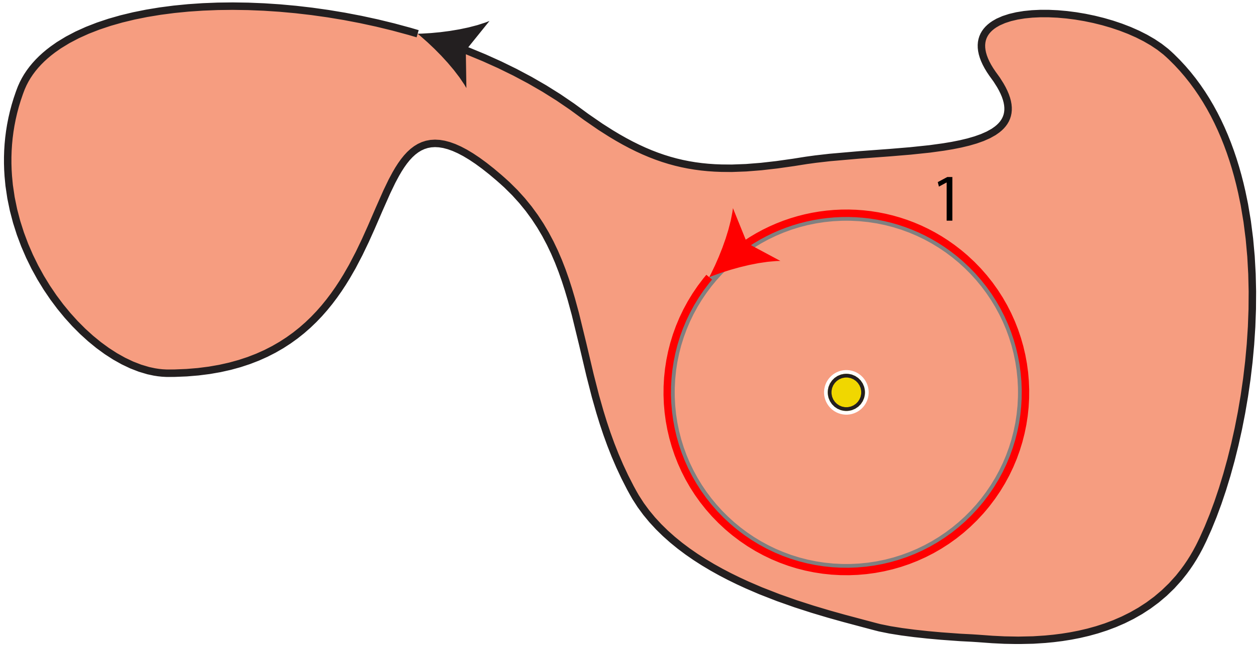


input triangles → CDT → measure of insideness → segment CDT → refine, etc.

# Idea: mesh entire convex hull, segment inside tets from outside ones

# If shape is watertight, winding number is perfect measure of inside

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$



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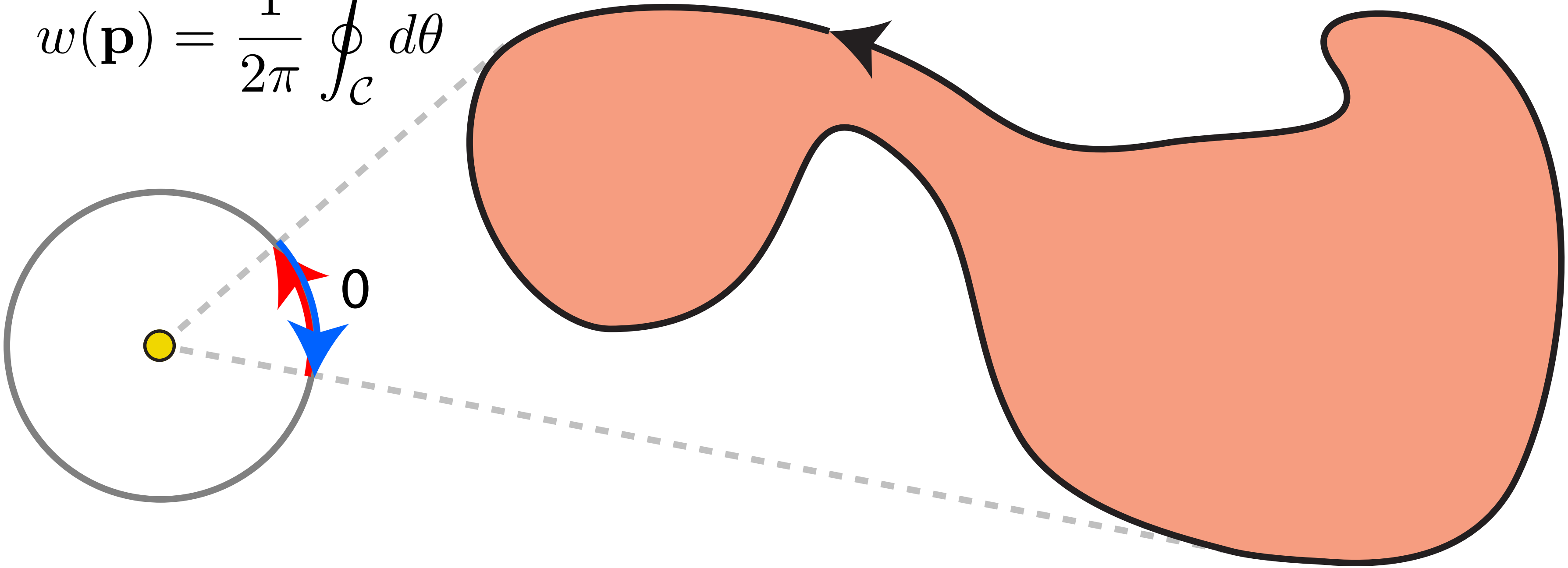
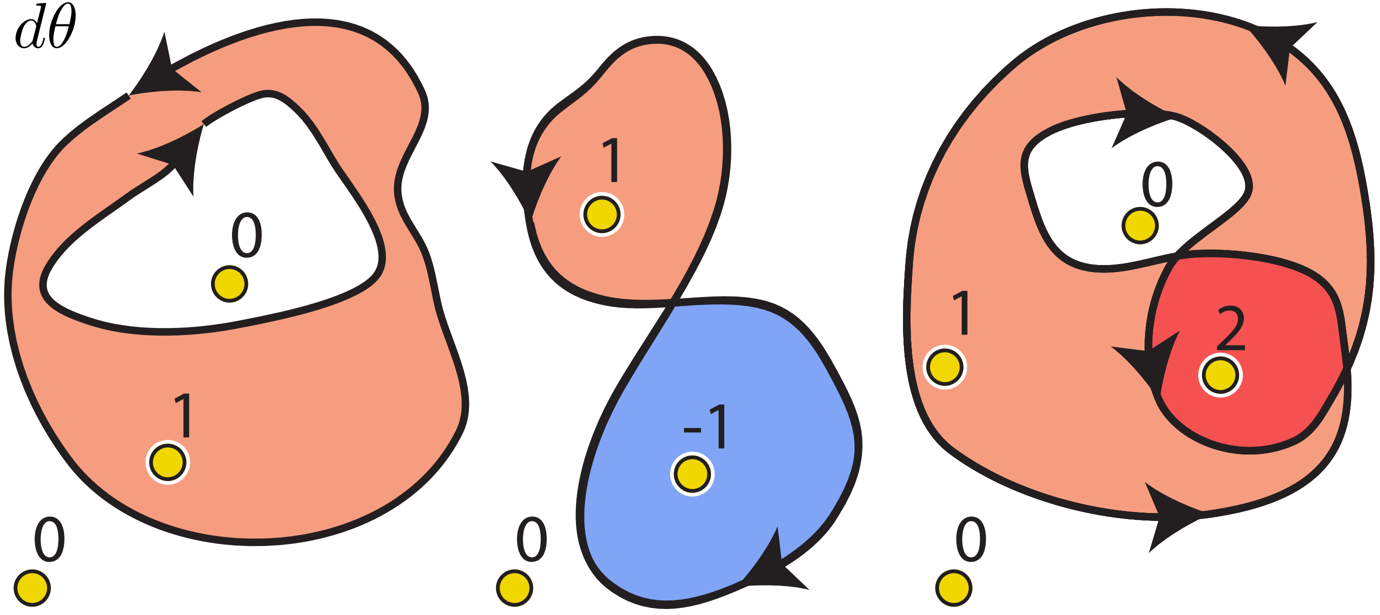


# Winding number uses orientation to treat insideness as *signed integer*

$$w(\mathbf{p}) = \frac{1}{2\pi}\oint_C d\theta$$

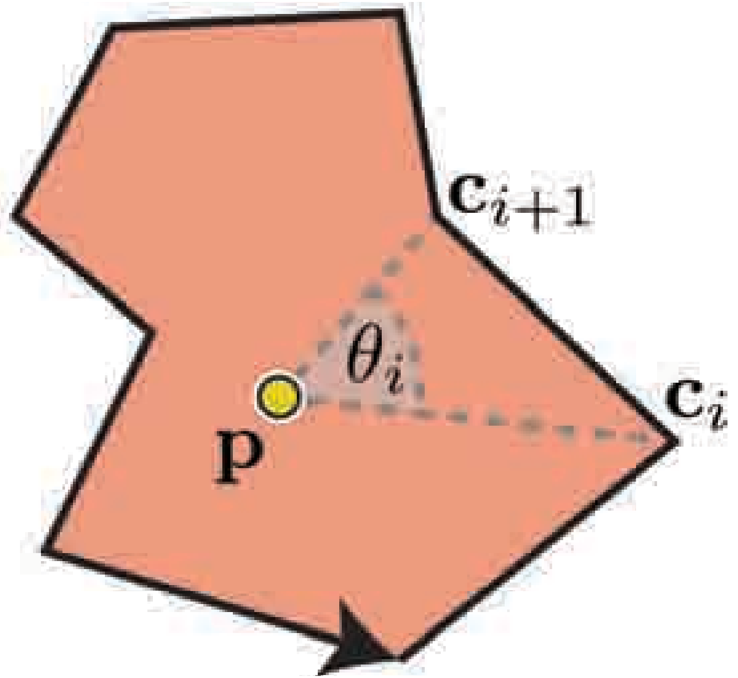


# Naive discretization is simple and exact

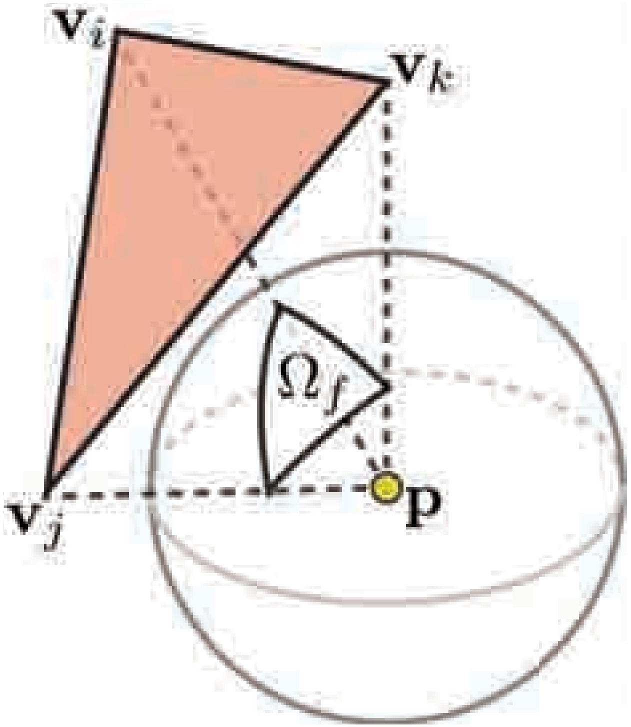
$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$

$$\Downarrow$$

$$w(\mathbf{p}) = \frac{1}{2\pi} \sum_{i=1}^{n} \theta_i$$



# Generalizes elegantly to 3D via solid angle

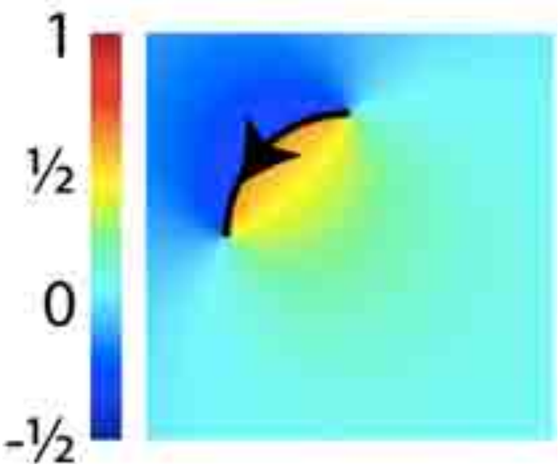



$$w(\mathbf{p}) = \frac{1}{4\pi} \iint_{S} \sin(\phi)\, d\theta\, d\phi$$

$$w(\mathbf{p}) = \frac{1}{4\pi} \sum_{f=1}^{m} \Omega_f$$

# What happens if the shape is open?

$$w(\mathbf{p}) = \frac{1}{2\pi}\oint_C d\theta$$



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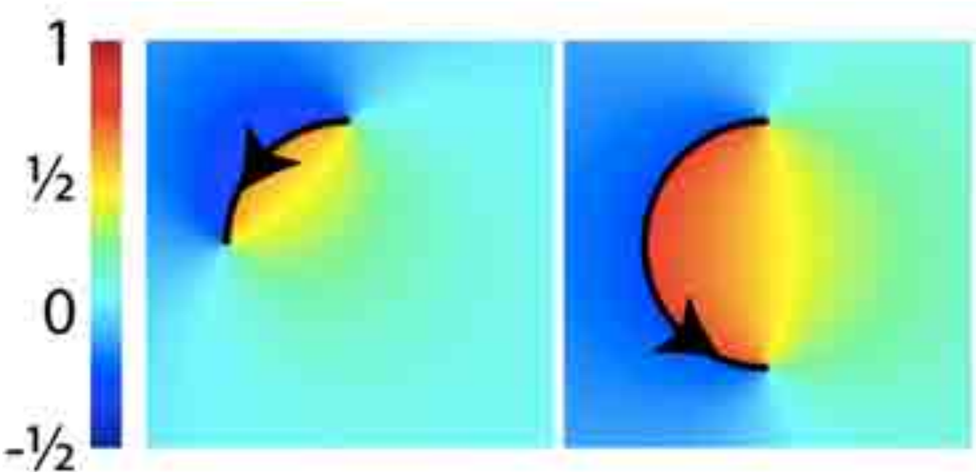




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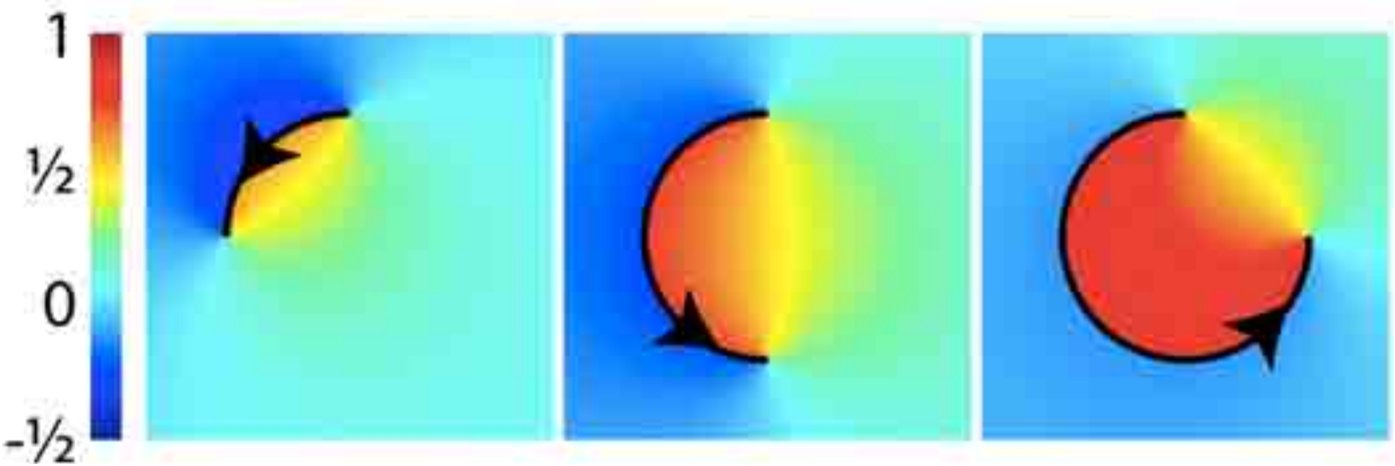

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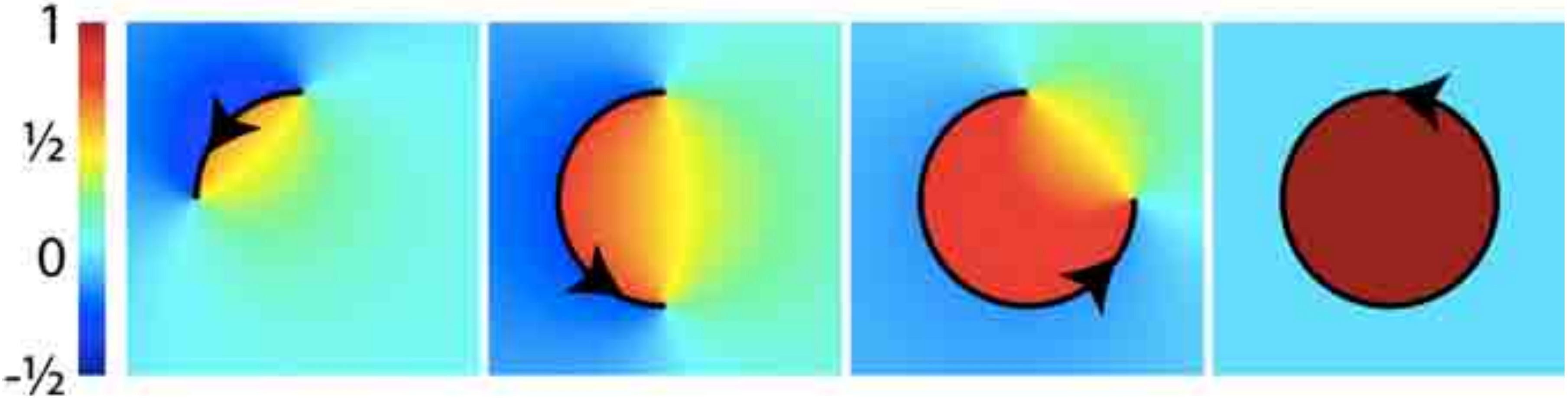

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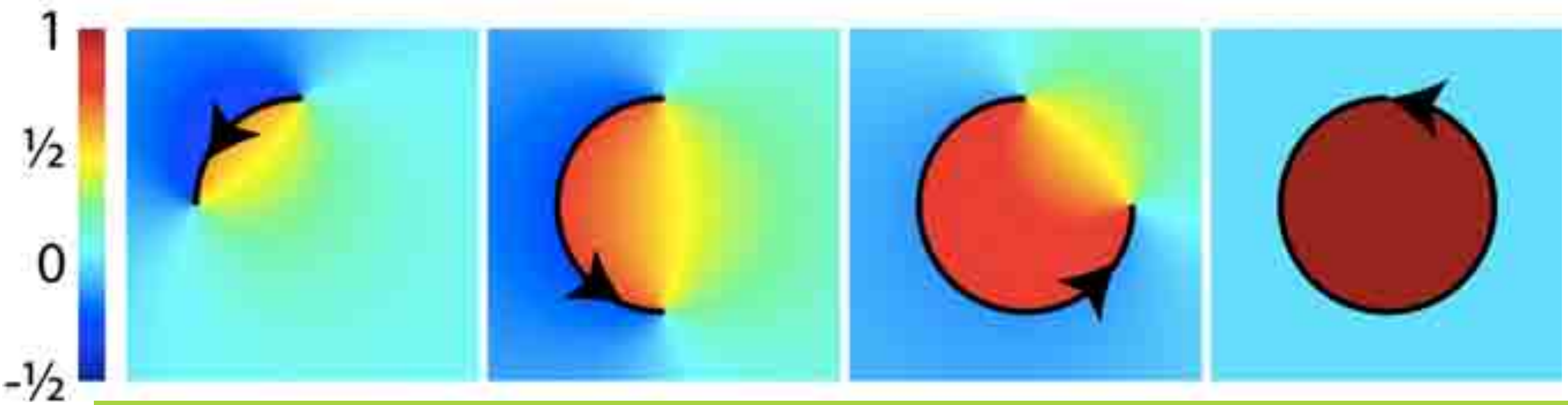
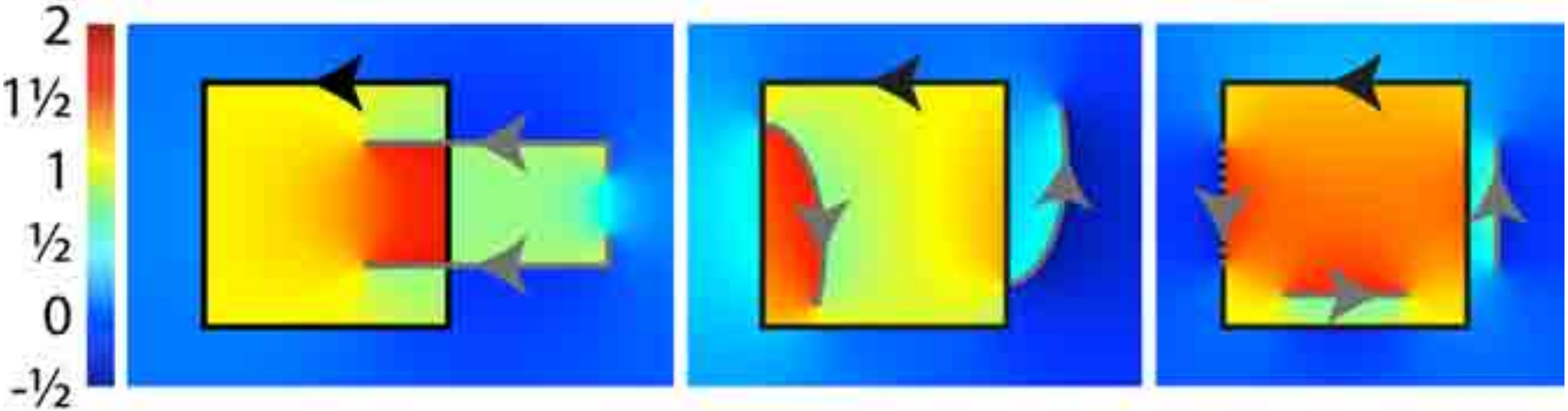



Gracefully tends toward perfect indicator as shape tends towards watertight

# What if shape is self-intersecting? Non-manifold?

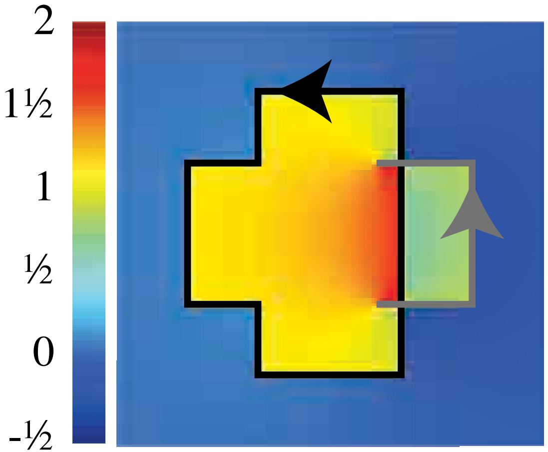
$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$

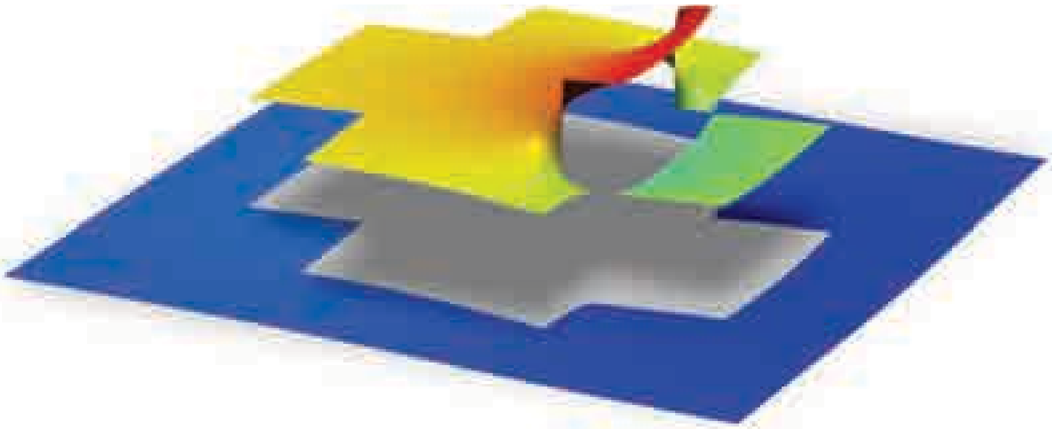



Jumps by ±1 across input facets

# Winding number jumps across boundaries, otherwise harmonic!

$$w(\mathbf{p}) = \frac{1}{2\pi}\oint_C d\theta$$



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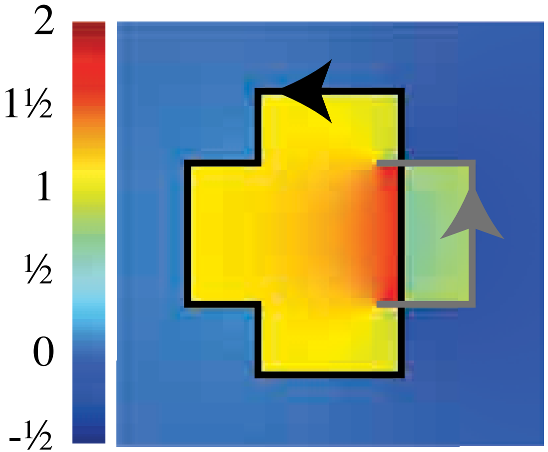
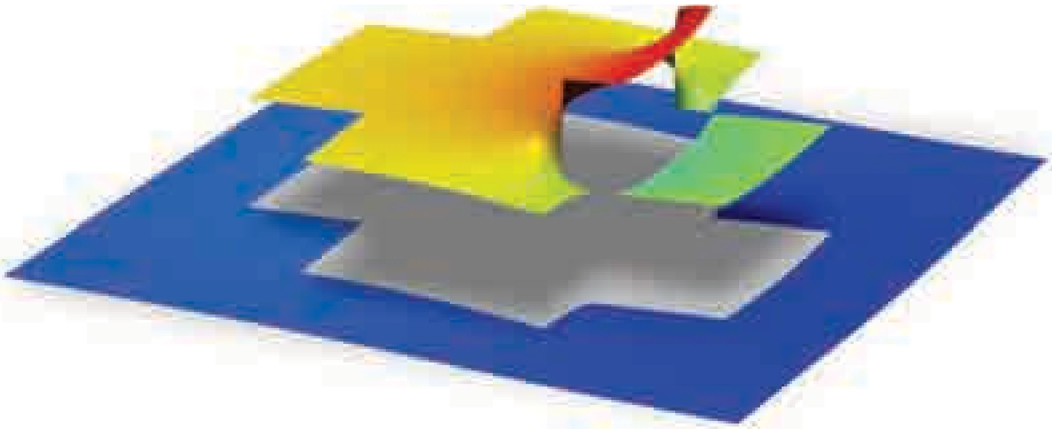
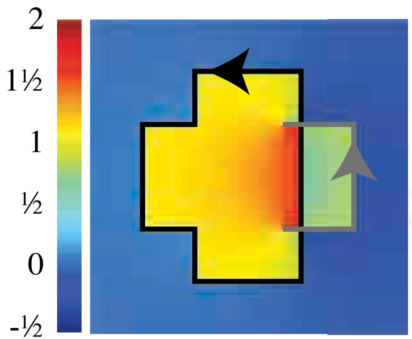
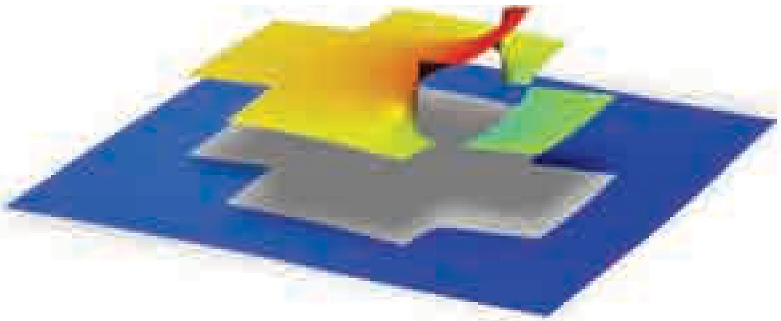
See MAPLE proof in paper or Rahul Narain's recent proof http://goo.gl/5LJWf



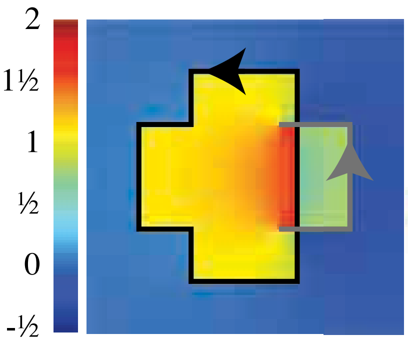
# Other interpolating implicit functions are confused by overlap…

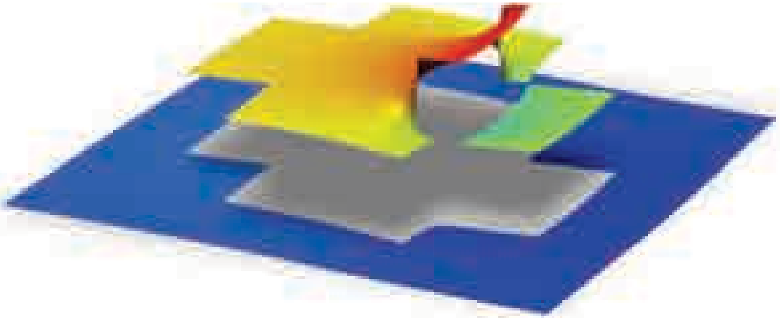


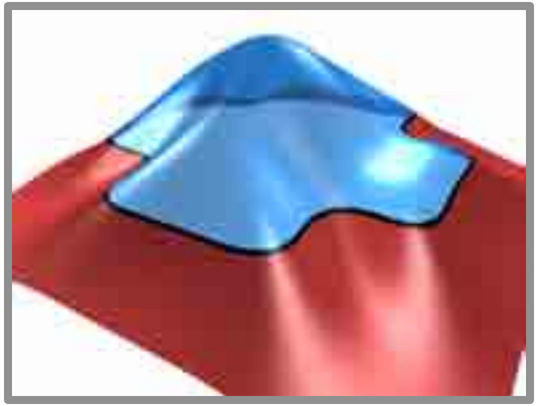



[Shen et al. 2004]

# …or resort to approximation

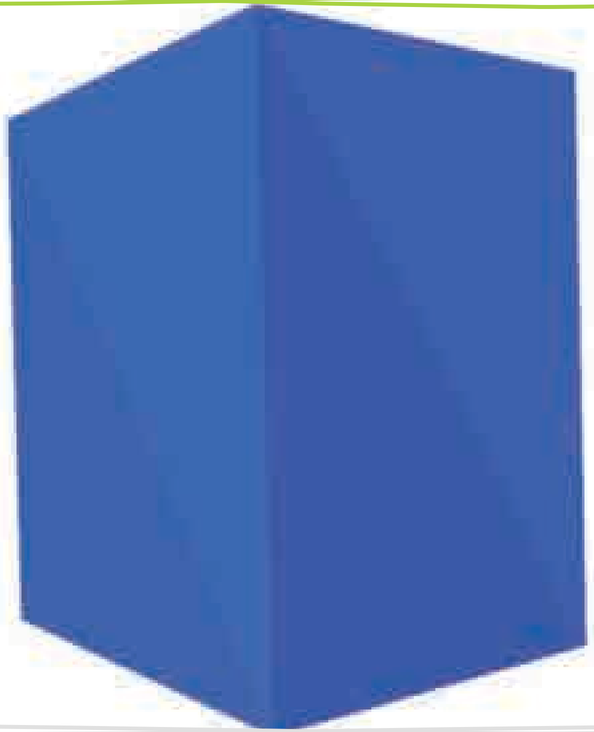




[Shen et al. 2004]

# Sharp discontinuity across input eases precise, *conformal* segmentation



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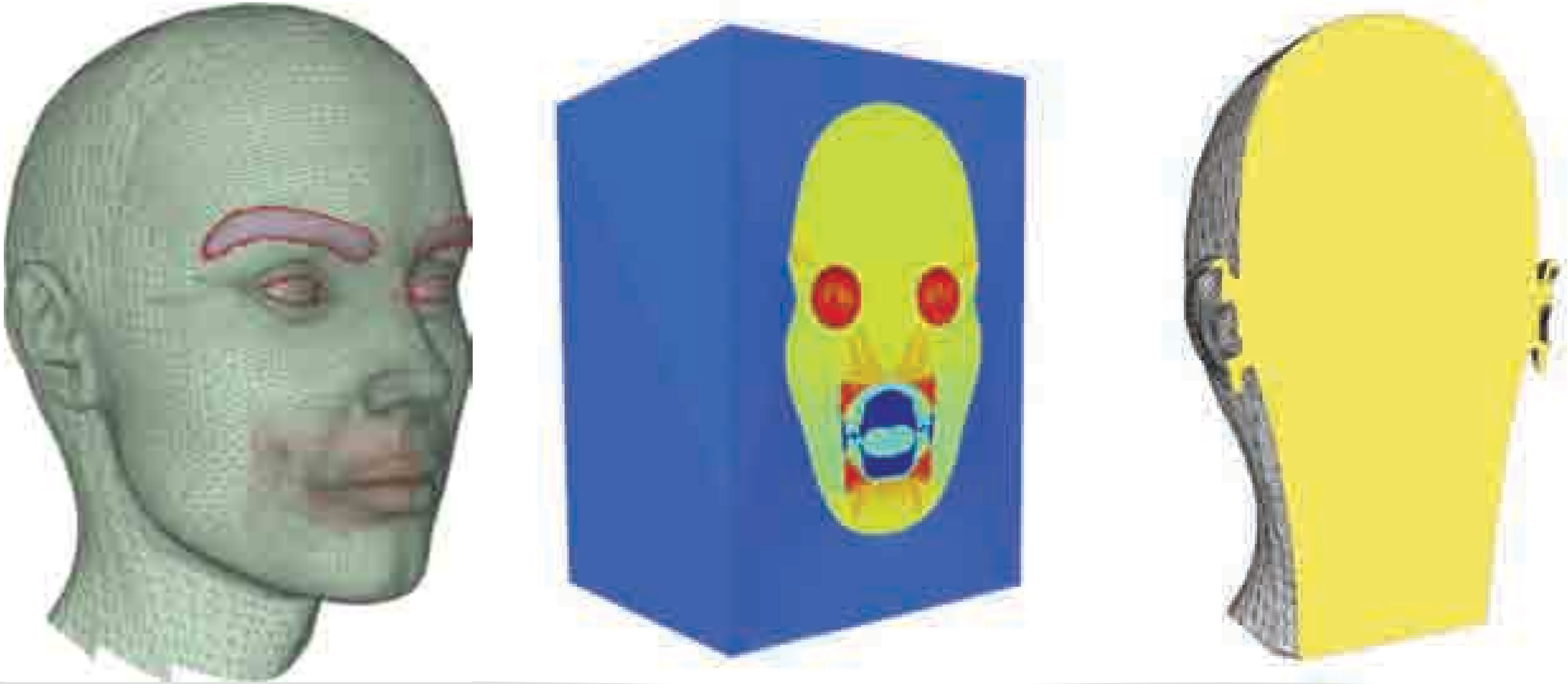
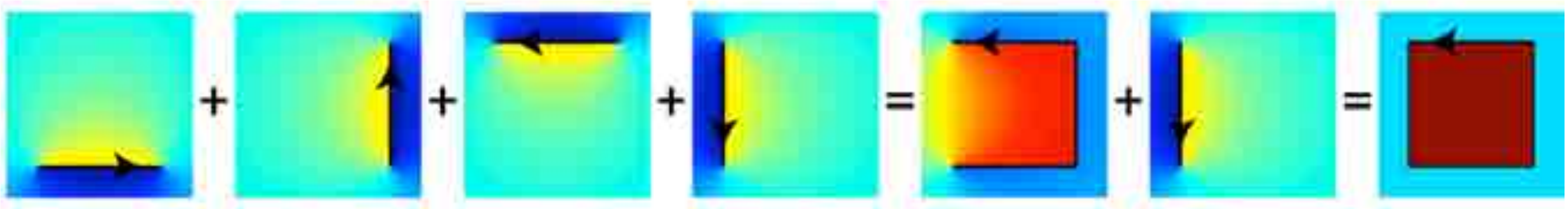


# Naive implementation is too expensive

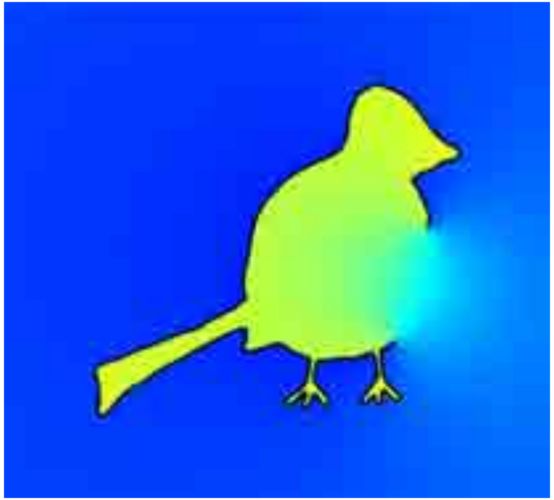
$$w(\mathbf{p}) = \frac{1}{2\pi}\sum_{i=1}^{n}\theta_i$$




Winding number is sum of winding numbers: *O(m)*

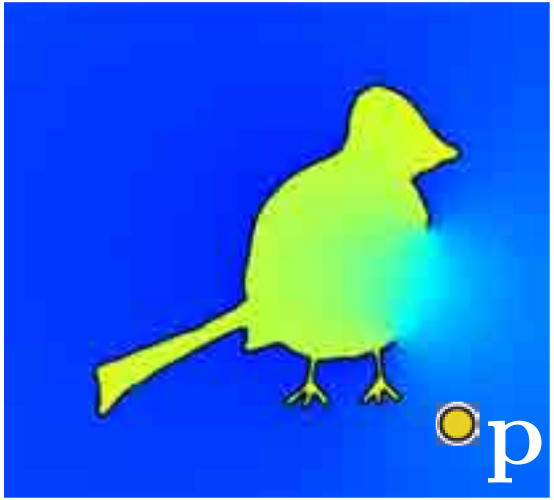
# Interesting fact reveals asymptotic speedup

$\mathcal{C}$



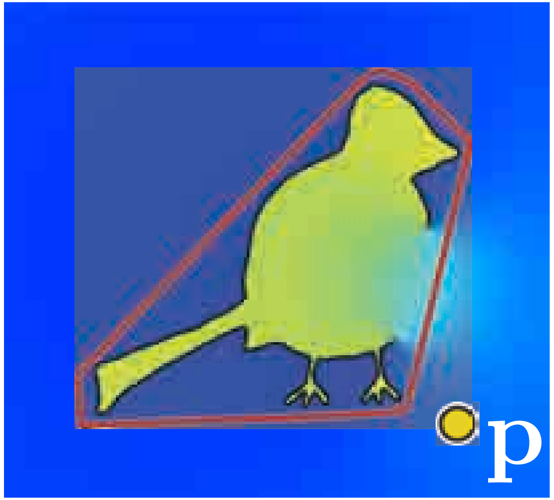
# Interesting fact reveals asymptotic speedup

$\mathcal{C}$

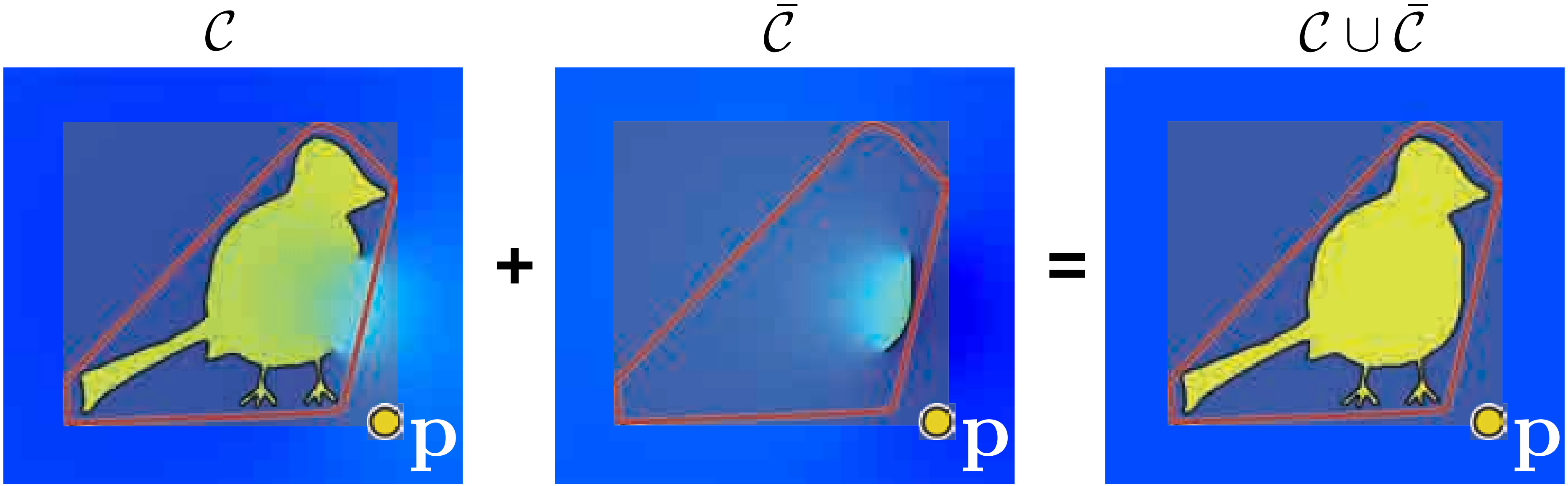


# Interesting fact reveals asymptotic speedup

$\mathcal{C}$

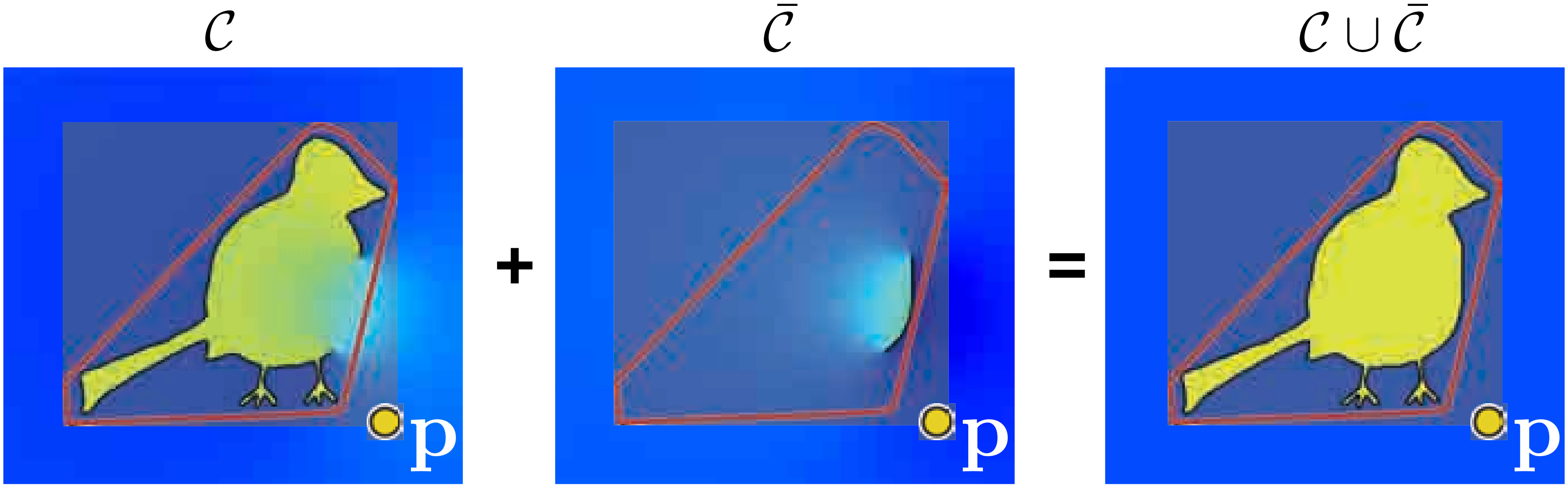


# Interesting fact reveals asymptotic speedup




$$w_{\mathcal{C}\cup\bar{\mathcal{C}}}(\mathbf{p}) = 0$$

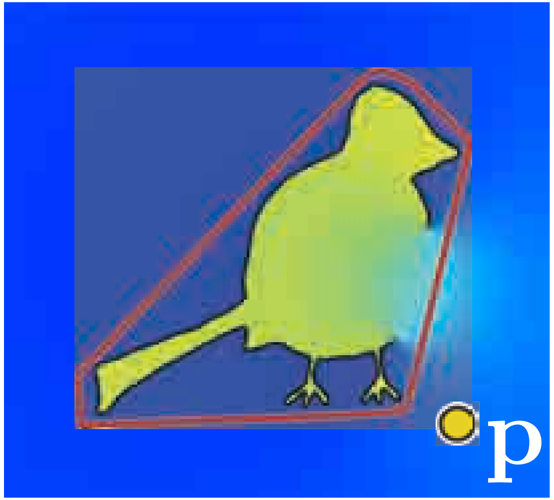
# Interesting fact reveals asymptotic speedup



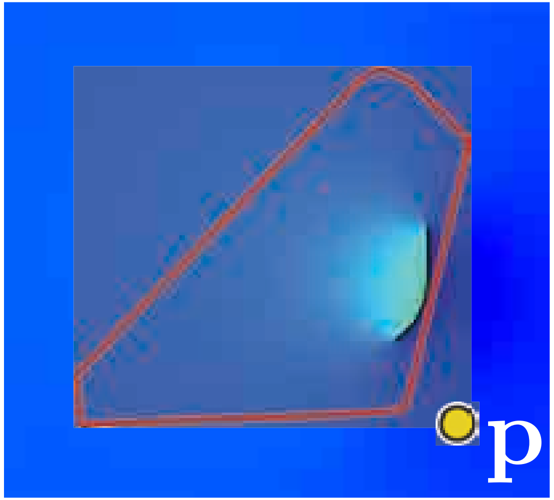

$$w_{\mathcal{C}}(\mathbf{p}) + w_{\bar{\mathcal{C}}}(\mathbf{p}) = w_{\mathcal{C}\cup\bar{\mathcal{C}}}(\mathbf{p}) = 0$$

# Interesting fact reveals asymptotic speedup

$\mathcal{C}$ $\bar{\mathcal{C}}$ $\mathcal{C} \cup \bar{\mathcal{C}}$




+

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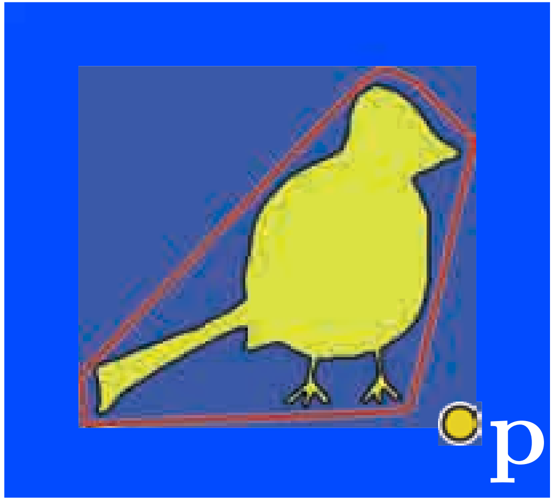
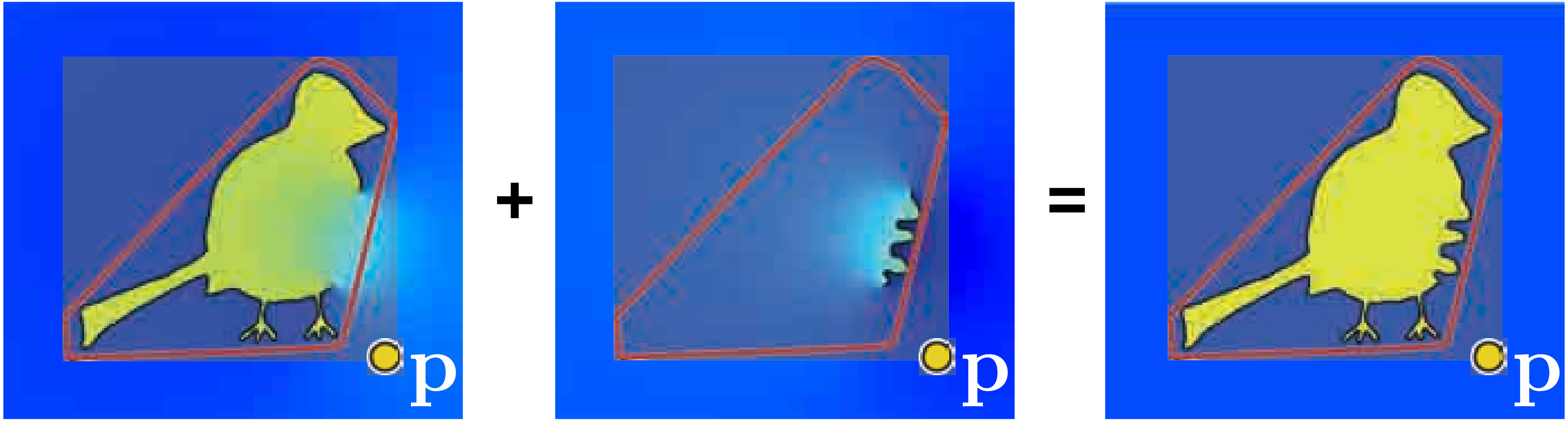



$$w_{\mathcal{C}}(\mathbf{p}) = -w_{\bar{\mathcal{C}}}(\mathbf{p})$$

# Interesting fact reveals asymptotic speedup

$\mathcal{C}$ $\qquad$ $\bar{\mathcal{C}}$ $\qquad$ $\mathcal{C} \cup \bar{\mathcal{C}}$

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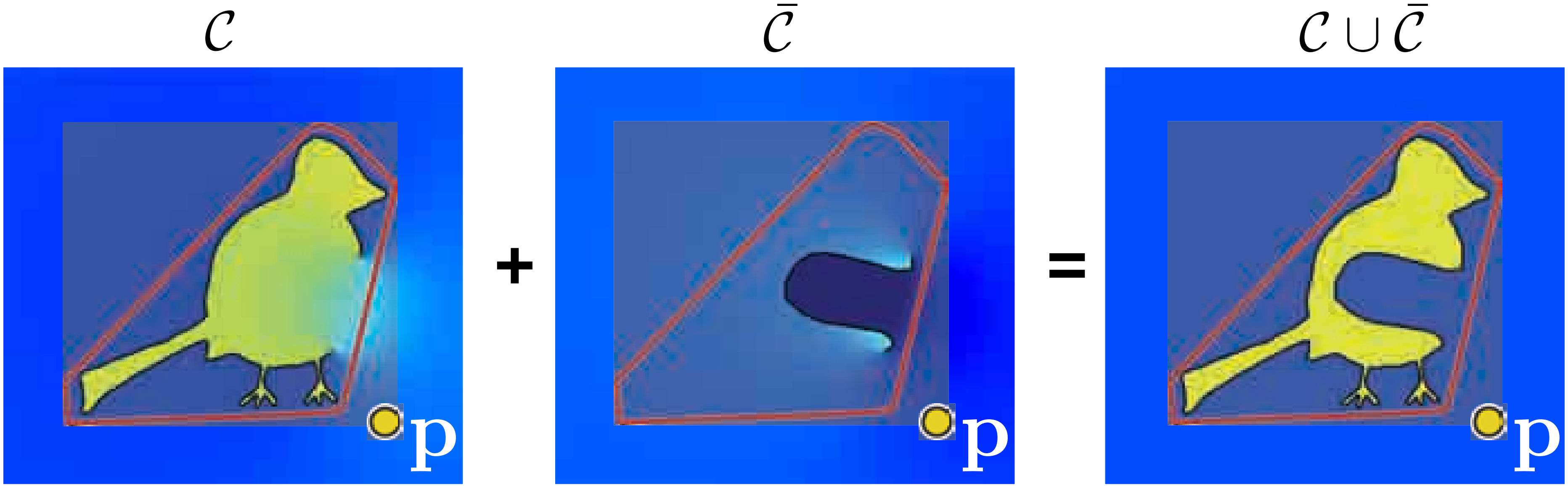
# Interesting fact reveals asymptotic speedup




$$w_{\mathcal{C}}(\mathbf{p}) = -w_{\bar{\mathcal{C}}}(\mathbf{p})$$

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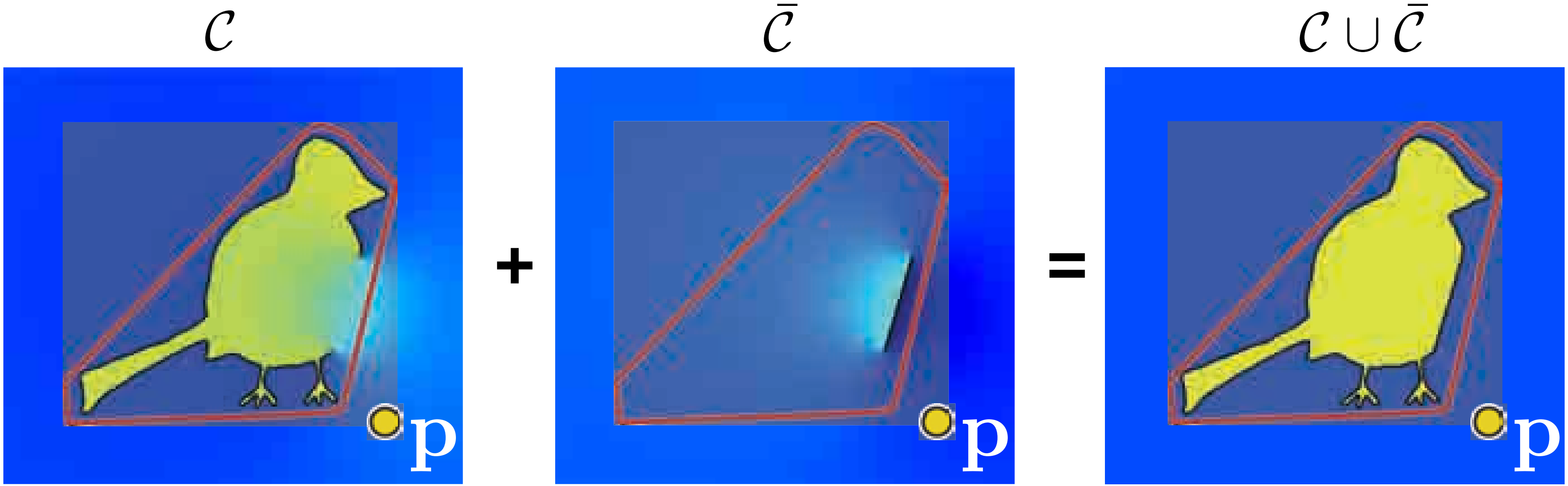


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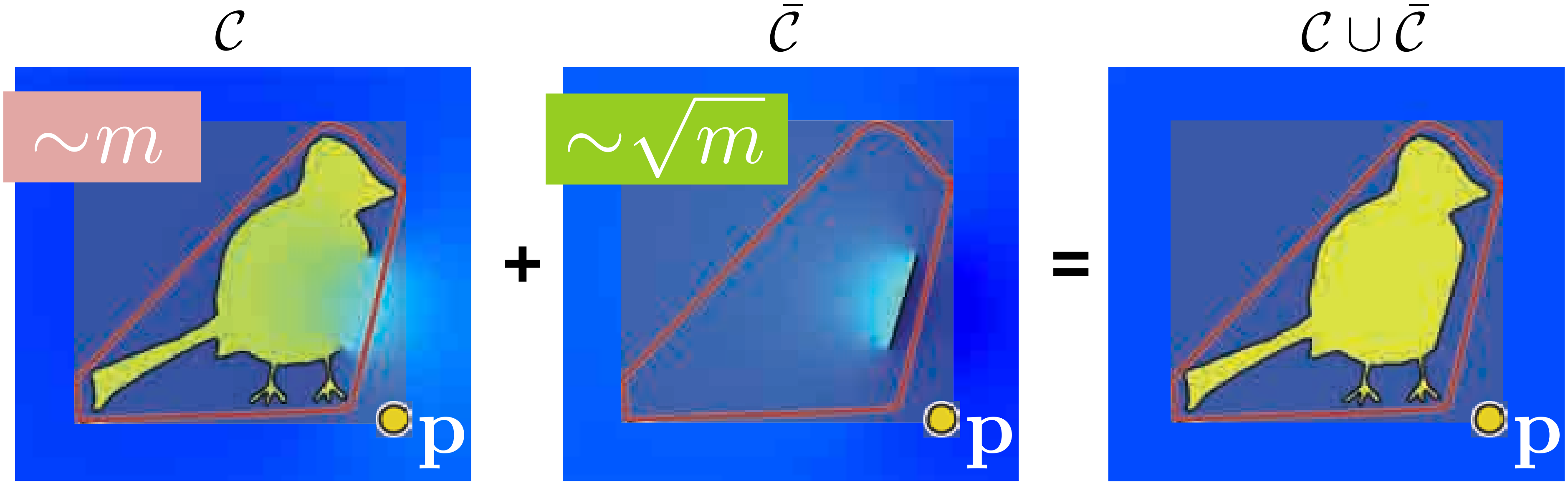
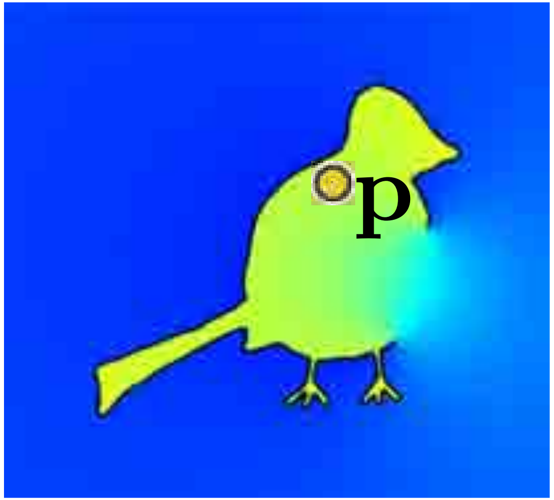
# Interesting fact reveals asymptotic speedup




$$w_{\mathcal{C}}(\mathbf{p}) = -w_{\bar{\mathcal{C}}}(\mathbf{p})$$

# Divide and conquer!



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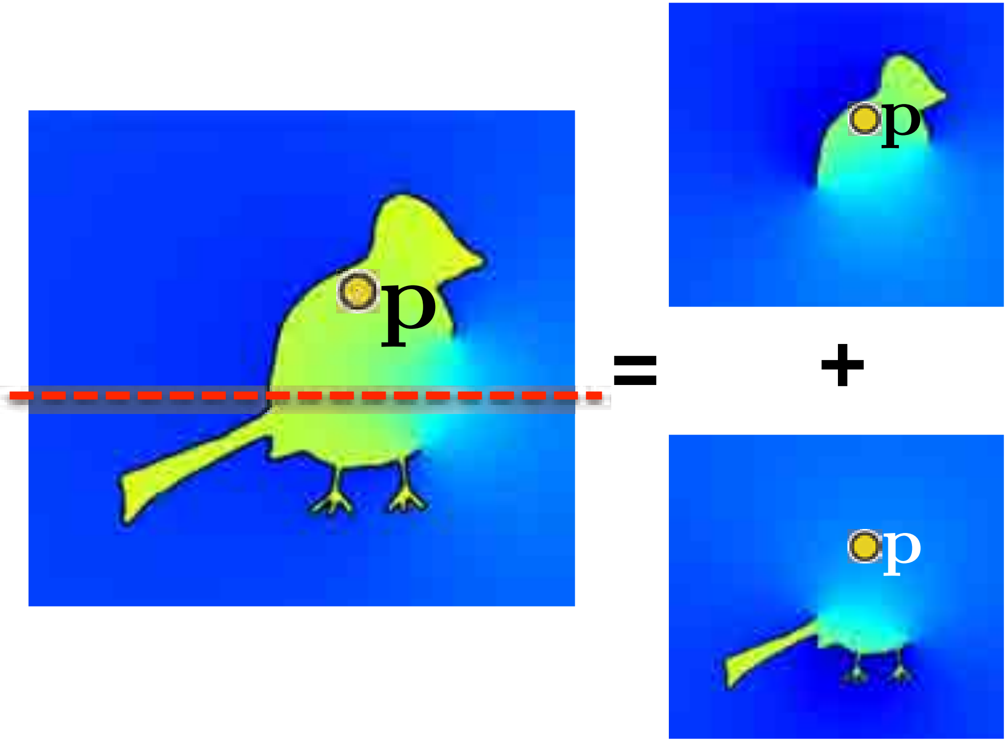

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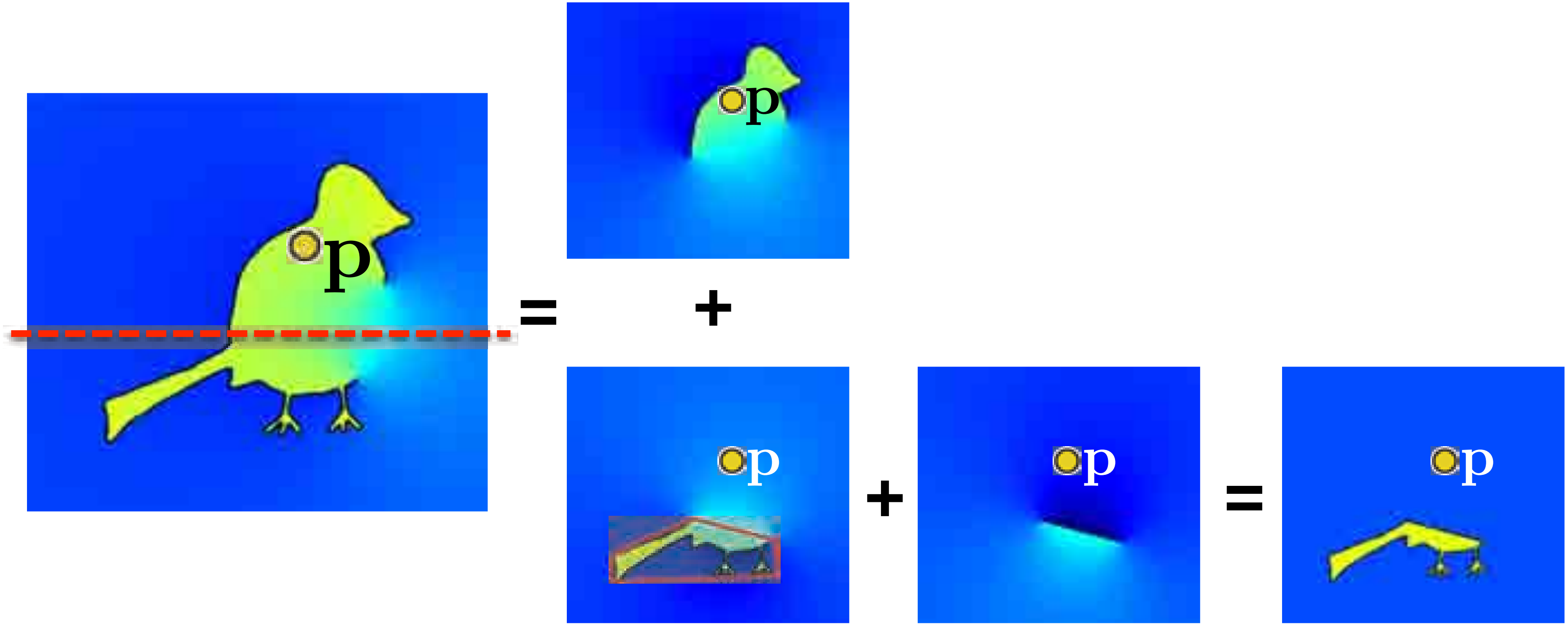

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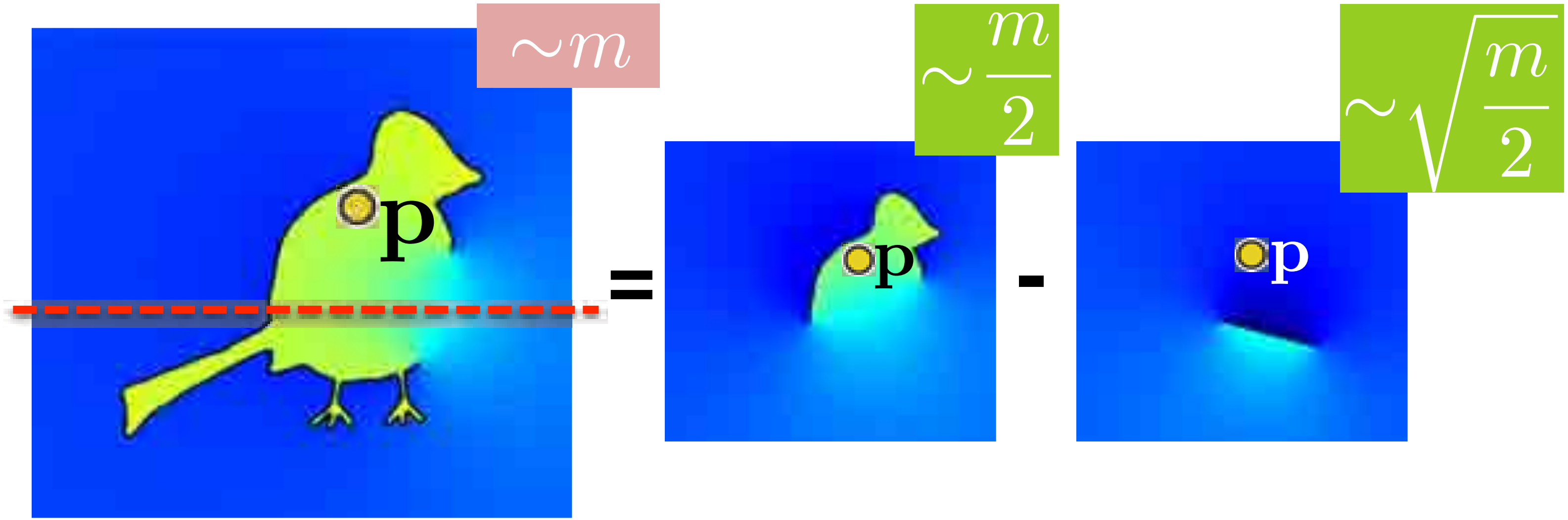
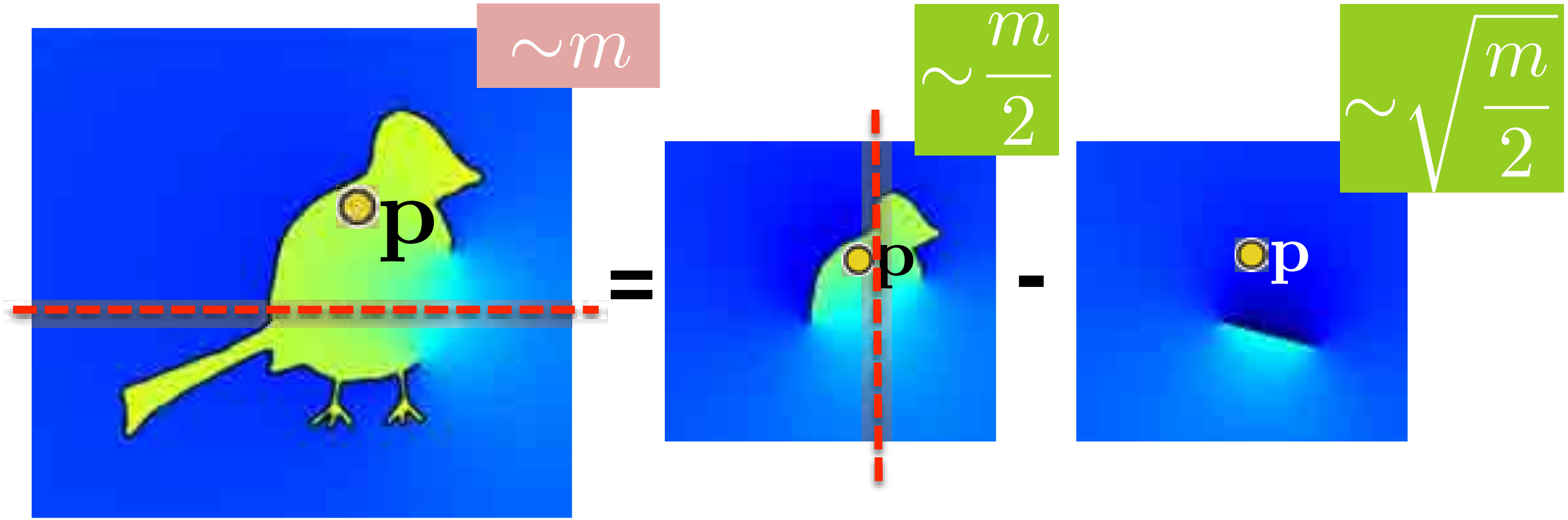
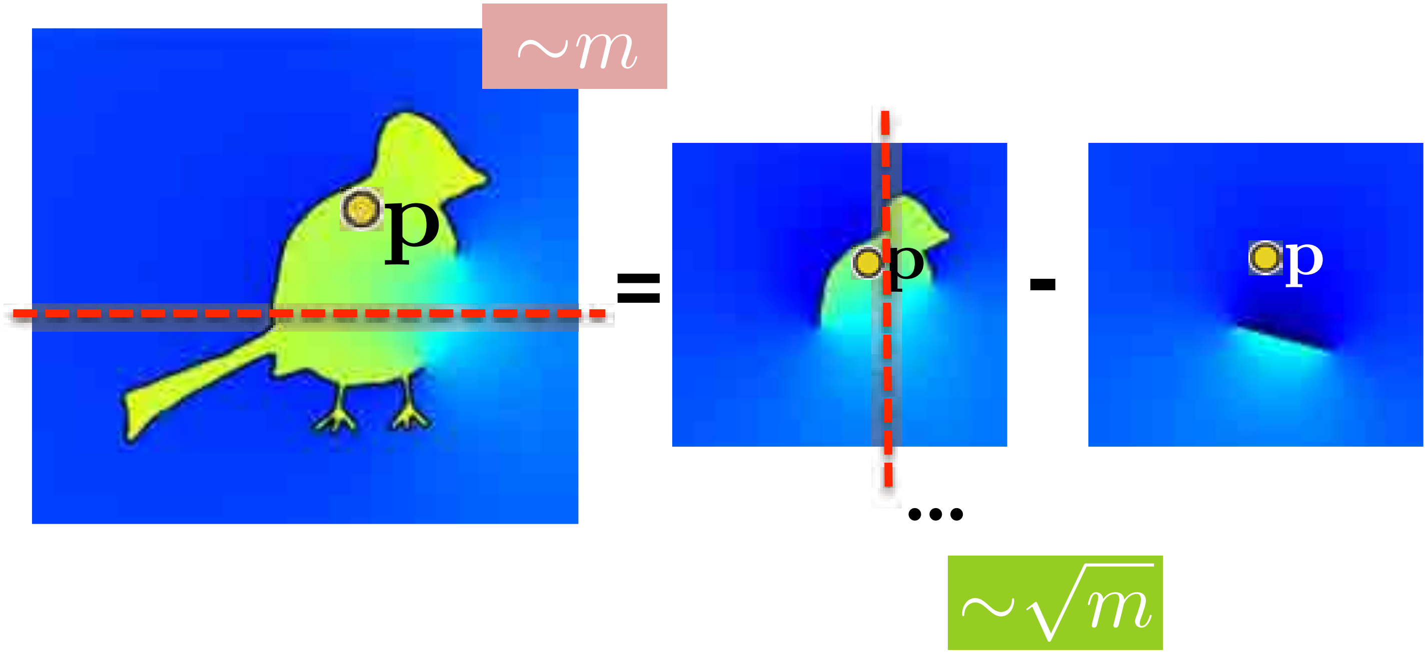


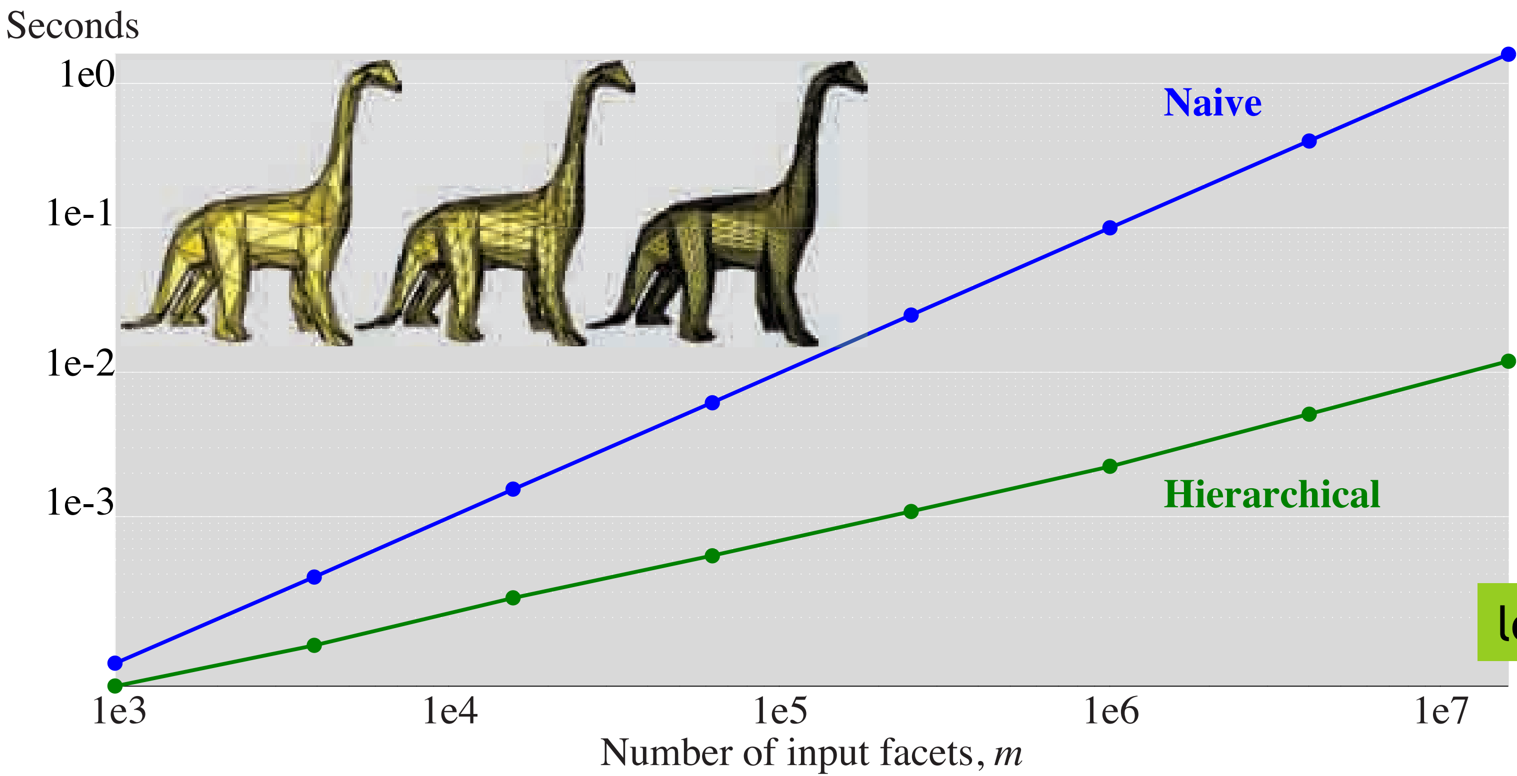
# Divide and conquer!



# Divide and conquer!

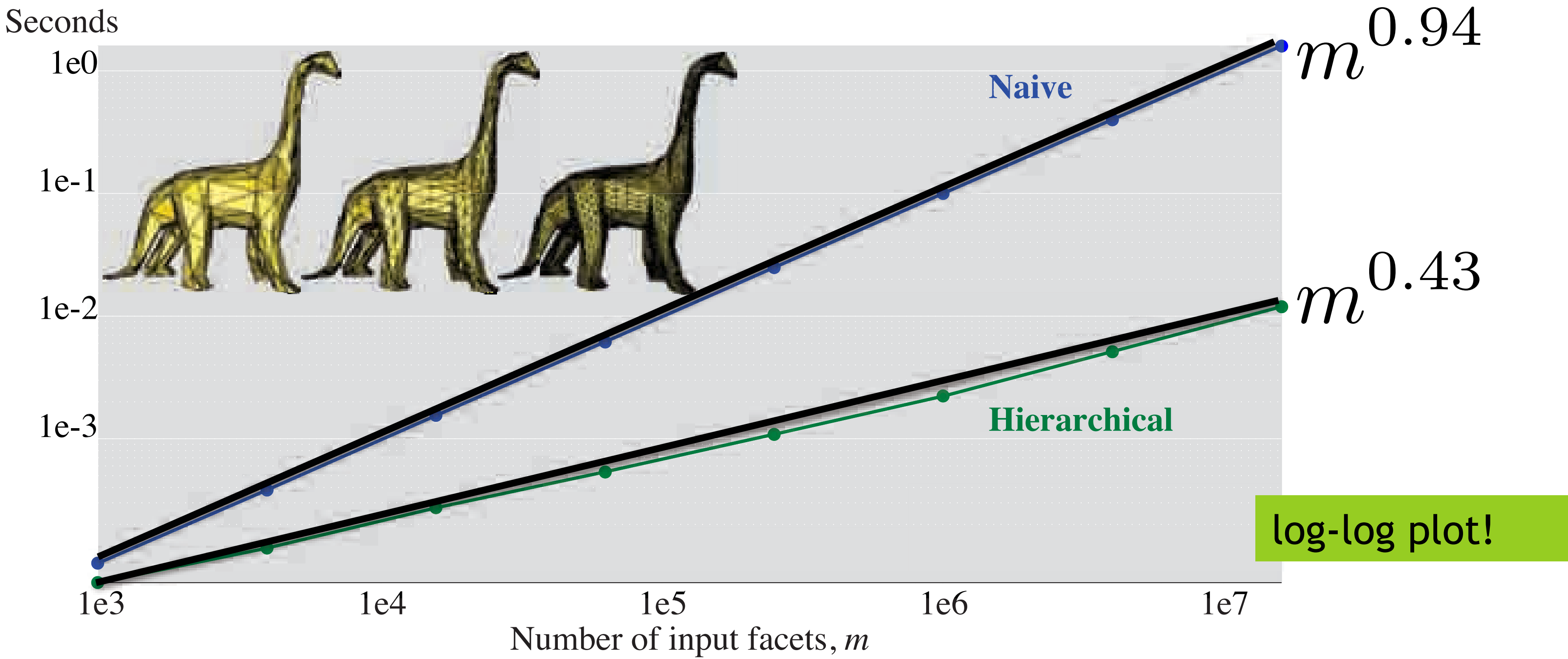


# Divide-and-conquer evaluation performs asymptotically better

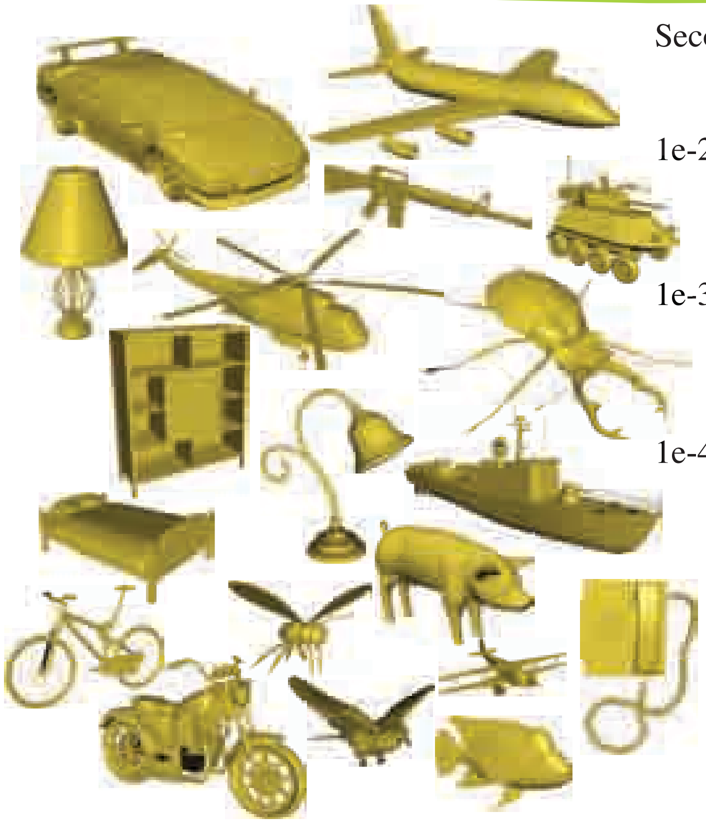


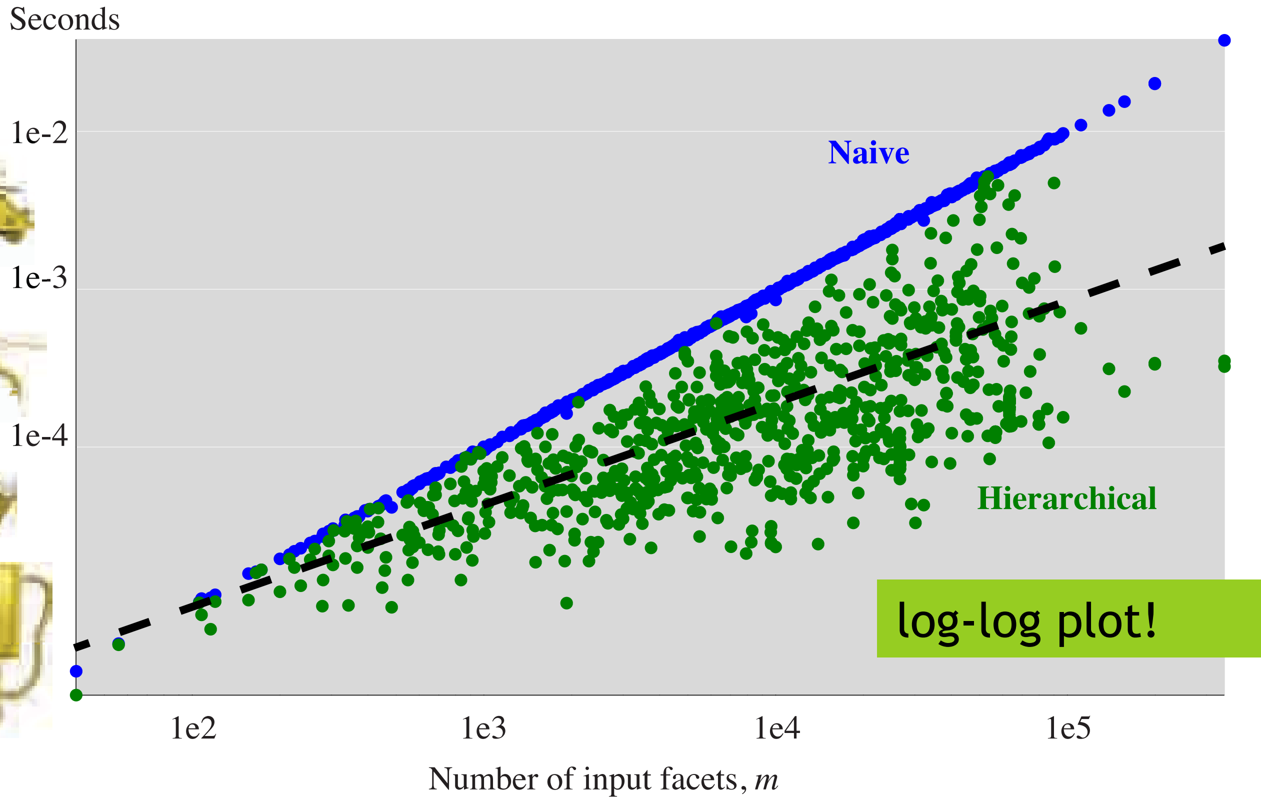

log-log plot!

# Divide-and-conquer evaluation performs asymptotically better



# Divide-and-conquer evaluation performs asymptotically better





# Idea: mesh entire convex hull, segment inside tets from outside ones

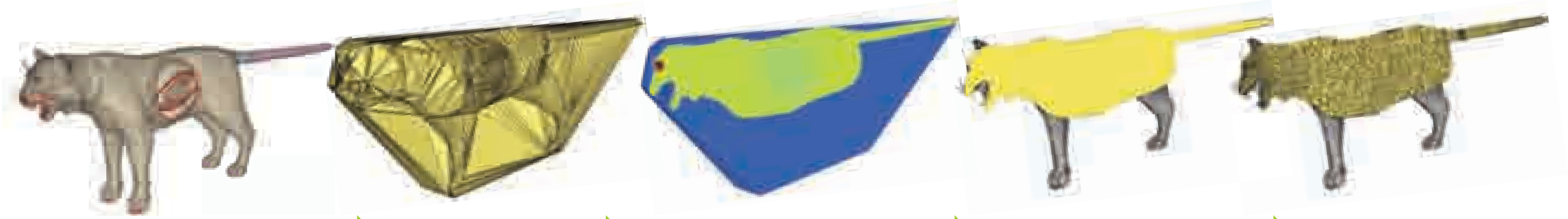
input triangles → CDT → measure of insideness → segment CDT → refine, etc.

# Segmentation is a labeling problem, labels should agree with w.n.

input triangles → CDT → measure of inside/outside | segment CDT | efine, etc.

graphcut energy optimization with nonlinear coherency term
+ optional facet or surface-manifoldness constraints

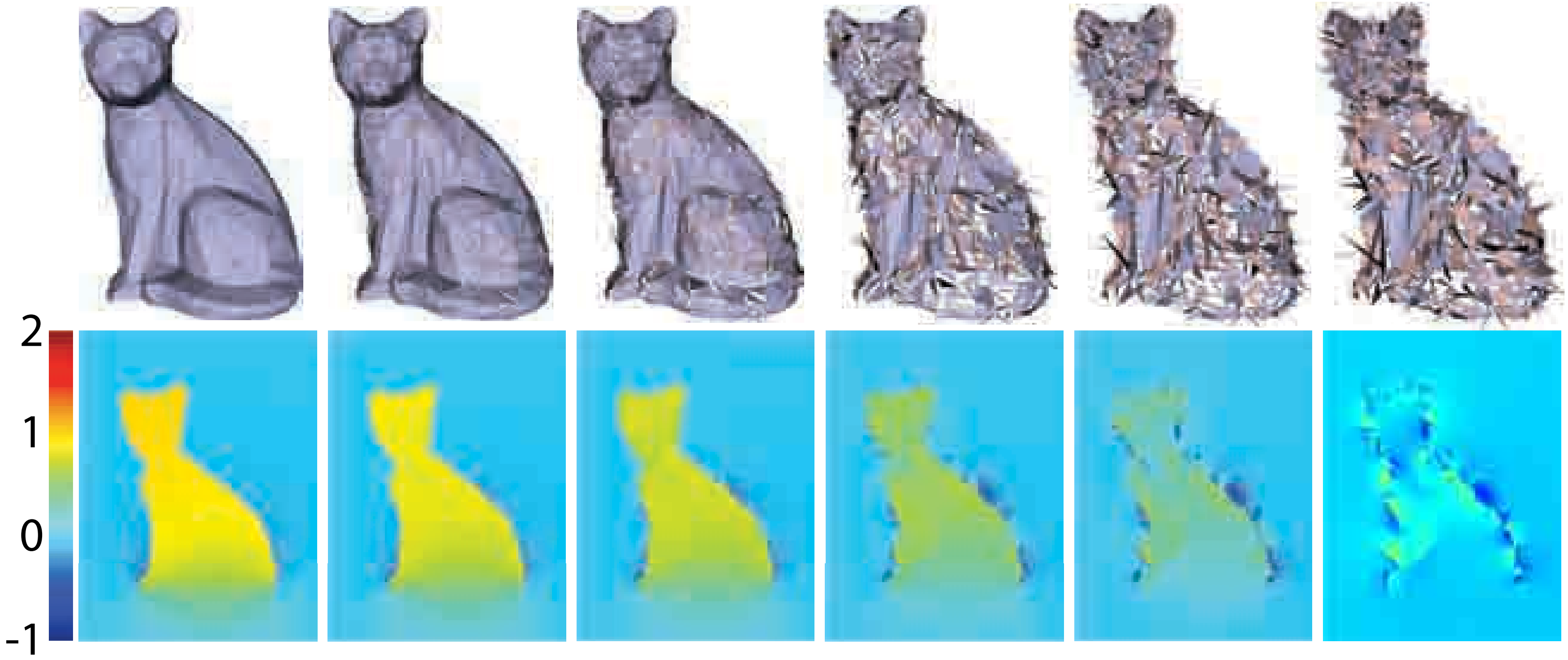
# Preprocessing and meshing convex hull dominates runtime



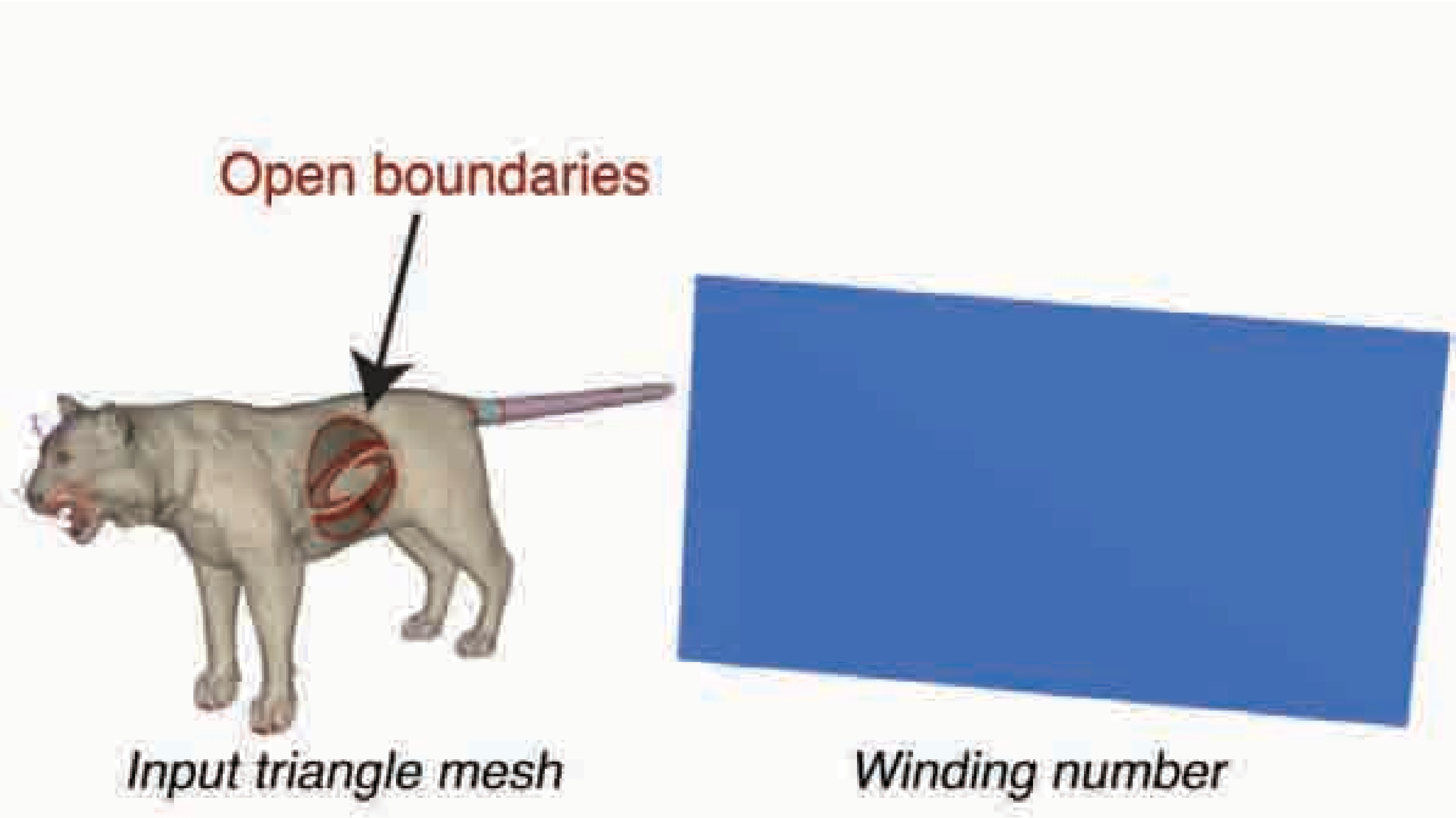
input triangles → CDT → measure of insideness → segment CDT → refine, etc.

| | | | |
|---|---|---|---|
| *60K facet mesh* | ~22 secs | ~10 secs | ~0.1 secs |

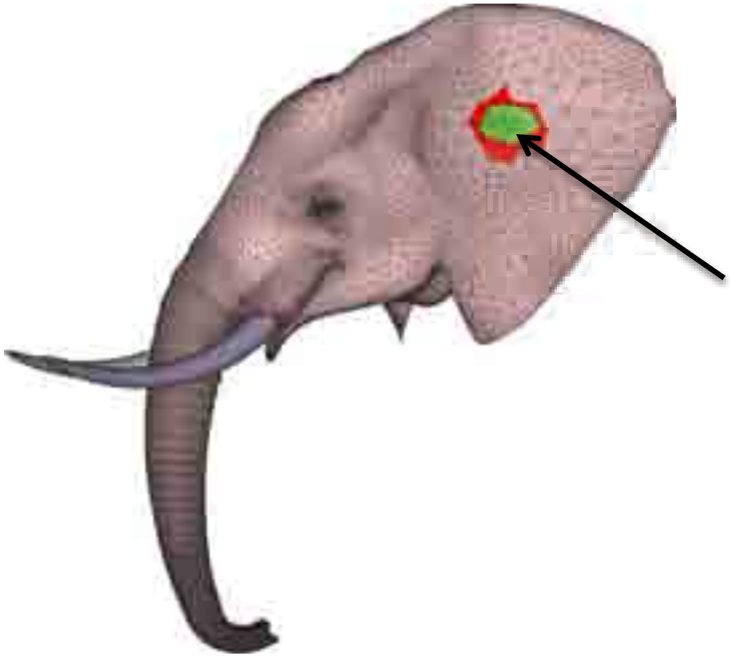
# Winding number degrades gracefully



# CDT maintains small features



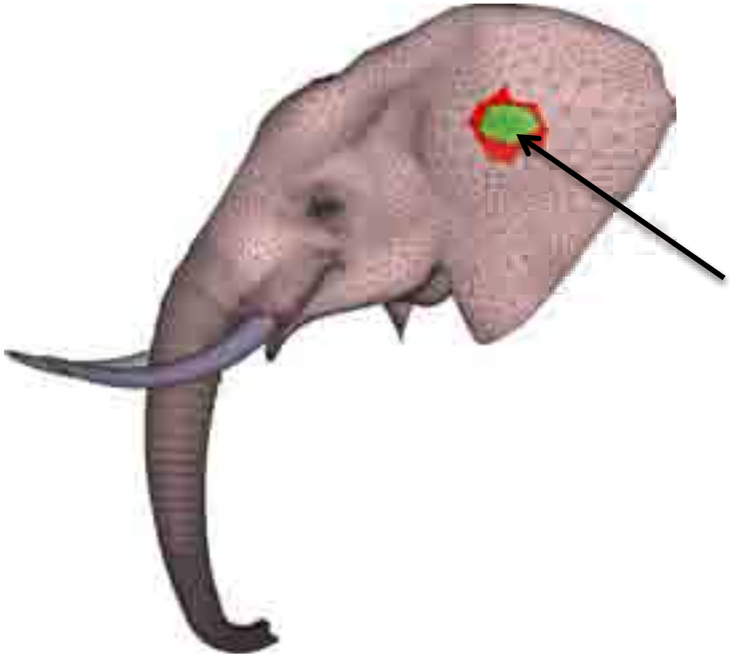
# We rely heavily on orientation



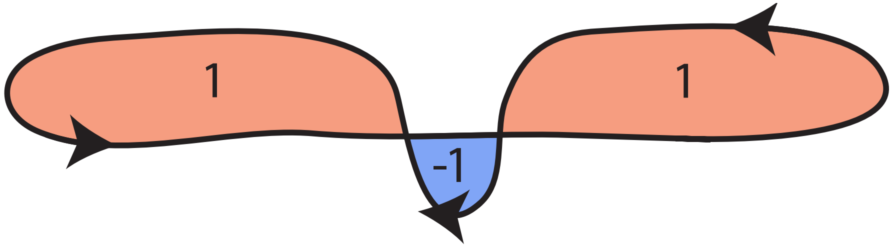
backside of ear penetrates front
(inside-out region)

input mesh

# We rely heavily on orientation



input mesh




backside of ear penetrates front
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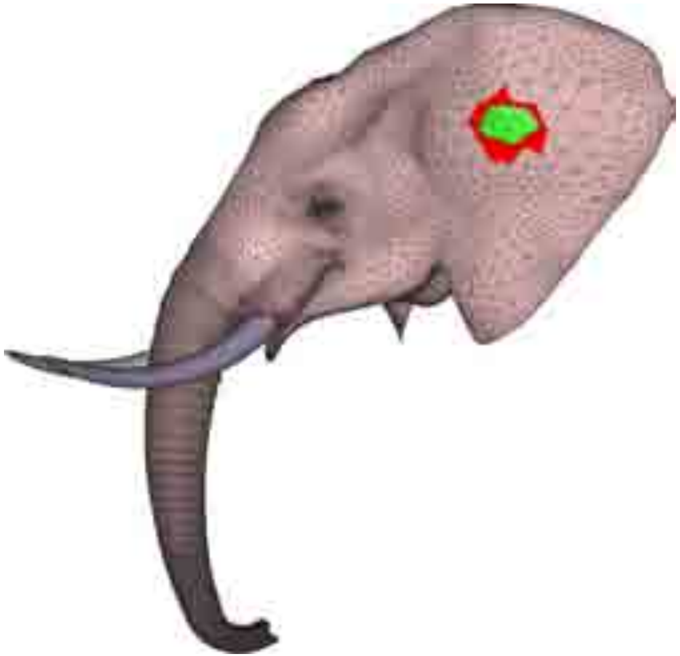
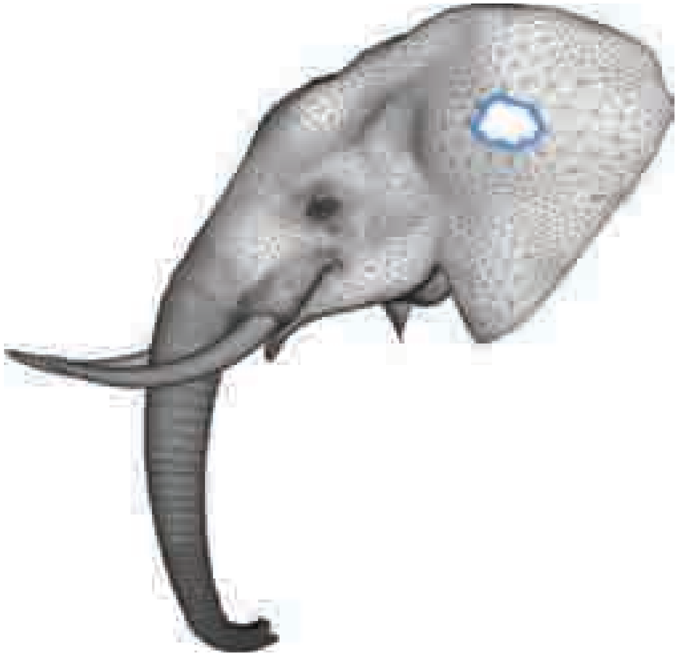


input mesh

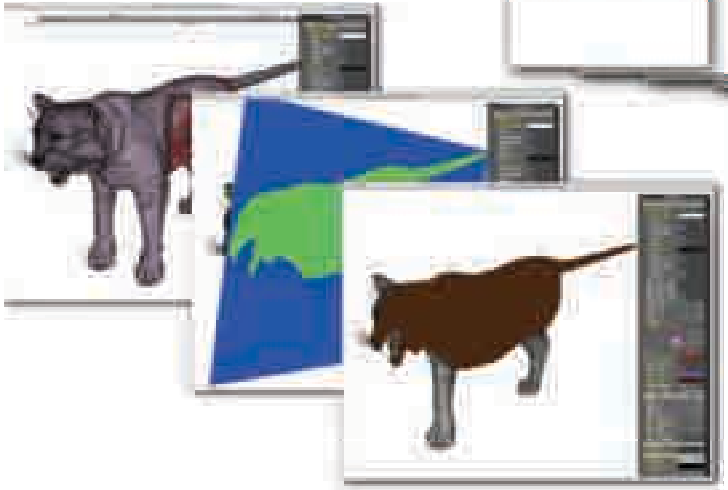


our output

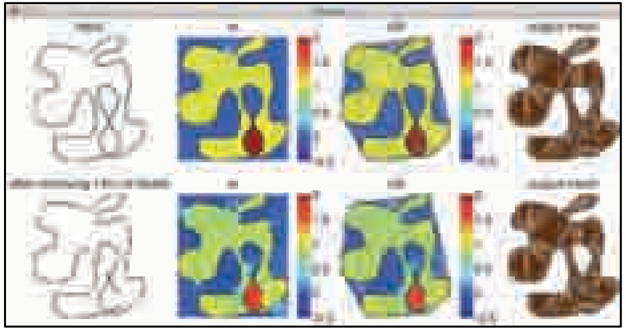
# Brings a new level of robustness to volume meshing for a variety of shapes

high res images and mesh files

3D demo



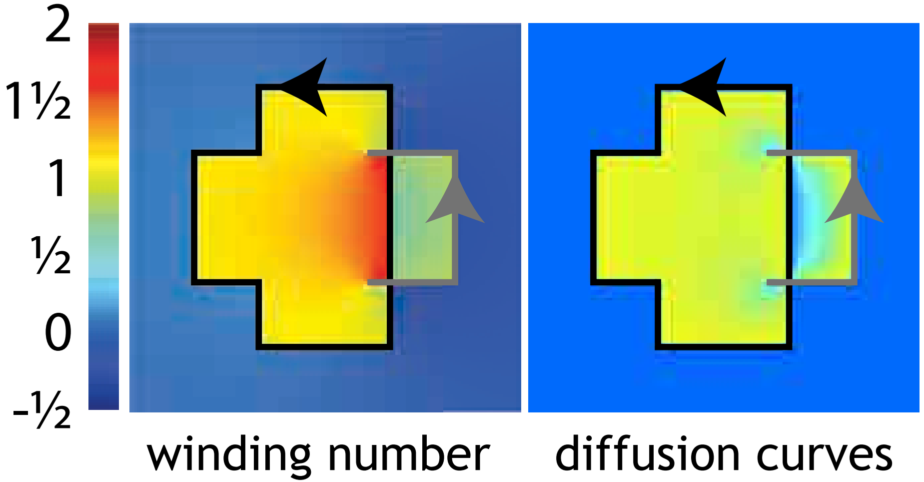
2D demo



**http://goo.gl/m0oL9**

# Future work

- Even faster approximation
- Relationship to:
  diffusion curves,
  Mean Value Coordinates,
  etc.



# Acknowledgements

Pierre Alliez, Ilya Baran, Leo Guibas, Fabian Hahn, James O'Brien, Daniele Panozzo, Leonardo Koller Sacht, Alexander Sorkine-Hornung, Josef Pelikan, Kenshi Takayama, Kaan Yücer

Marco Attene for MESHFIX

Hang Si for TETGEN

This work was supported in part by the ERC grant iModel (StG-2012-306877), by an SNF award 200021 137879 and the Intel Doctoral Fellowship.

# Robust Inside-Outside Segmentation using Generalized Winding Numbers

http://igl.ethz.ch/projects/winding-number/
(paper, code, video)

Alec Jacobson
jacobson@inf.ethz.ch
Ladislav Kavan
Olga Sorkine-Hornung

INTERACTIVE GEOMETRY LAB

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

October 9, 2013

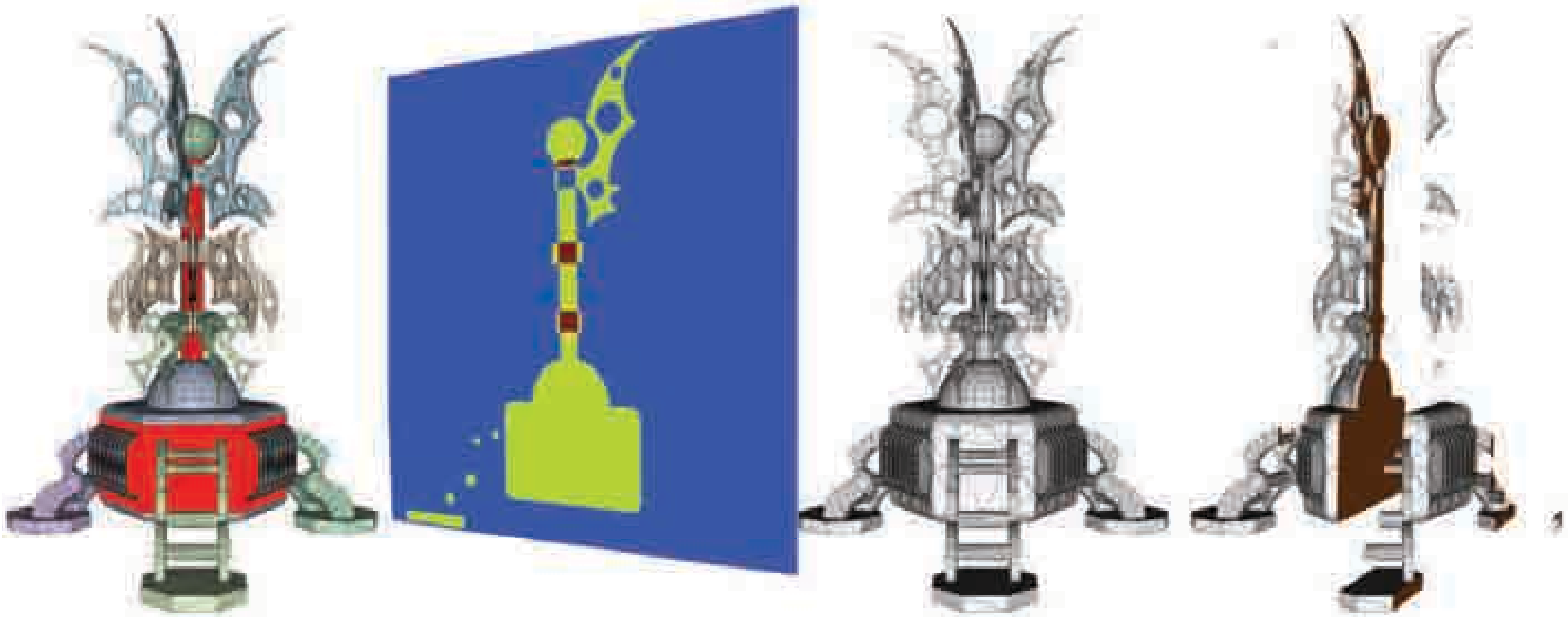
# Additional material

# Surface processing is distinct from volumetric

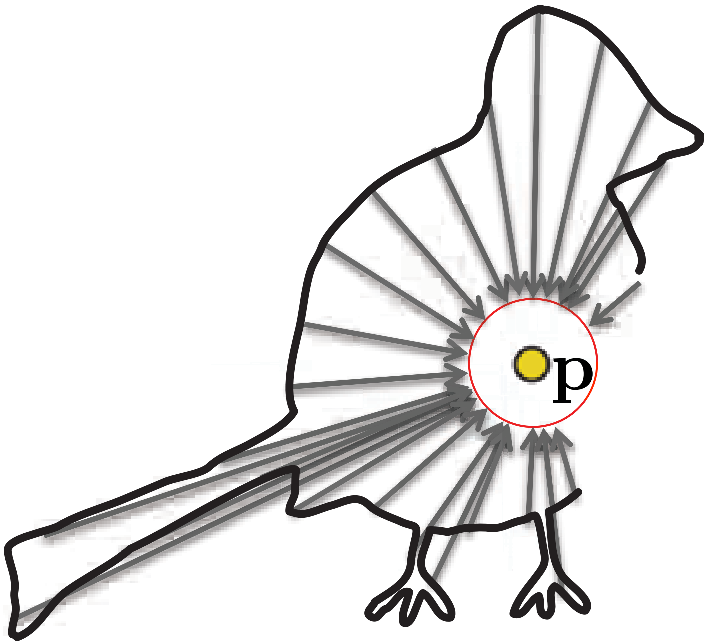
surface distance

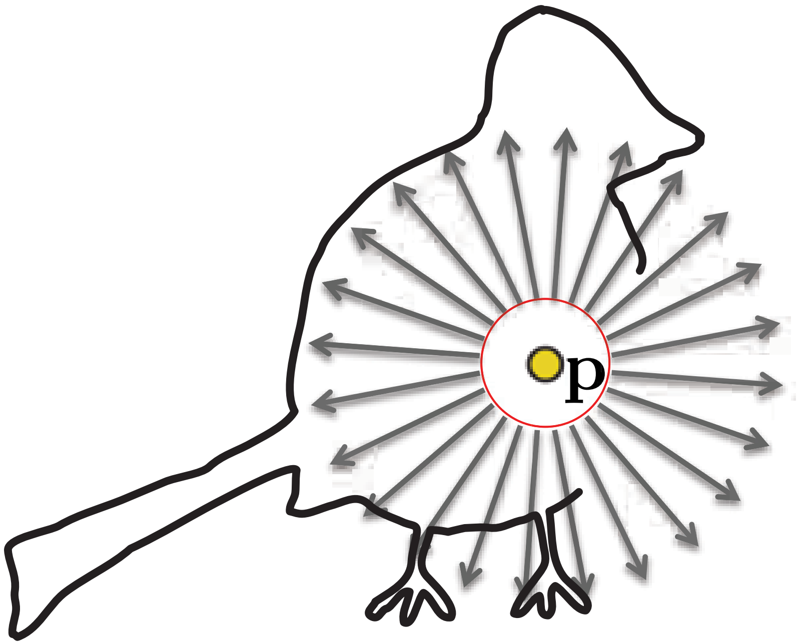
volumetric distance

# Brings a new level of robustness to volume meshing for a variety of shapes



# We rasterize the winding number, rather than ray cast



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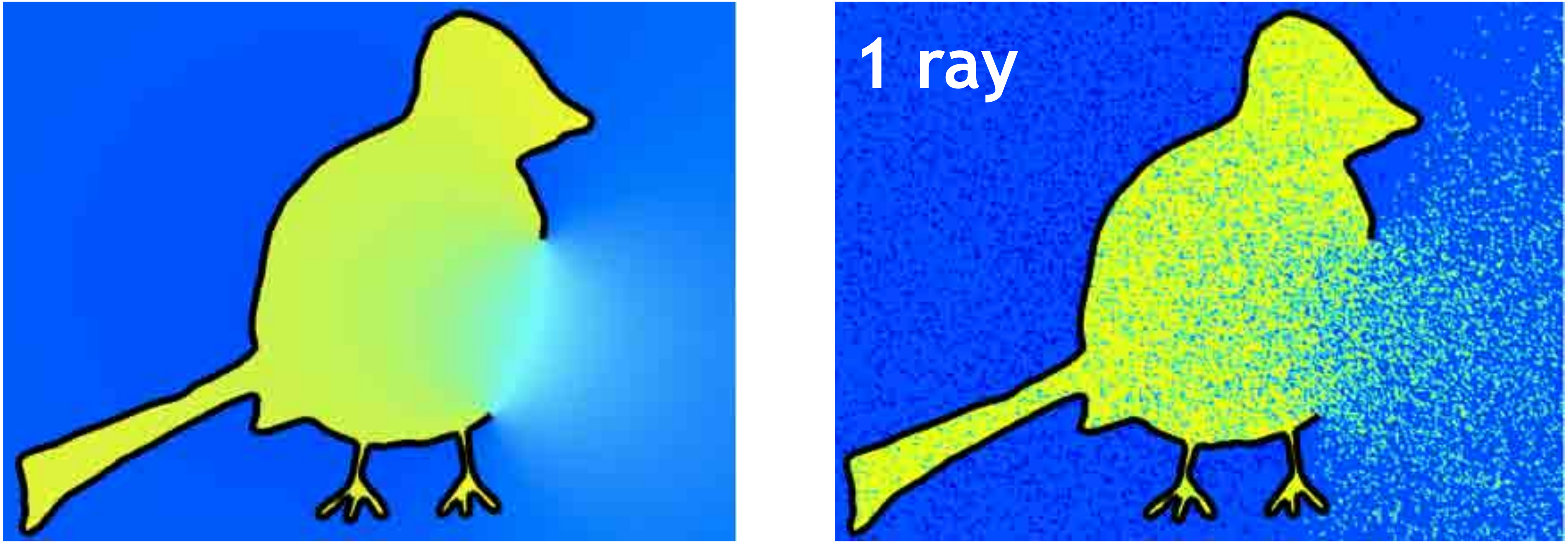

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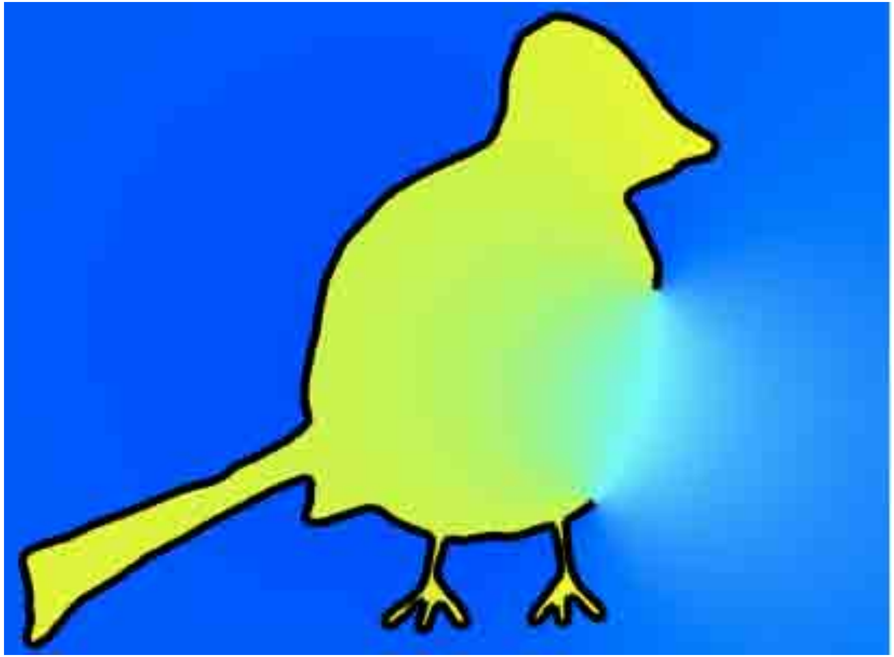

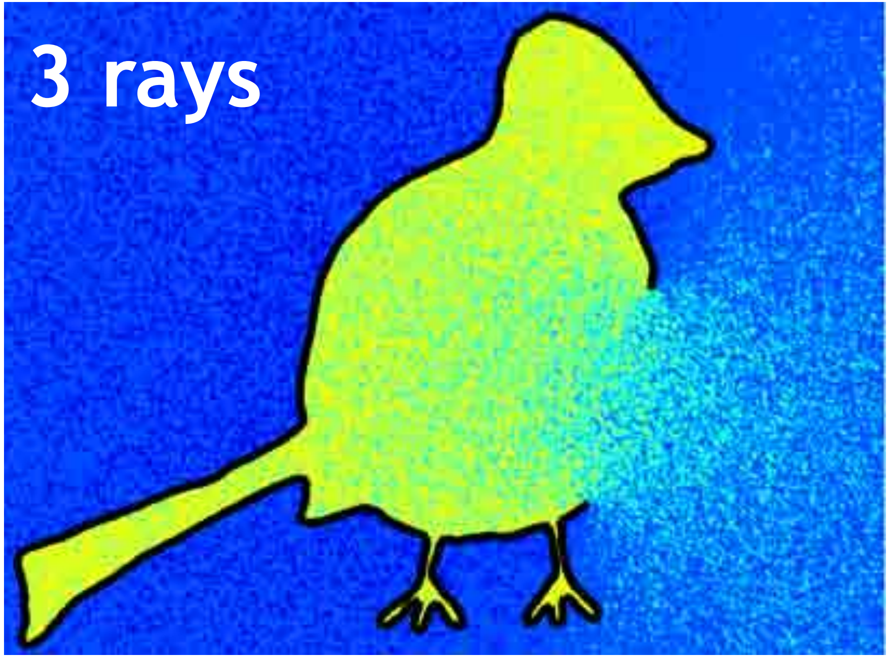

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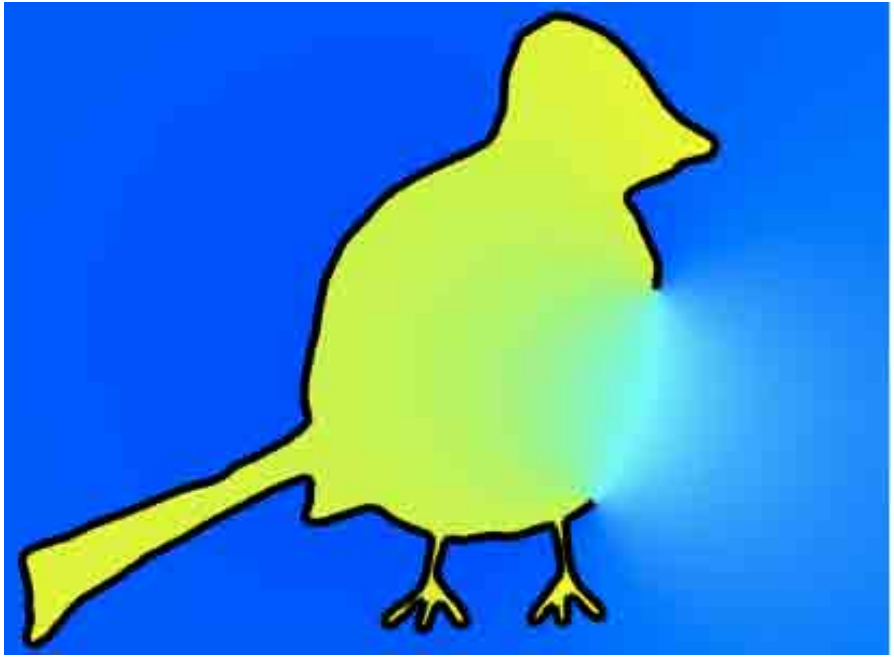

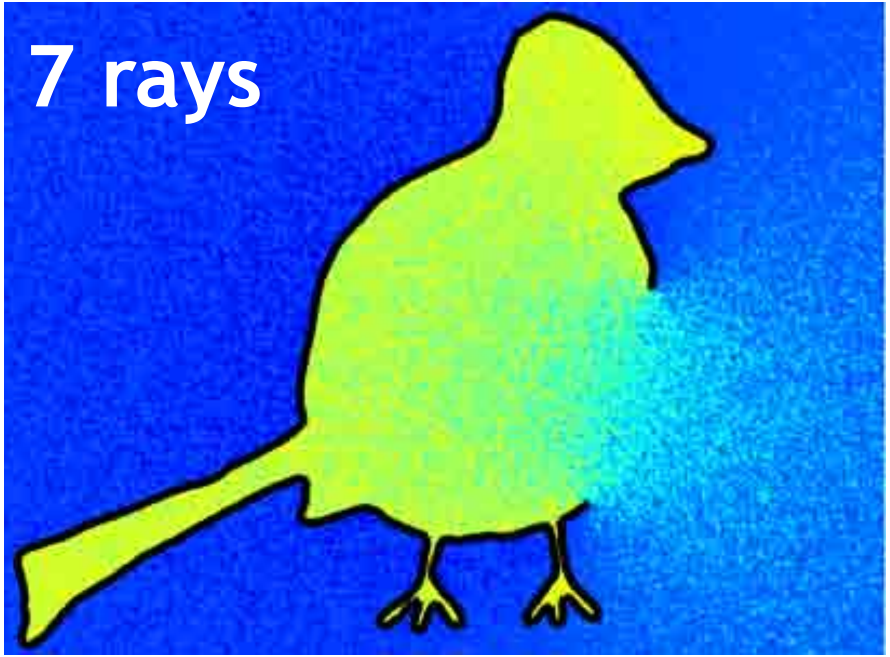

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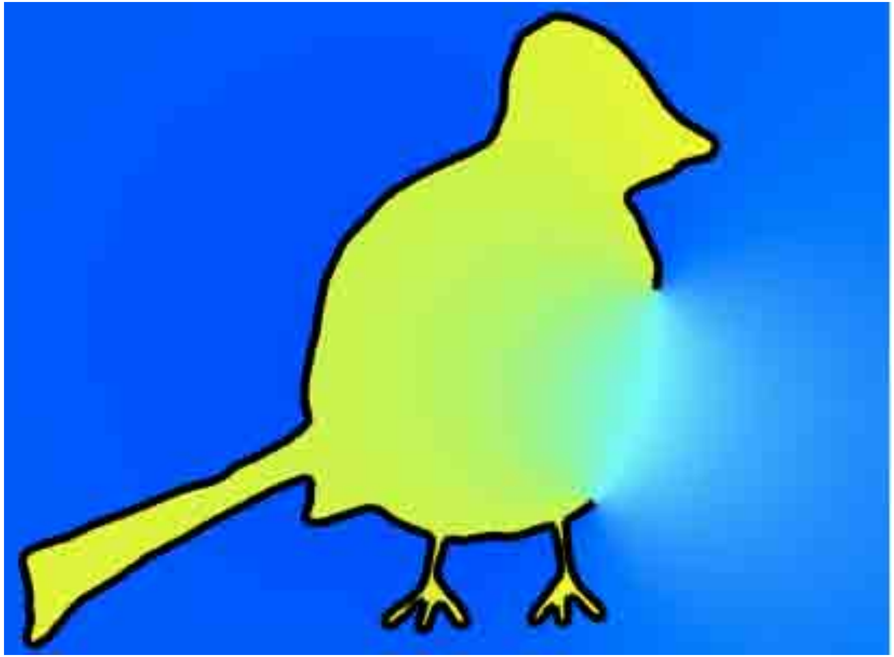

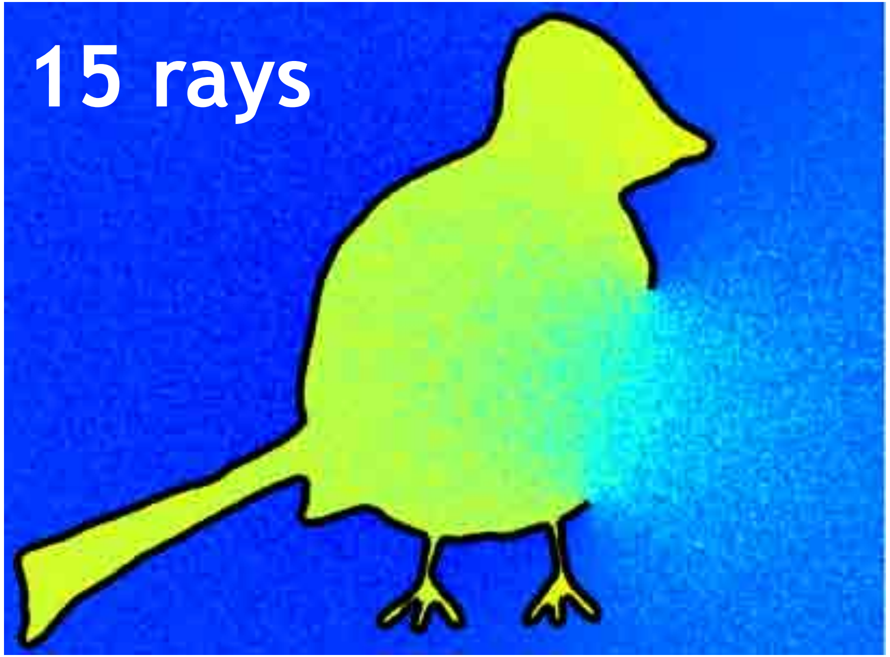

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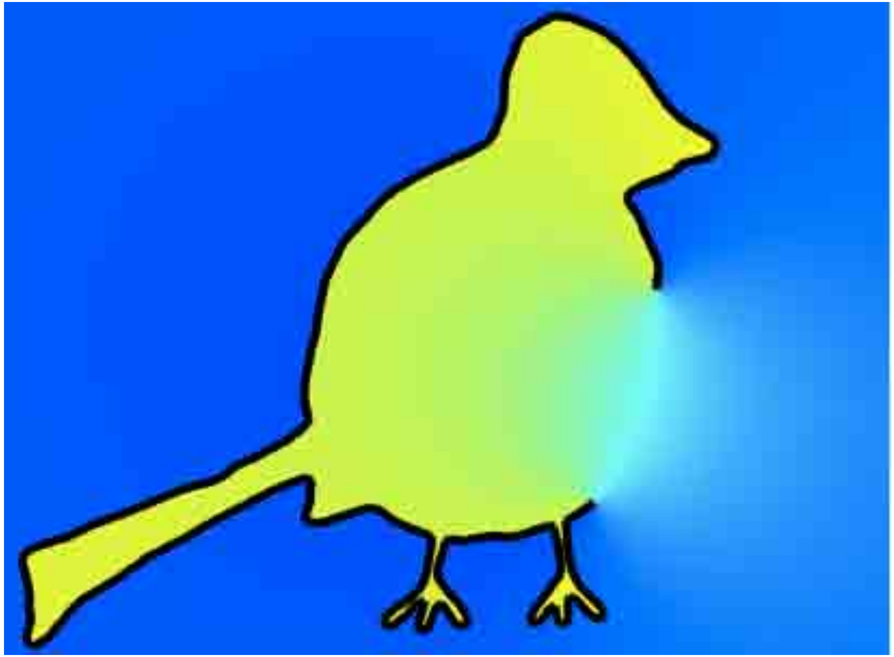

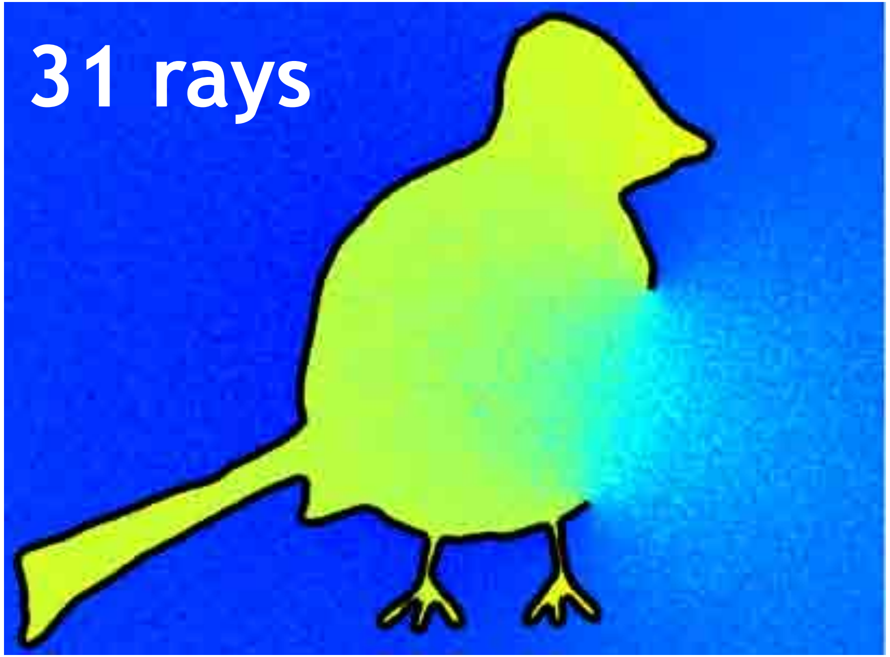

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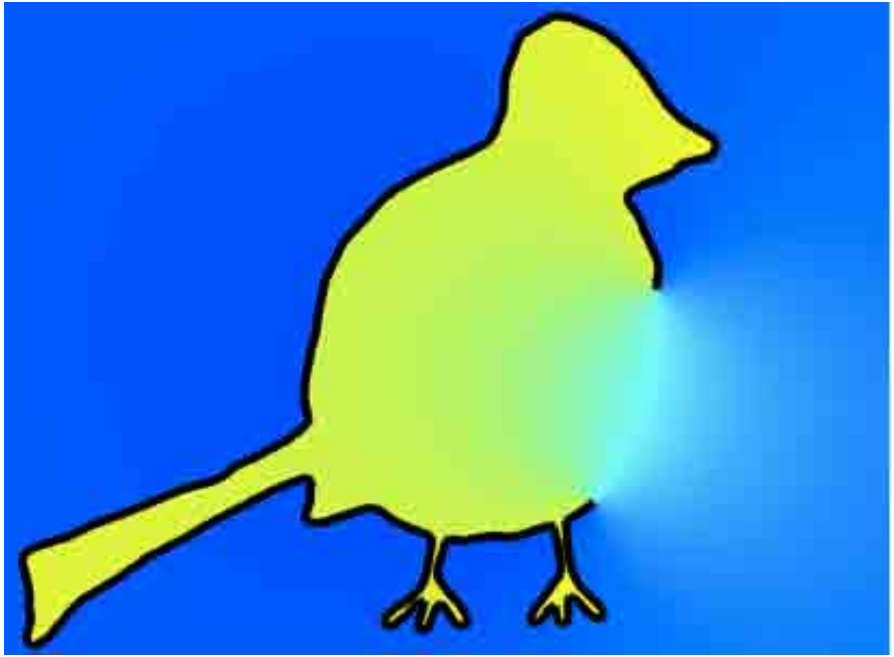

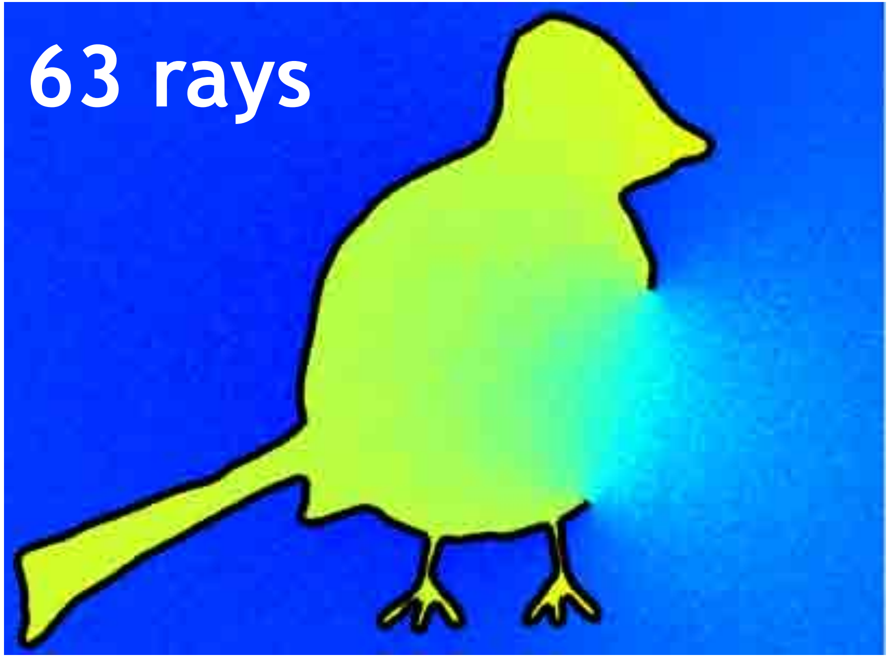

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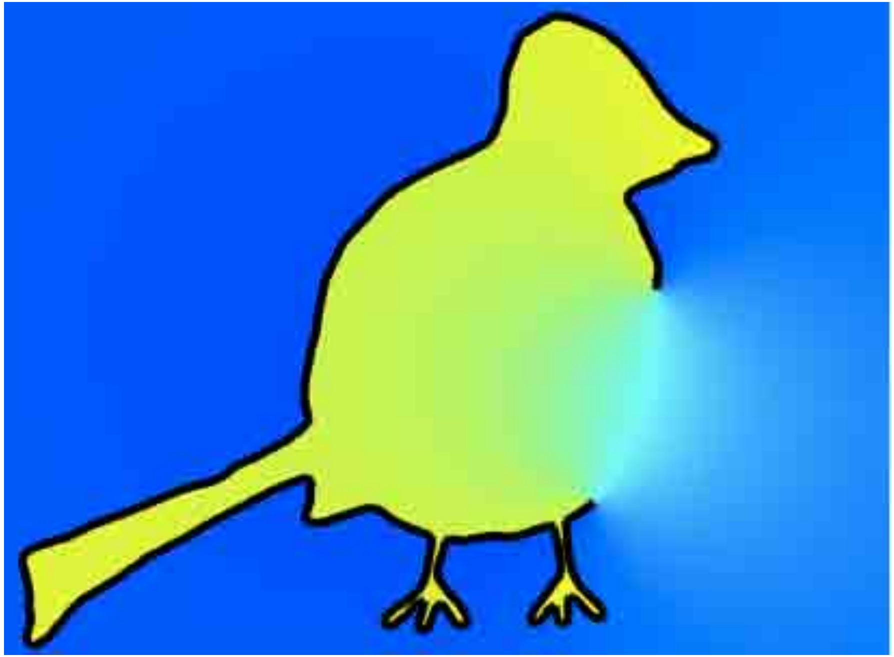

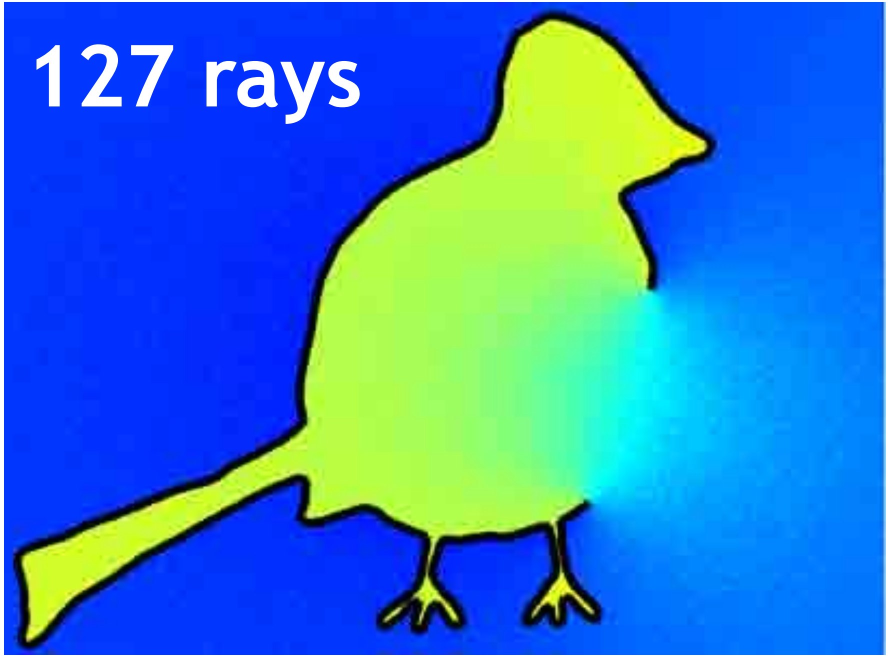

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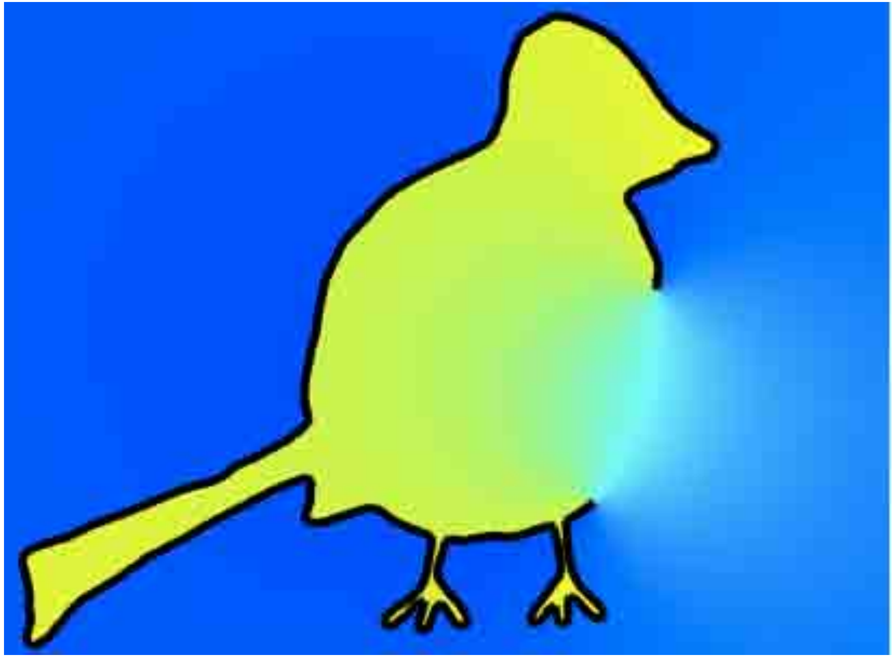

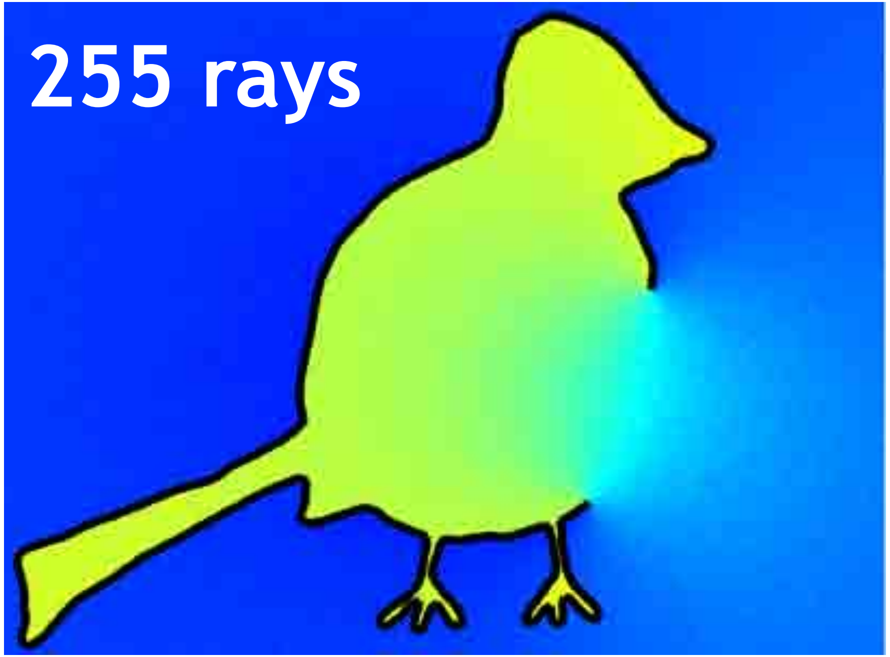

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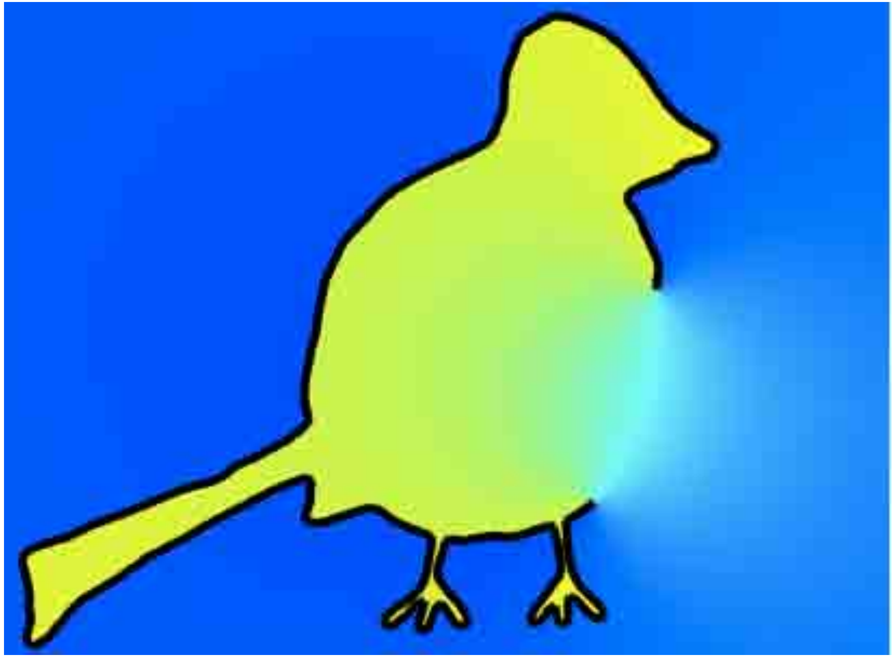

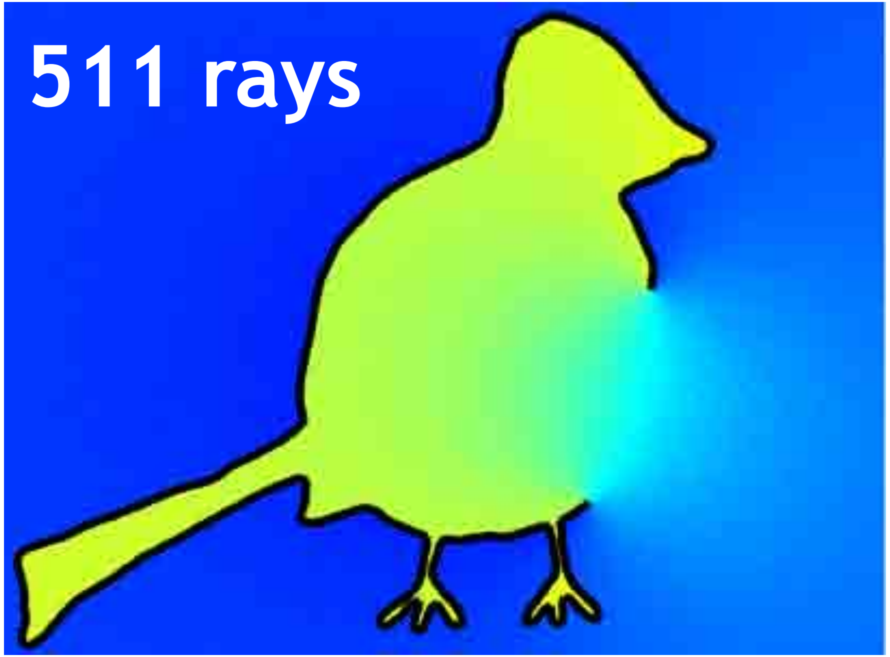

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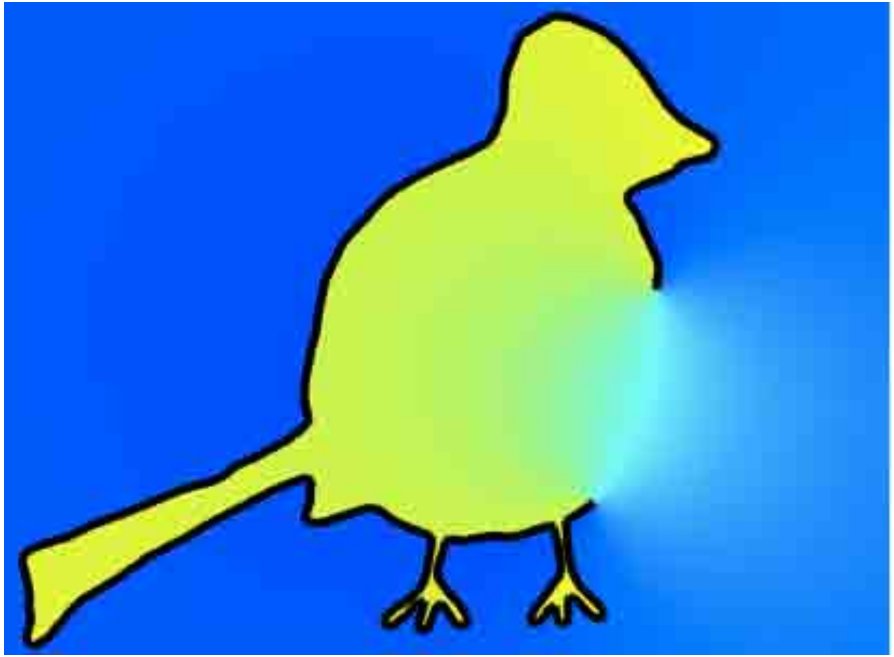

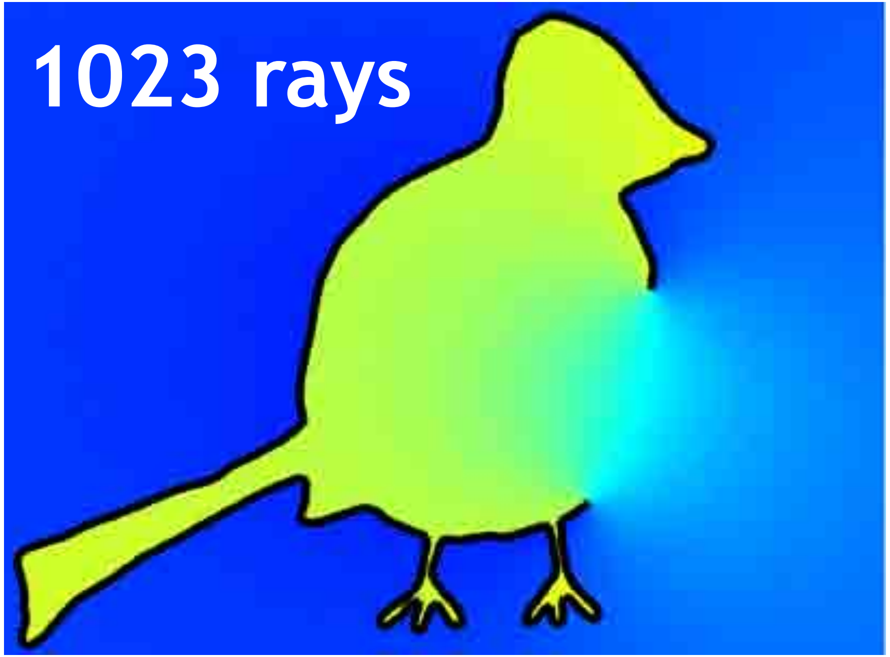

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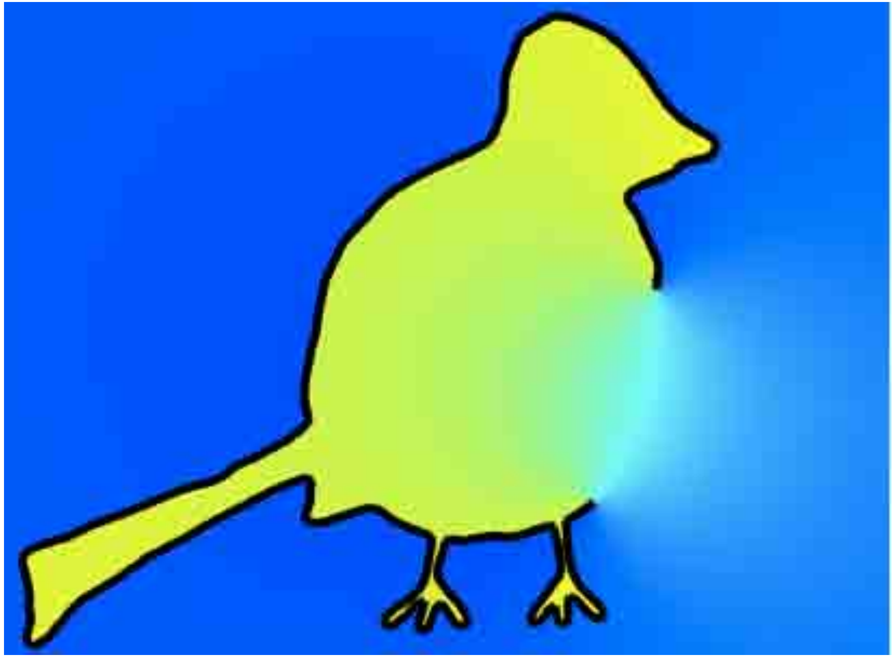
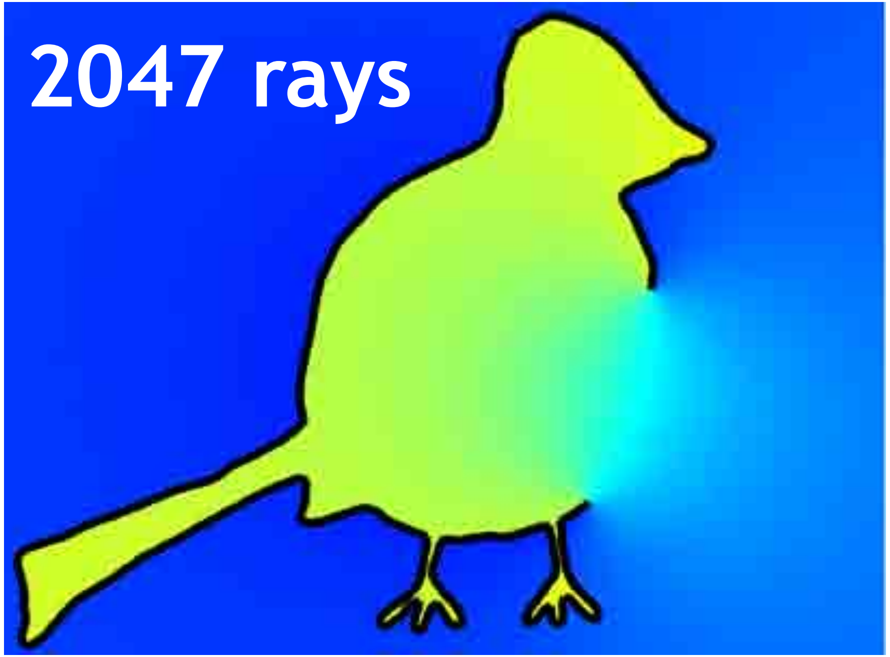
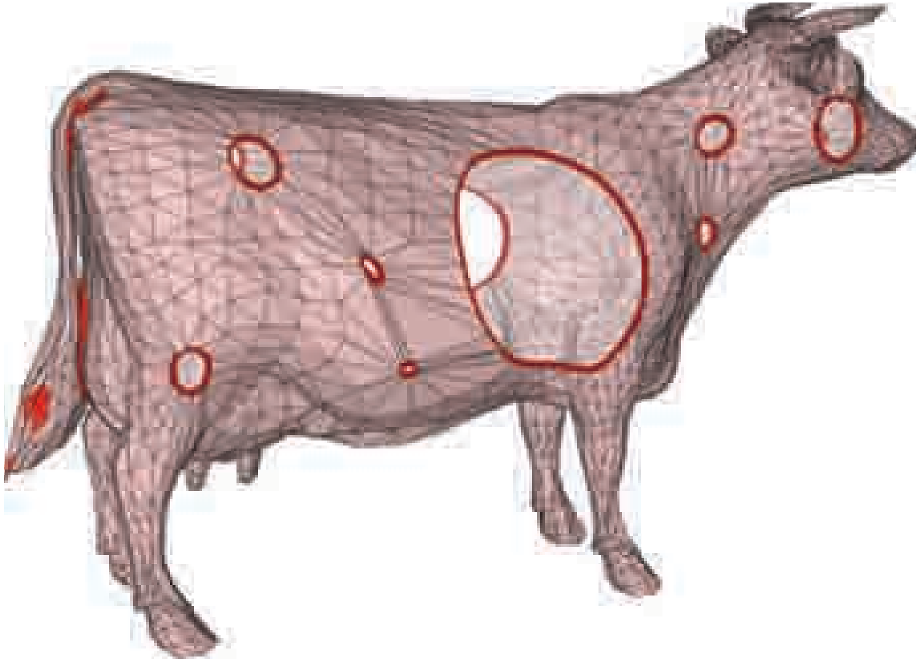




# Surface cleanup methods modify the input too much



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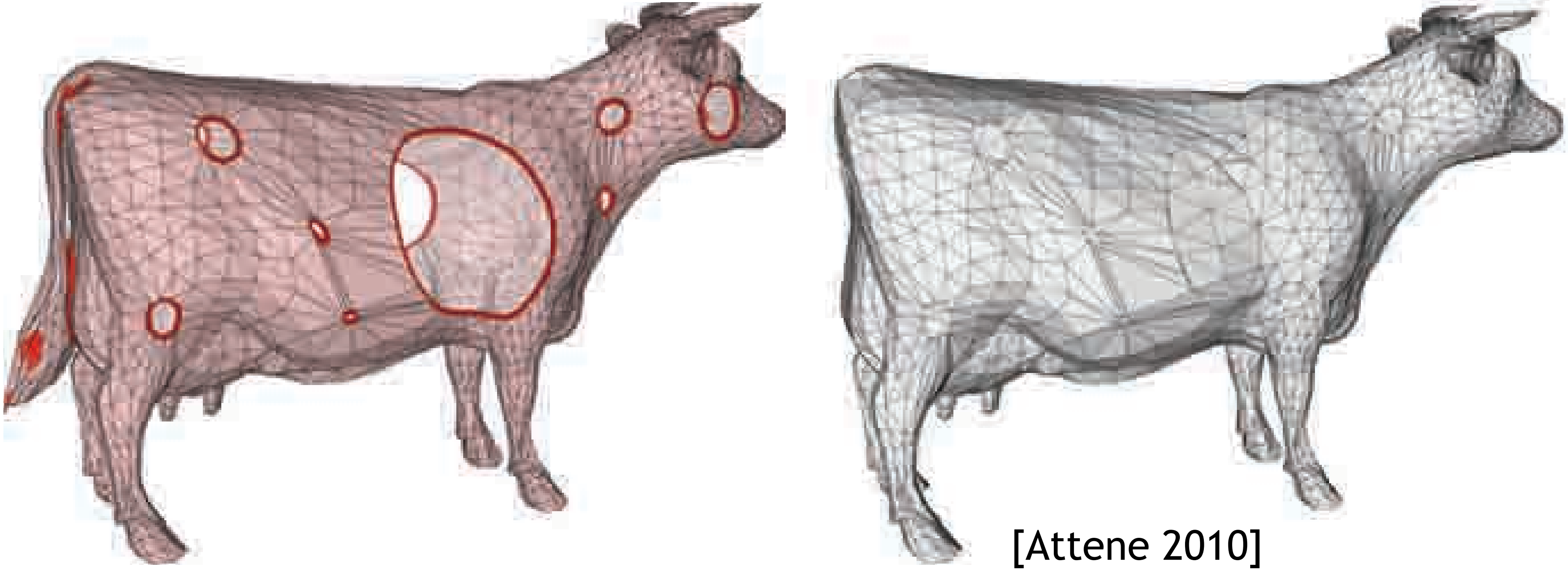
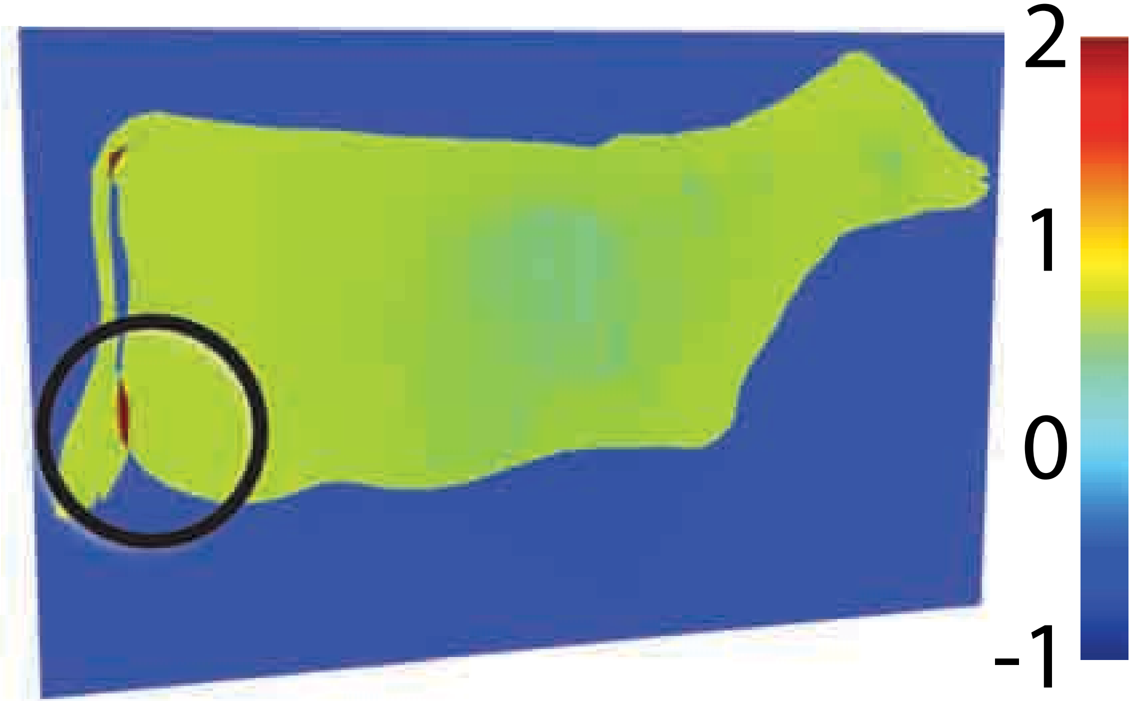


[Attene 2010]

# Winding number tells more than just inside: *how many times inside*



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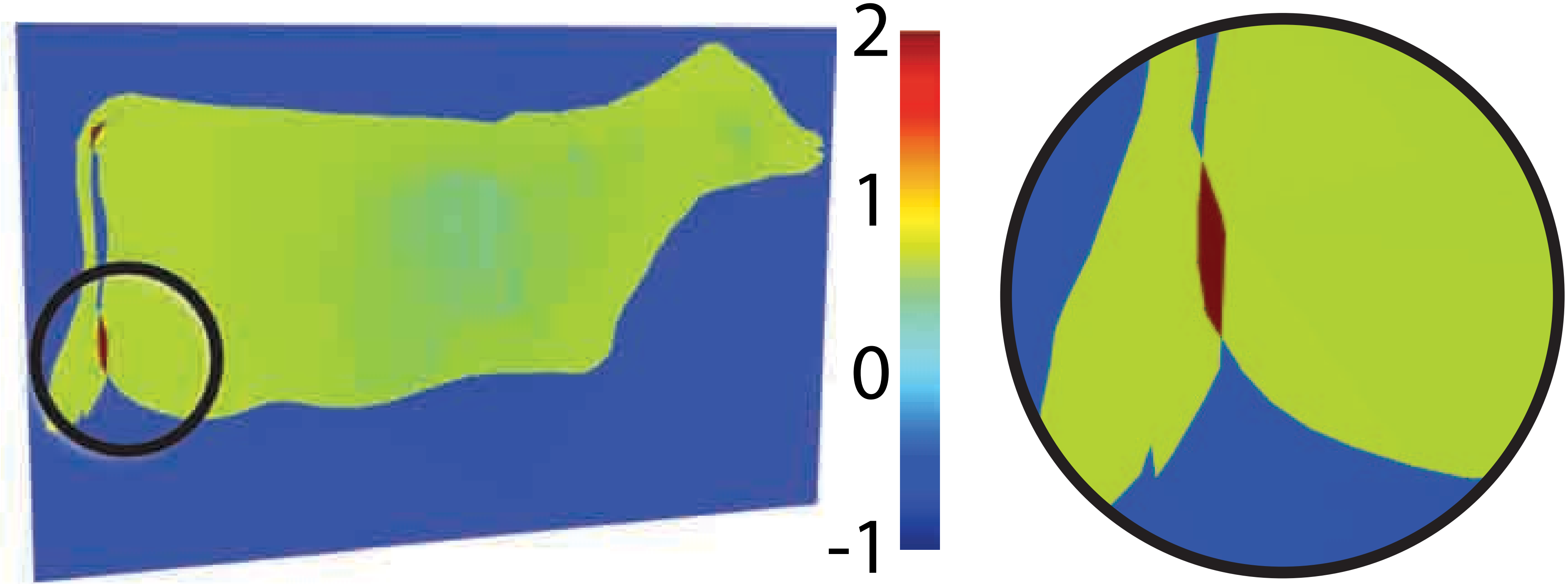
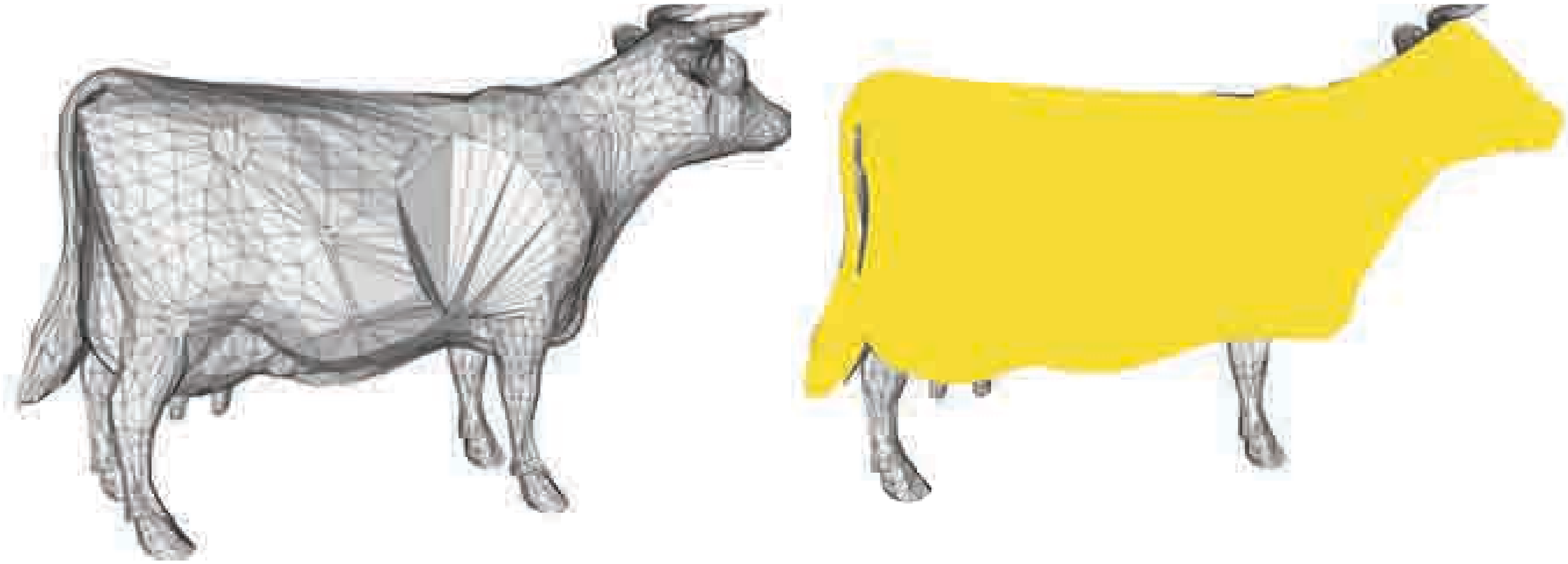
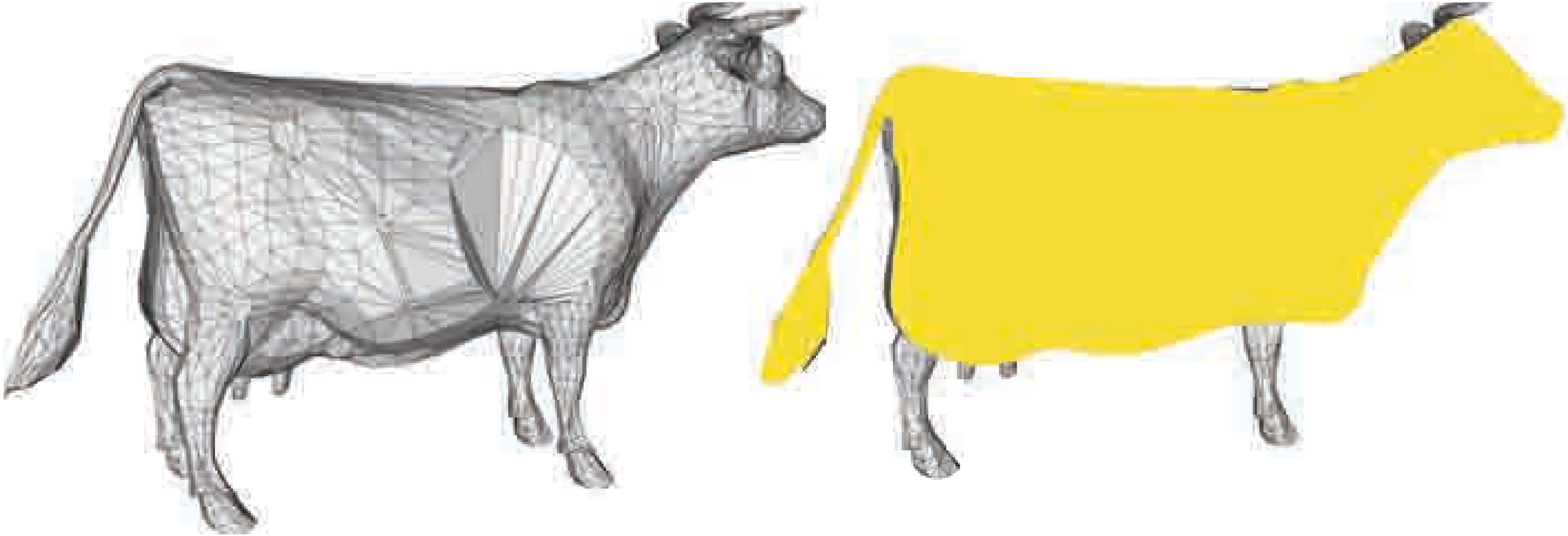


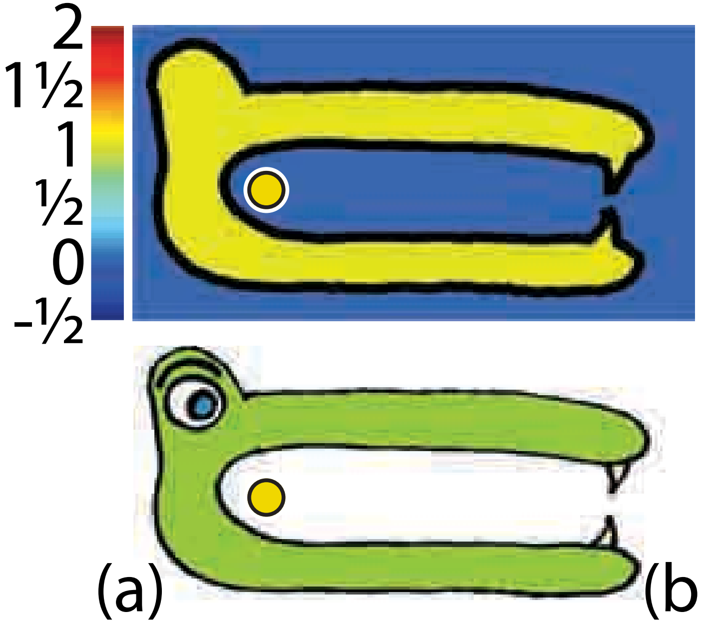
# Duplicate any multiply inside parts: consistently overlapping tet mesh



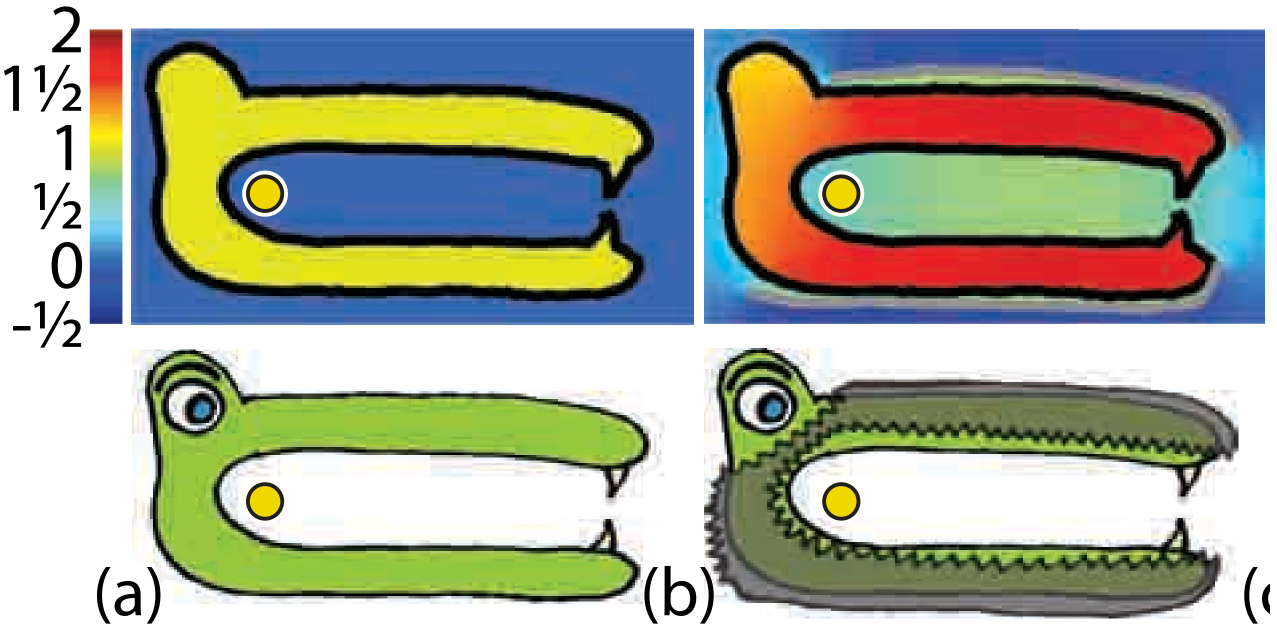
# Duplicate any multiply inside parts: consistently overlapping tet mesh



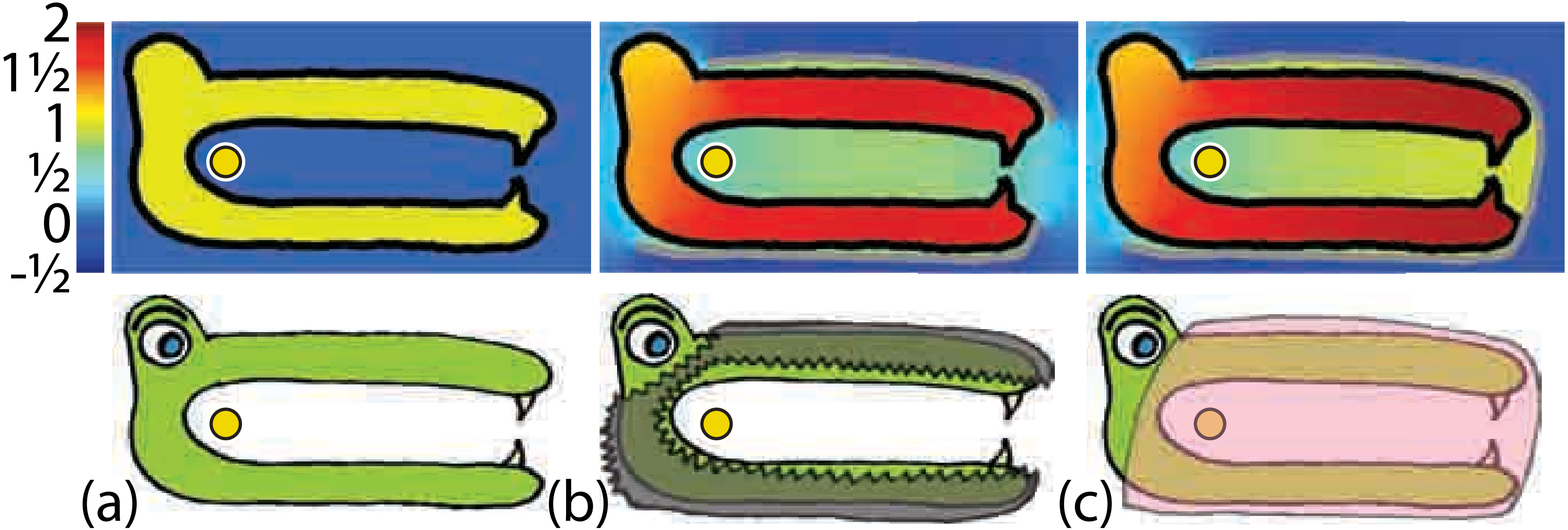
# Some ambiguities are just semantics



# Some ambiguities are just semantics



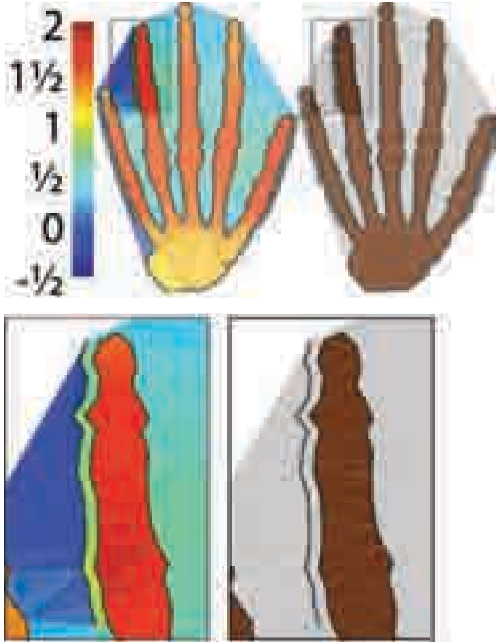
# Some ambiguities are just semantics



# Simple thresholding is not enough

$$\text{is\_outside}(\boxed{e_i}) = \begin{cases} \text{true} & \text{if } w(e_i) < 0.5 \\ \text{false} & \text{otherwise} \end{cases}$$

Each element in CDT



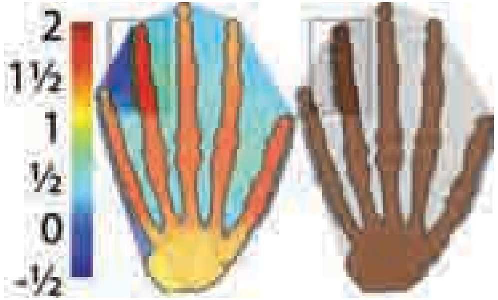

winding number

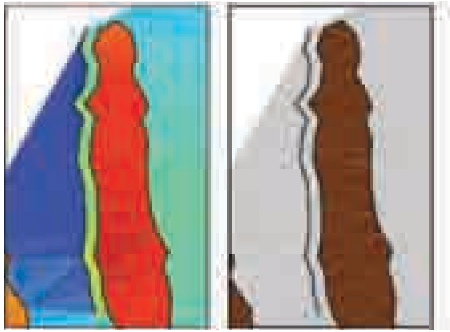
threshold

# Graphcut encourages coherency

$$E = \sum_{i=1}^{m} \left[ \boxed{u(x_i)} + \gamma \frac{1}{2} \sum_{j \in N(i)} \boxed{v(x_i, x_j)} \right]$$

data coherency



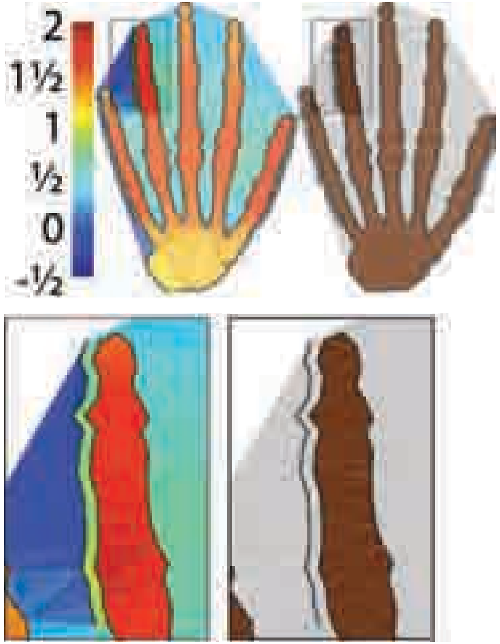



winding number threshold

# Graphcut encourages coherency

$$E = \sum_{i=1}^{m} \left[ \boxed{u(x_i)} + \gamma \frac{1}{2} \sum_{j \in N(i)} v(x_i, x_j) \right]$$

$$u(x_i) = \begin{cases} \max(w(e_i) - 0, 0) & \text{if } x_i = \texttt{outside} \\ \max(1 - w(e_i), 0) & \text{otherwise} \end{cases}$$

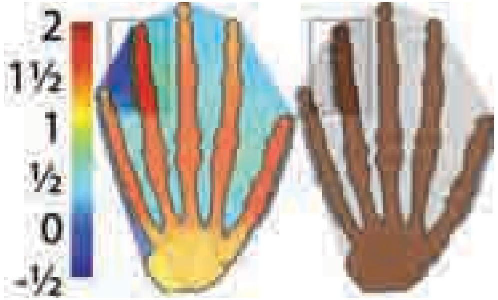


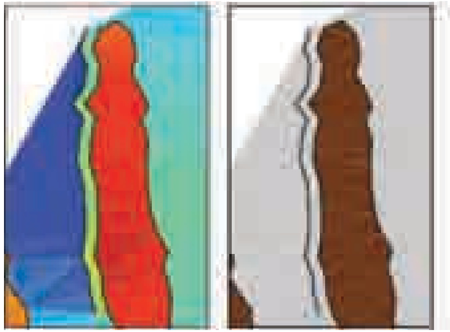

winding number threshold

# Graphcut encourages coherency

$$E = \sum_{i=1}^{m} \left[ u(x_i) + \gamma \frac{1}{2} \sum_{j \in N(i)} v(x_i, x_j) \right]$$

$$v(x_i, x_j) = \begin{cases} 0 & \text{if } x_i = x_j \\ \frac{a_{ij} \exp\left(|w(e_i) - w(e_j)|^2\right)}{2\sigma^2} & \text{otherwise} \end{cases}$$



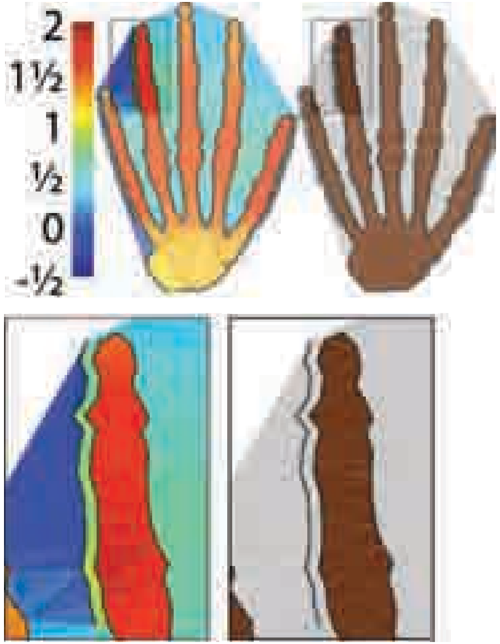



winding number threshold

# Graphcut encourages coherency

$$E = \sum_{i=1}^{m} \left[ u(x_i) + \gamma \frac{1}{2} \sum_{j \in N(i)} v(x_i, x_j) \right]$$

$$\underset{\mathbf{x} | x_i \in [0,1]}{\operatorname{argmin}} \; E(\mathbf{x})$$ use graphcut (maxflow)




winding number threshold

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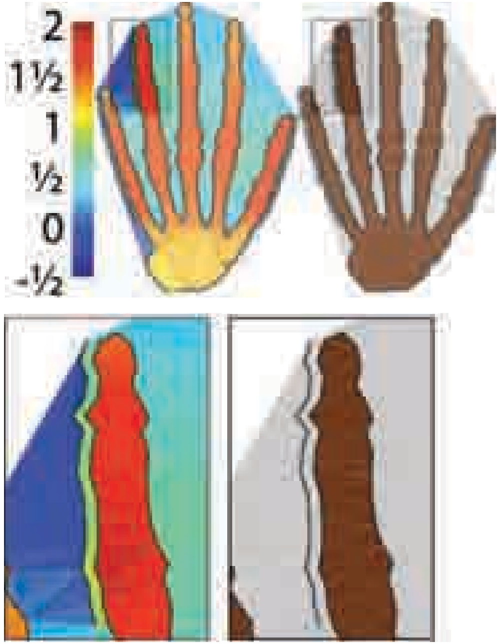
$$\underset{\mathbf{x} | x_i \in [0,1]}{\operatorname{argmin}} \; E(\mathbf{x})$$ use graphcut (maxflow)

subject to hard *facet constraints*




winding number    threshold

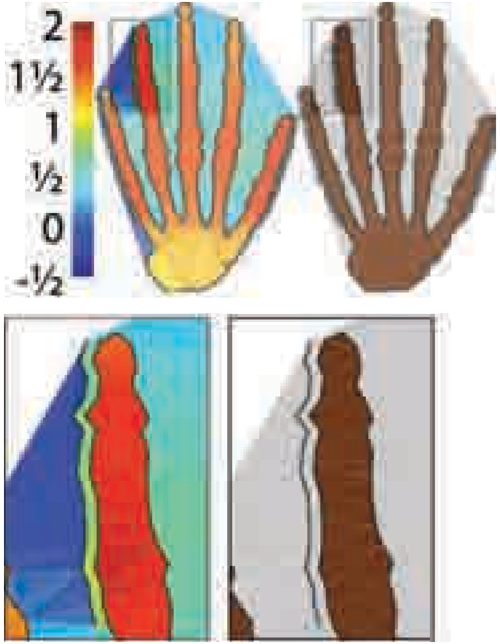
# Graphcut encourages coherency

$$E = \sum_{i=1}^{m} \left[ u(x_i) + \gamma \frac{1}{2} \sum_{j \in N(i)} v(x_i, x_j) \right]$$

$\underset{\mathbf{x}|x_i \in [0,1]}{\operatorname{argmin}} \ E(\mathbf{x})$ use graphcut (maxflow)

subject to hard *facet constraints*

"nonregular"
[Kolmogorov & Zabin 2004]



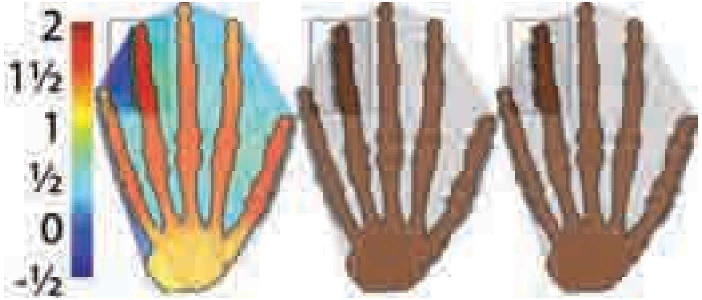

winding number threshold

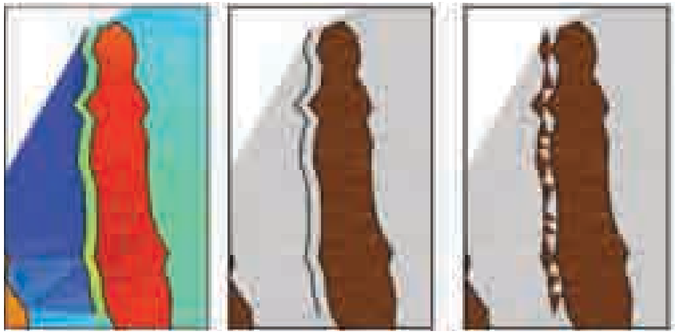
# Graphcut encourages coherency

$$E = \sum_{i=1}^{m} \left[ u(x_i) + \gamma \frac{1}{2} \sum_{j \in N(i)} v(x_i, x_j) \right]$$

$$\underset{\mathbf{x} | x_i \in [0,1]}{\operatorname{argmin}} \; E(\mathbf{x})$$ use graphcut (maxflow)

subject to hard *facet constraints*



winding number | threshold | naive constraints

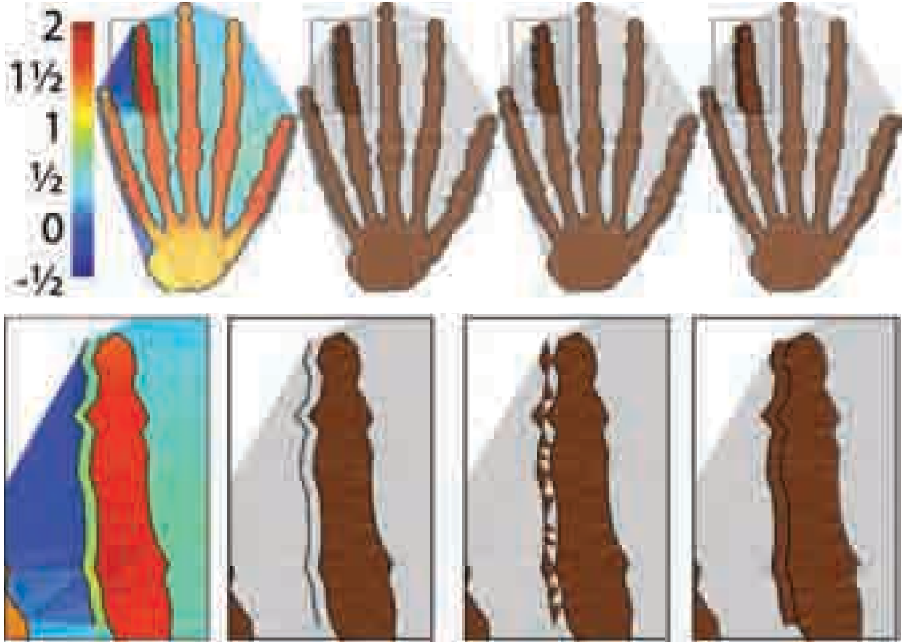
# Graphcut encourages coherency

$$E = \sum_{i=1}^{m} \left[ u(x_i) + \gamma \frac{1}{2} \sum_{j \in N(i)} v(x_i, x_j) \right]$$

$$\underset{\mathbf{x} | x_i \in [0,1]}{\operatorname{argmin}} \; E(\mathbf{x})$$ use graphcut (maxflow)

**subject to hard *facet constraints***

use heuristic → local min.




winding number | threshold | naive constraints | local min.

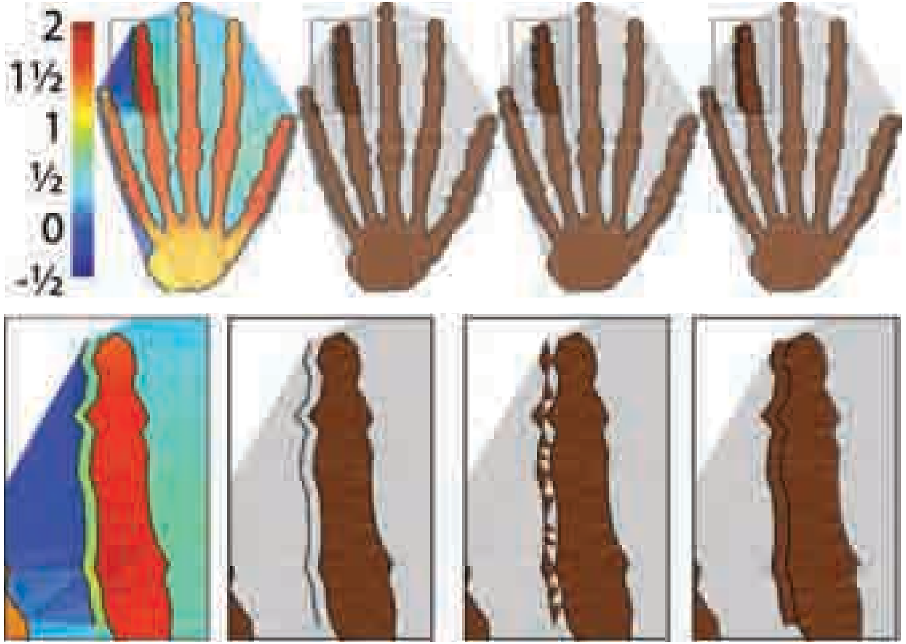
# Graphcut encourages coherency

$$E = \sum_{i=1}^{m} \left[ u(x_i) + \gamma \frac{1}{2} \sum_{j \in N(i)} v(x_i, x_j) \right]$$

$$\underset{\mathbf{x} | x_i \in [0,1]}{\operatorname{argmin}} \; E(\mathbf{x})$$ use graphcut (maxflow)

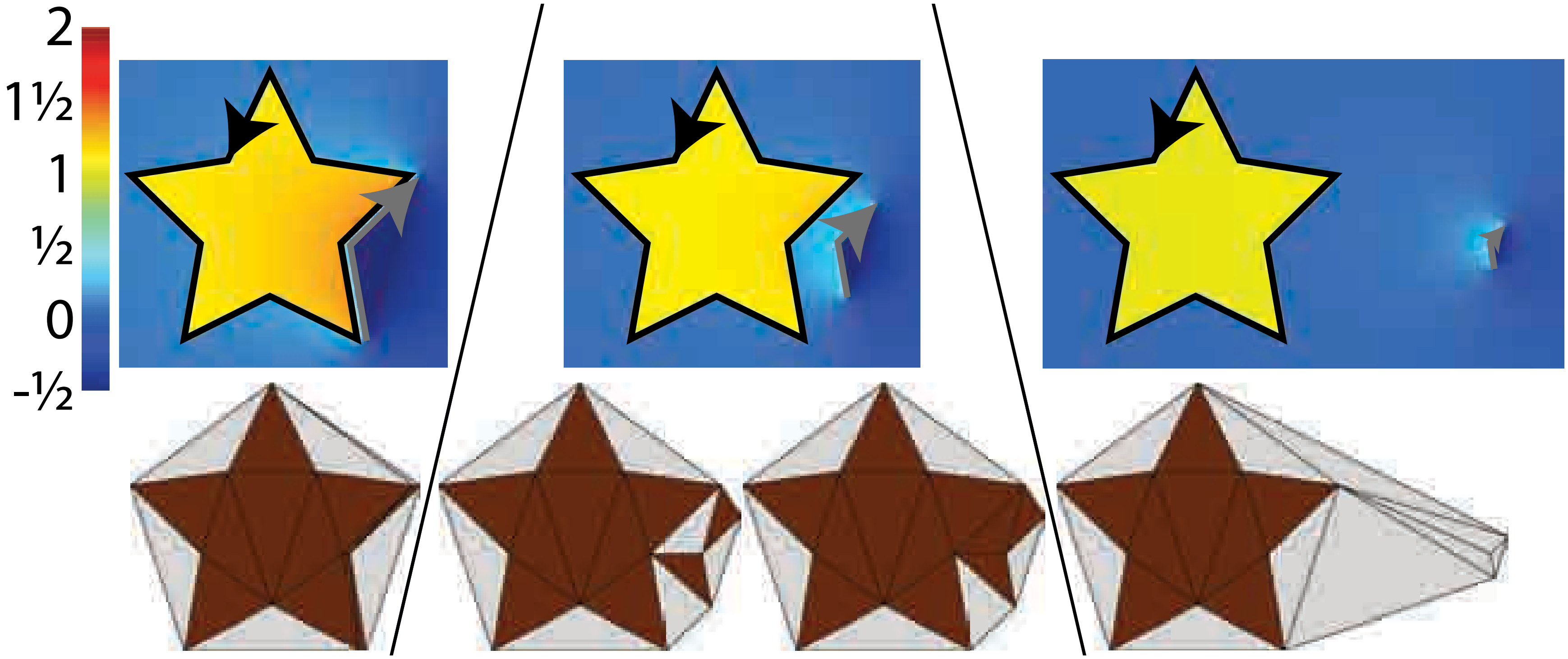
subject to hard *facet constraints*

+subject to hard *manifoldness constraints*

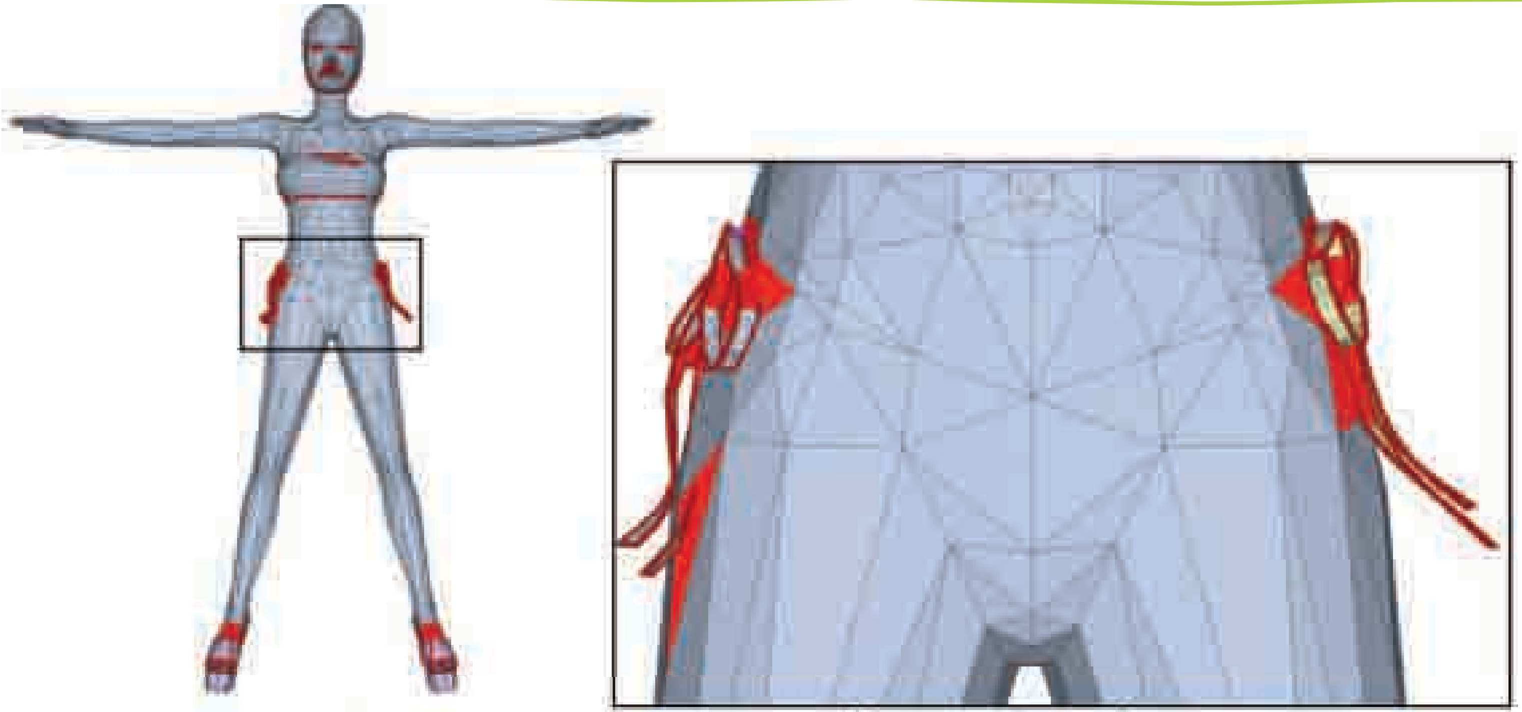



winding number | threshold | naive constraints | local min.

# Hard constraints are optional: outliers

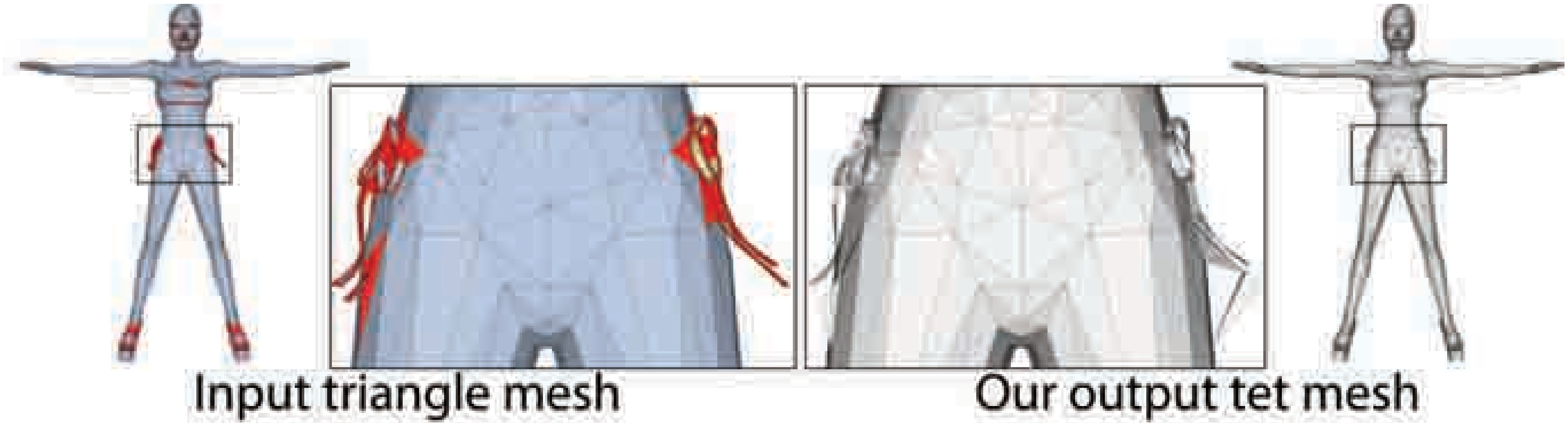


# Even failure to create beautiful *surface*, can be success as volume representation

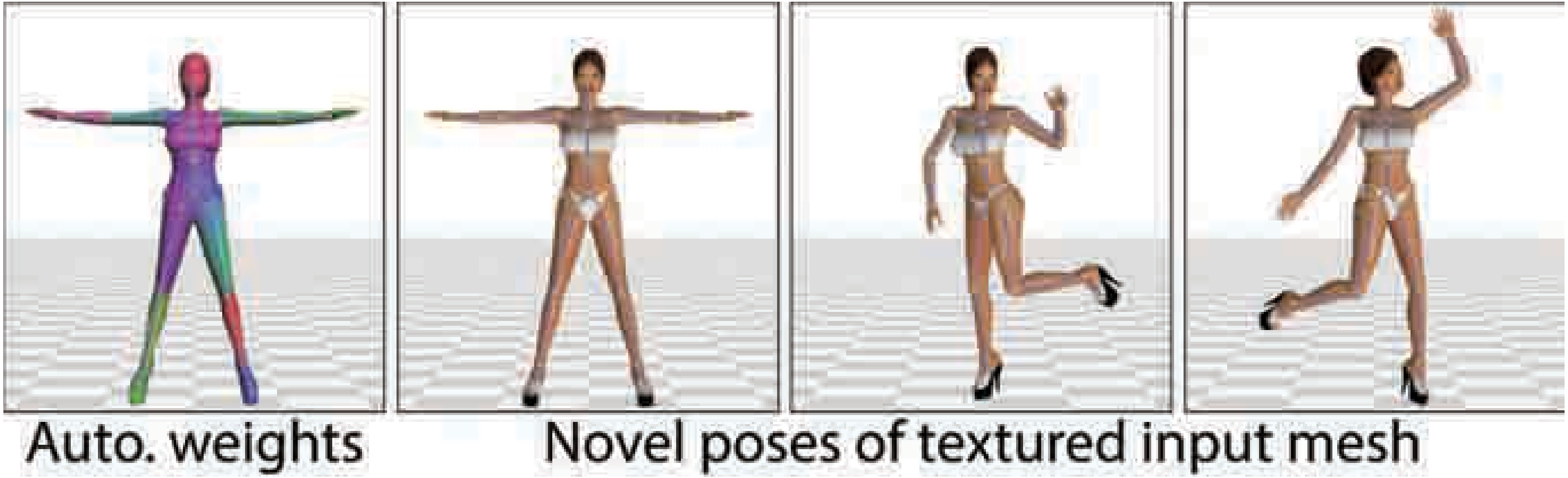


Input triangle mesh

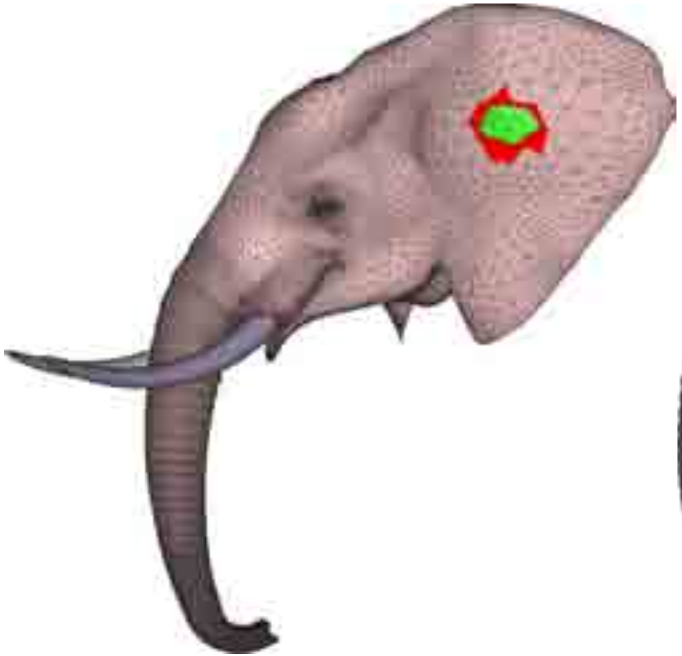
# Even failure to create beautiful *surface*, can be success as volume representation



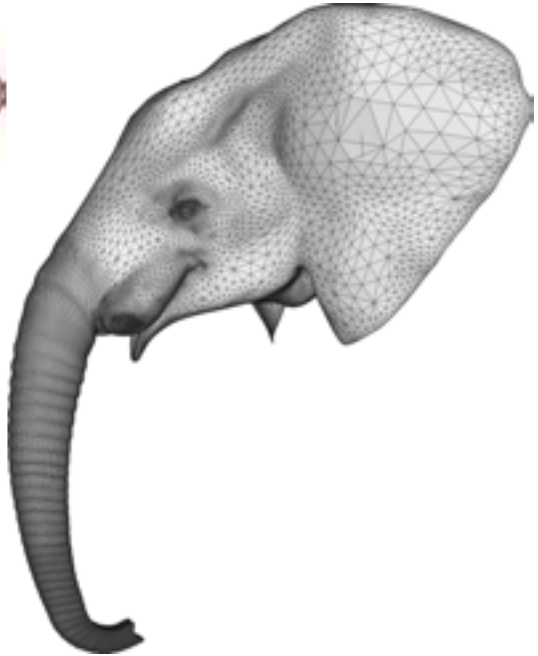
# Even failure to create beautiful *surface*, can be success as volume representation



# Cleanup methods modify input too much, …

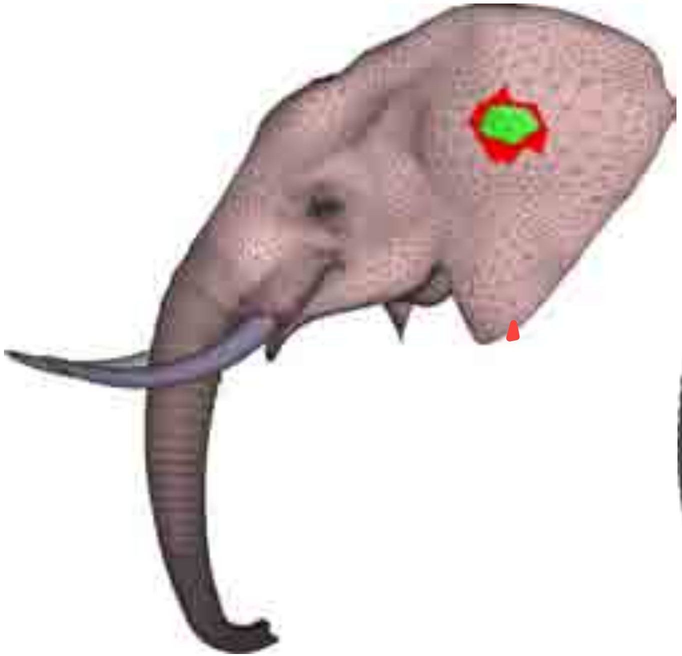


input mesh

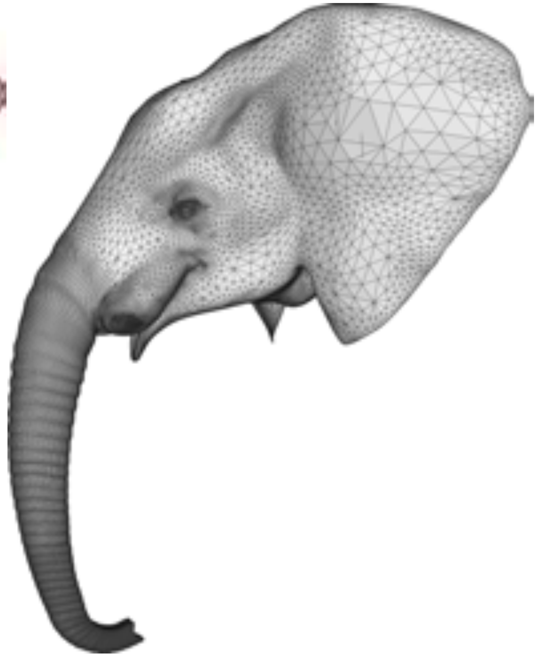


[Attene 2010]

# Cleanup methods modify input too much, …

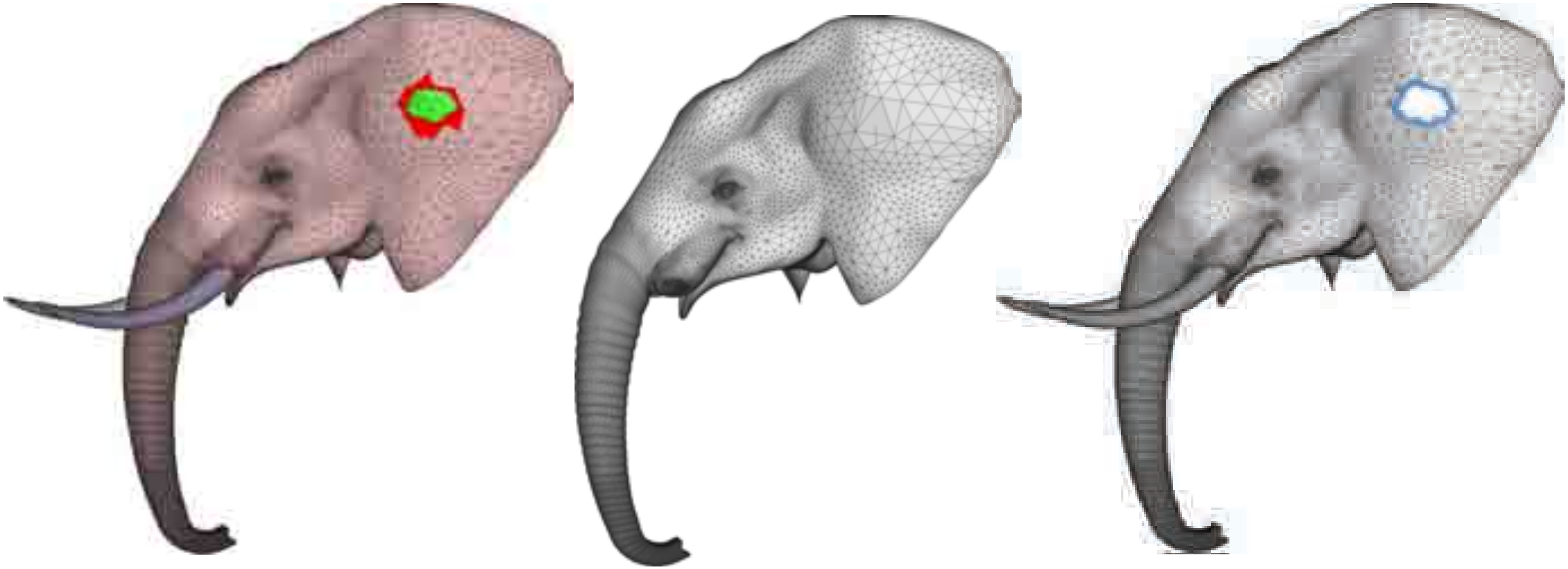


input mesh



[Attene 2010]

# … but we rely heavily on orientation



input mesh [Attene 2010] our output