252-0538-00L, Spring 2025

Shape Modeling and Geometry Processing

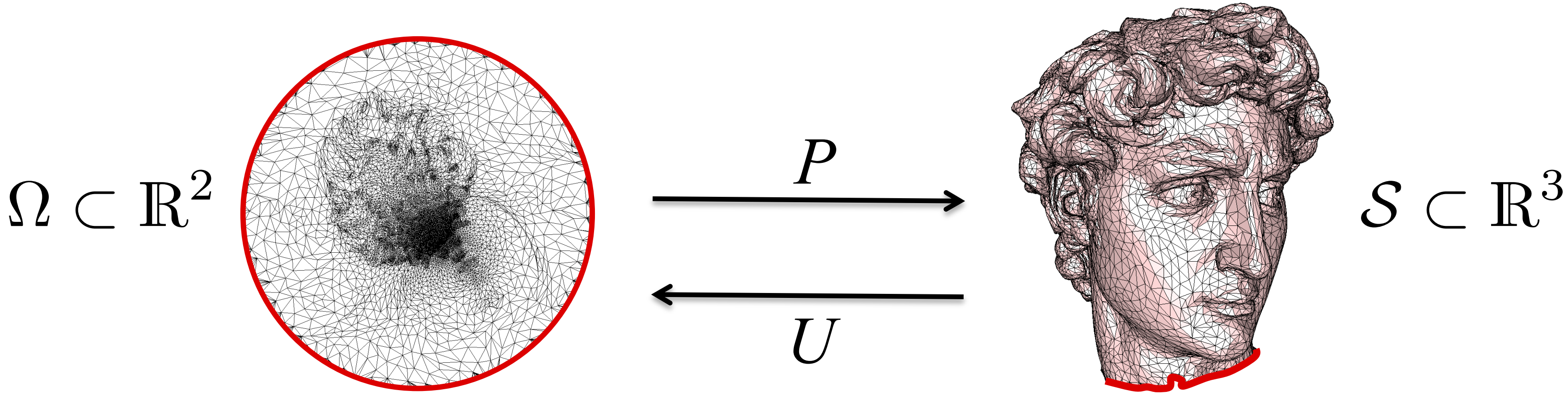
Least Squares Conformal Maps
As-Rigid-As-Possible Parameterization
Remeshing

INTERACTIVE GEOMETRY LAB

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

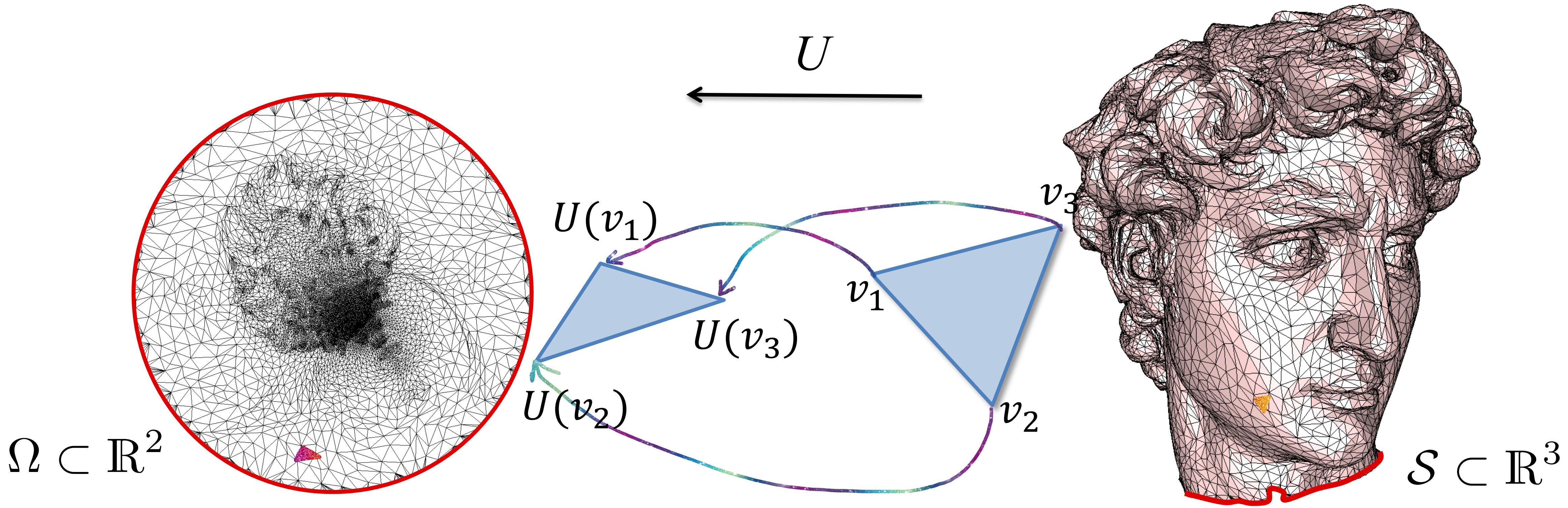
Parameterization – Definition

- Parameterization mapping P : 2D domain Ω → 3D mesh S
- Flattening mapping U : 3D mesh S → 2D domain Ω $\quad U(\mathbf{v}_i) = (u_i, v_i)$
- Inverse(P) = flattening U.
- (sometimes we also call U as the "parameterization" of input mesh S)



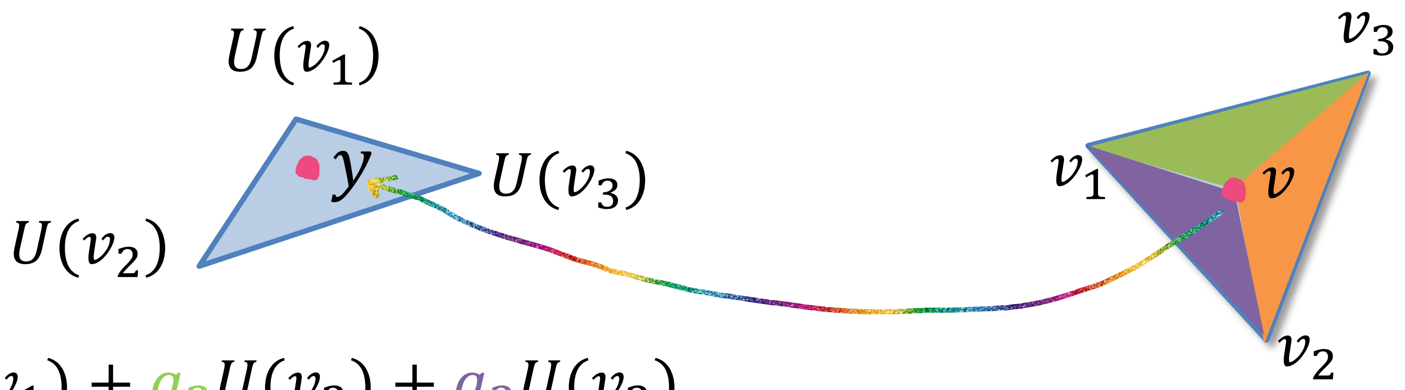
Parameterization – Definition

- Flatting mapping U: 3D mesh $S \rightarrow$ 2D domain Ω
- Each mesh vertex has a corresponding 2D position: $U(\mathbf{v}_i) = (u_i, v_i)$



Parameterization – Definition

- Flatting mapping U : 3D mesh $S \rightarrow$ 2D domain Ω
- Each mesh vertex has a corresponding 2D position: $U(\mathbf{v}_i) = (u_i, v_i)$
- Inside each triangle, the mappings P and U are affine (barycentric coordinates)

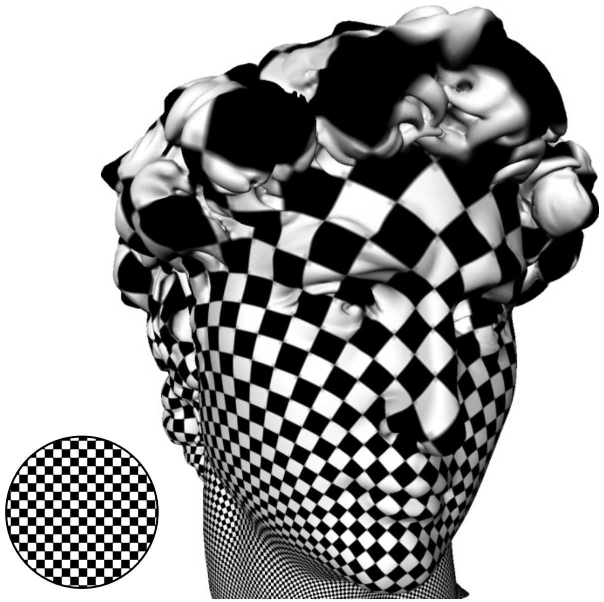


$$v = a_1 v_1 + a_2 v_2 + a_3 v_3$$

$$y = a_1 U(v_1) + a_2 U(v_2) + a_3 U(v_3)$$

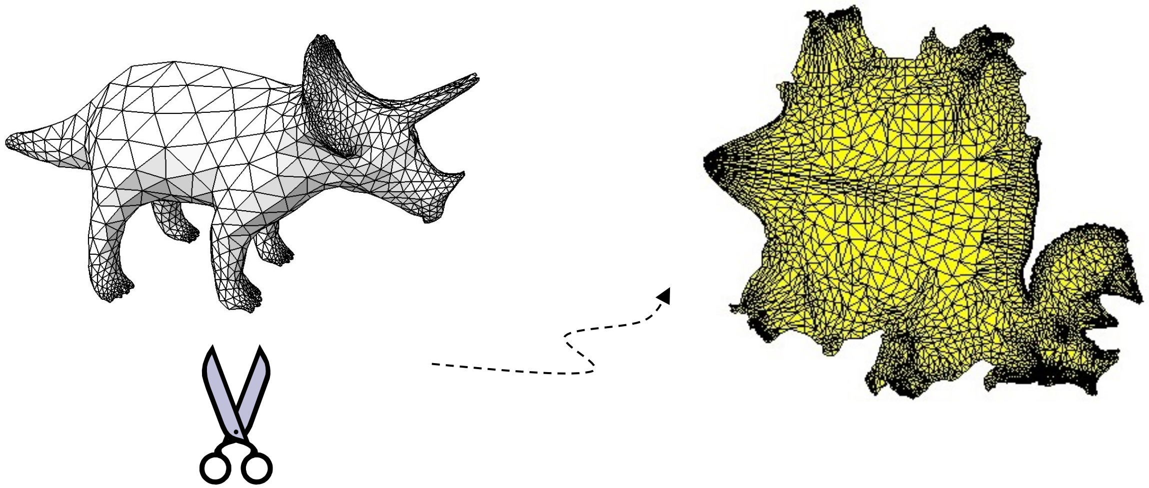
Parameterization – quality

- Minimize distortion E



minimize angular/conformal distortion

- Minimize/hide cuts

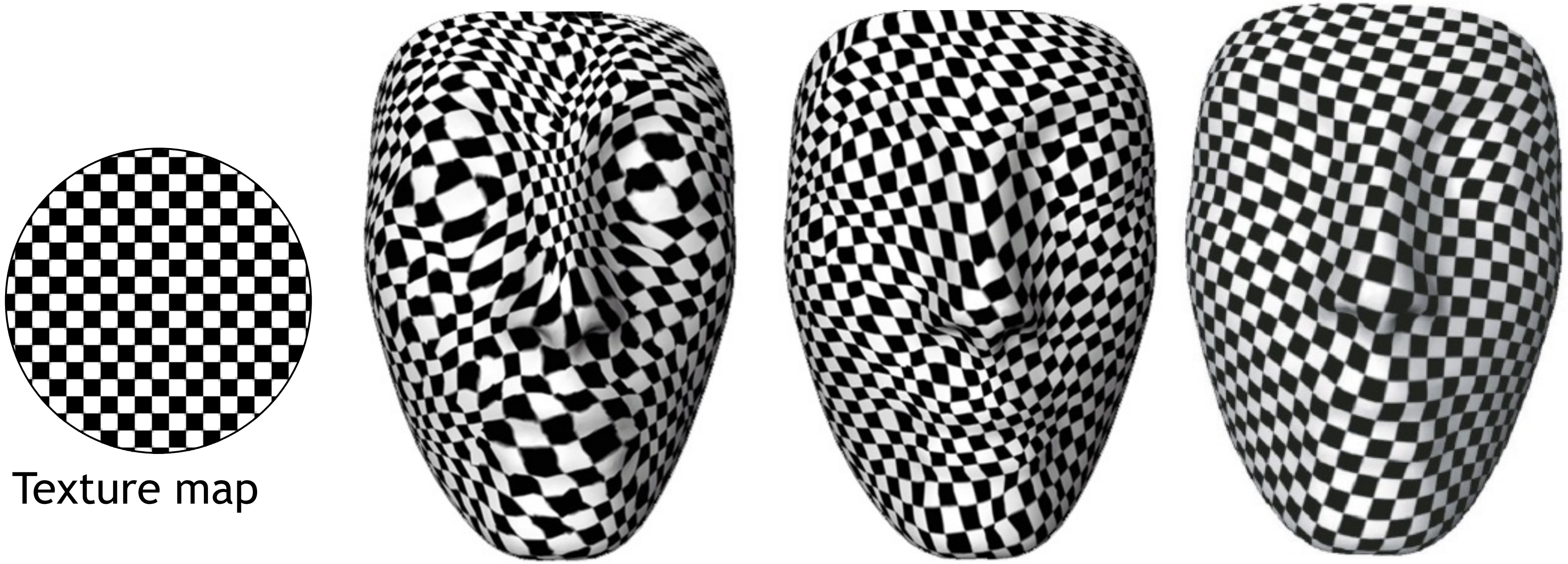


free boundary helps distortion minimization

Distortion on Triangle Meshes?

- Triangle in 3D $\Longleftrightarrow$ triangle in 2D
- Unique affine mapping

Distortion Minimization



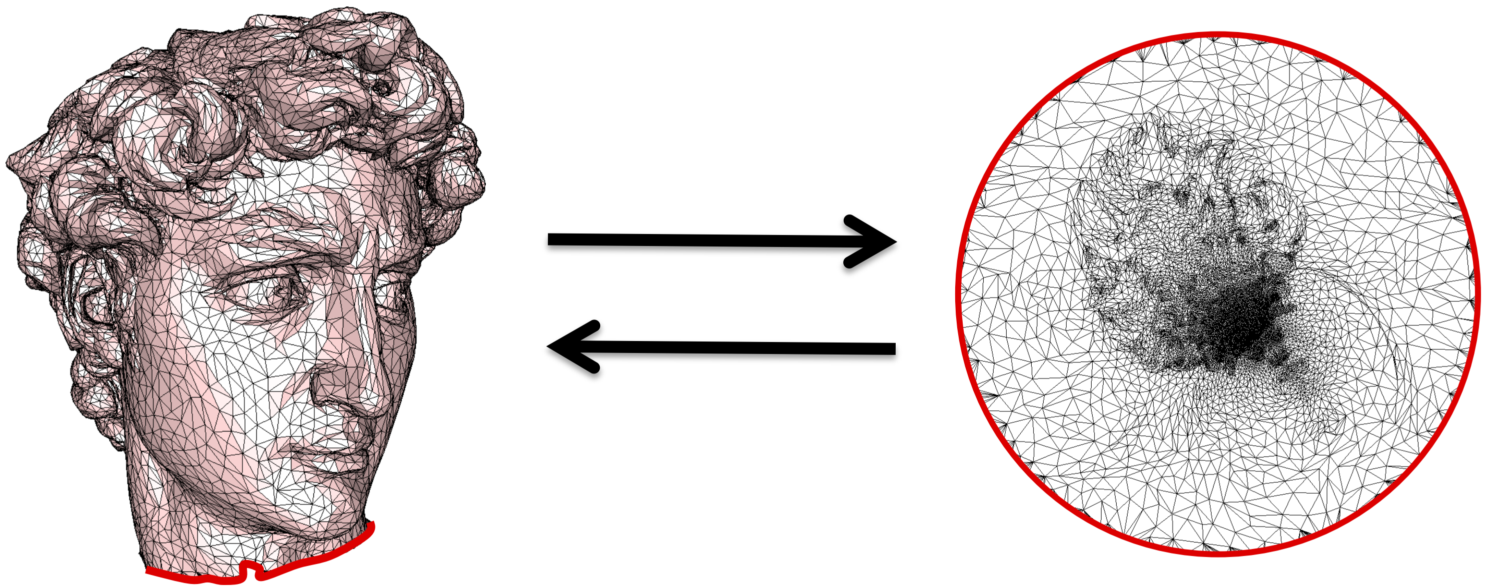
$$\underset{(u_1,v_1),\ldots,(u_n,v_n)}{\operatorname{argmin}} \sum_T A_T\, E(T)$$

triangle area in 3D

Recap: Harmonic mapping

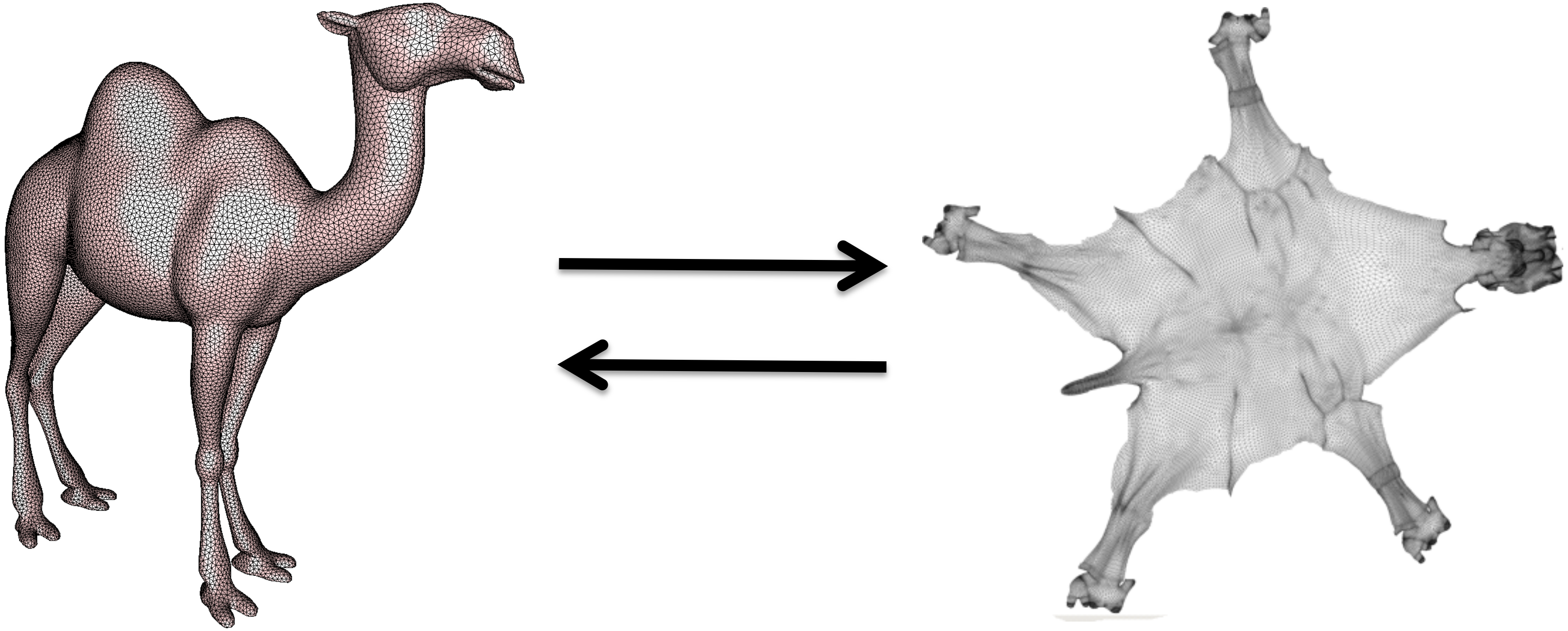
- Minimize linearized spring energy
- Boundary vertices must be fixed a priori

  ➔ Less freedom, more distortion

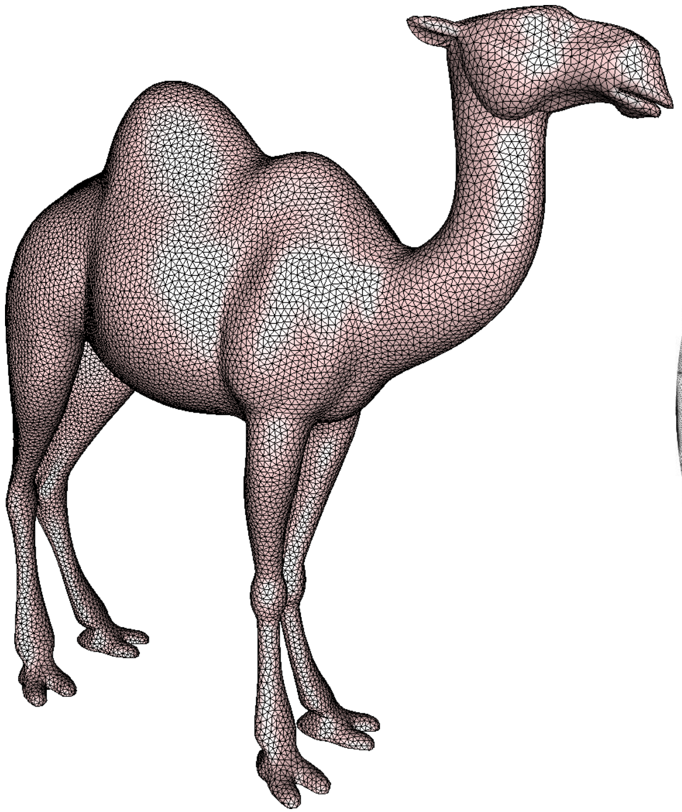


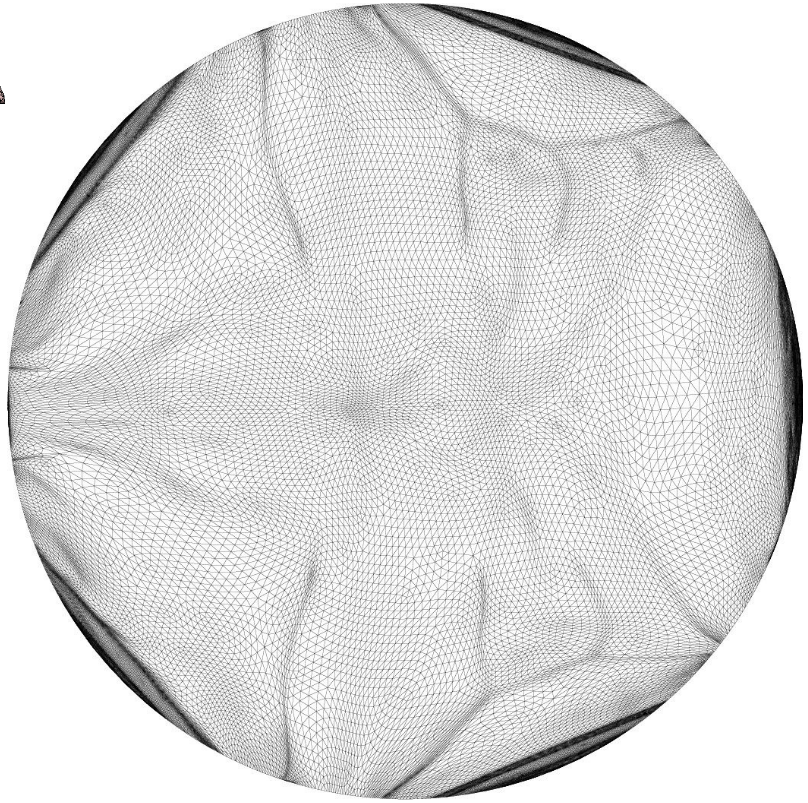
Today: free boundary param.

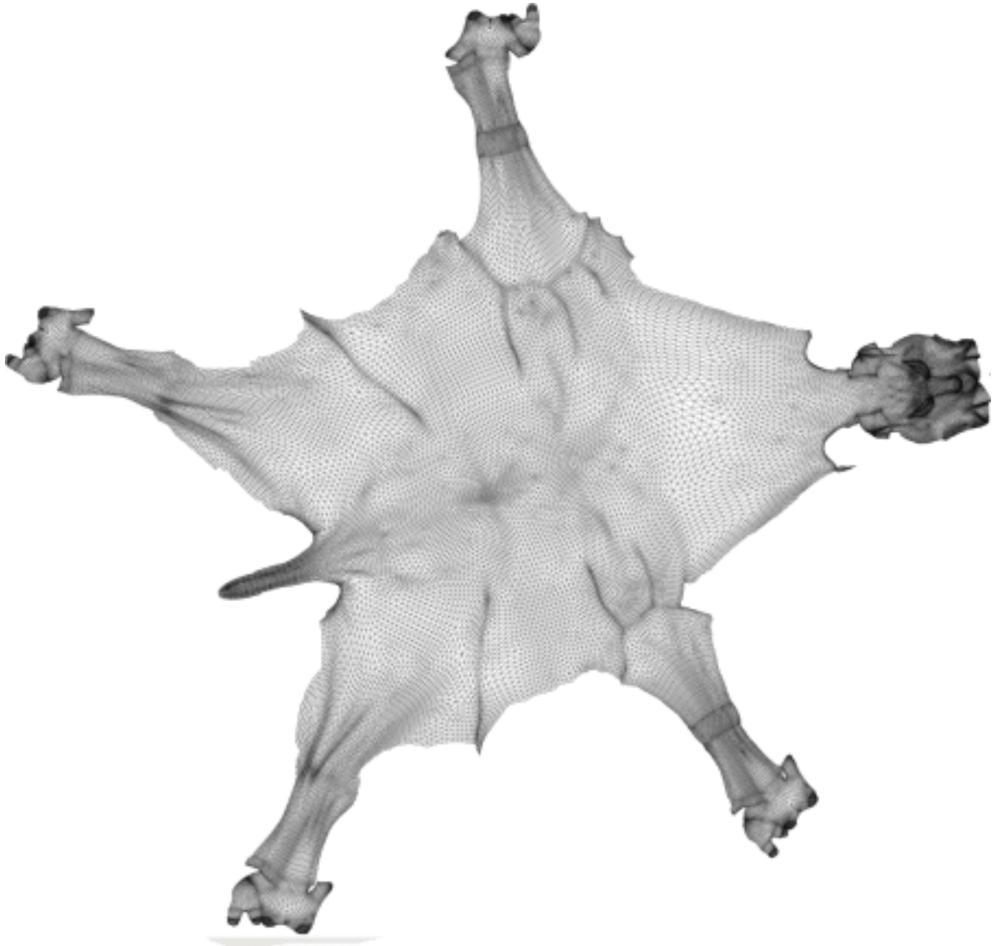
- Boundary vertices are free in optimization
  - ➔ "Good" boundary positions automatically obtained ☺



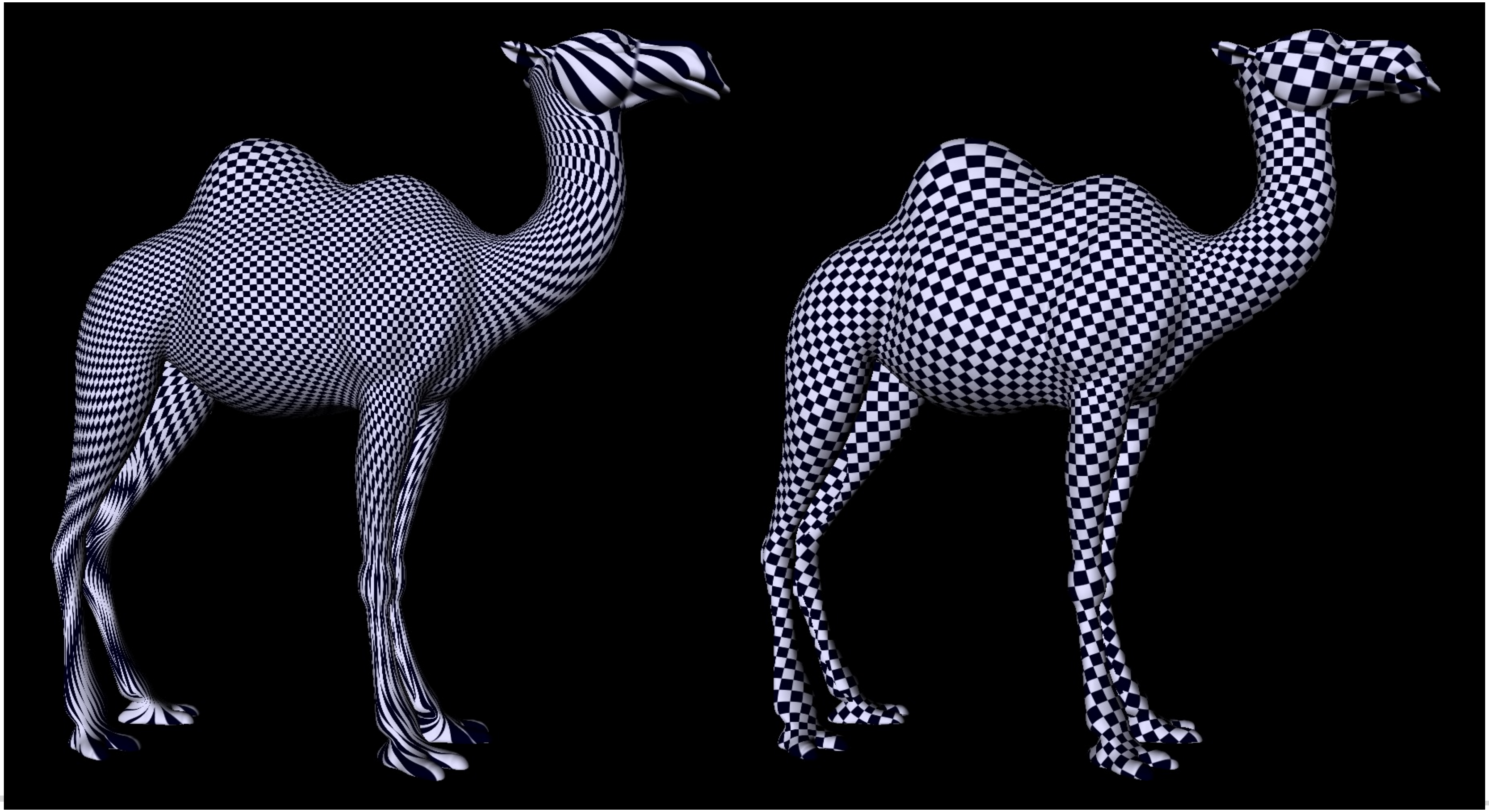
Fixed vs. free boundary







Fixed vs. free boundary



Literature for today

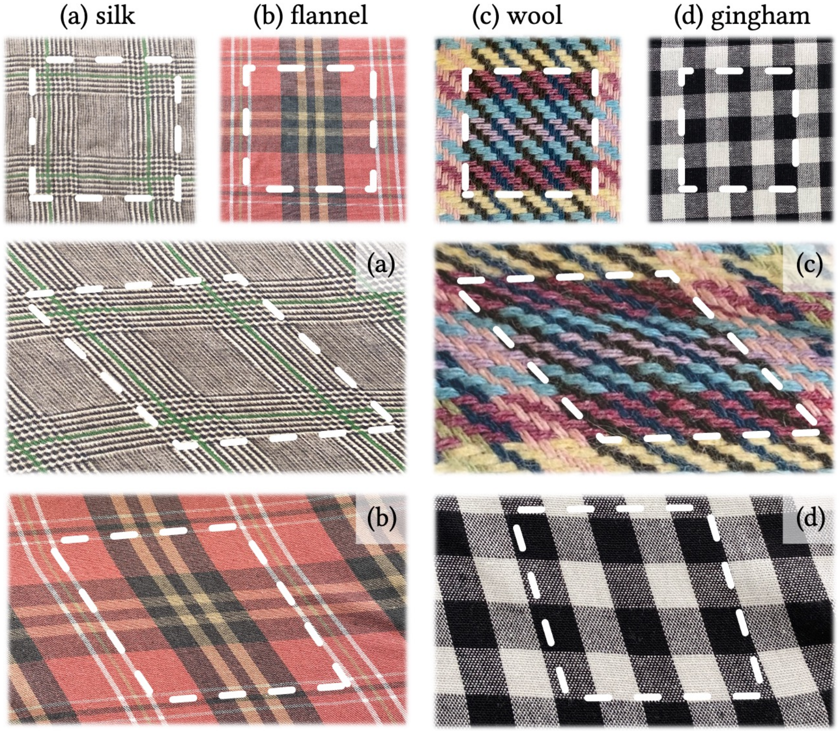
1. Angle-preserving parameterization(**LSCM**)
   - "Least Squares Conformal Maps", Lévy et al., SIGGRAPH 2002
2. Isometric parameterization (**ARAP**)
   - "A Local/Global Approach to Mesh Parameterization ",Liu et al., SGP 2008

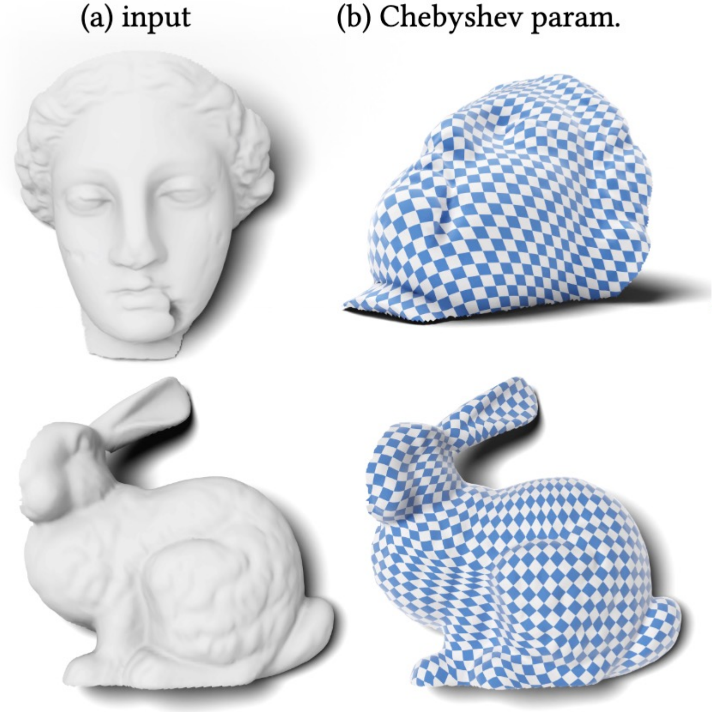
LSCM : "Conformal"

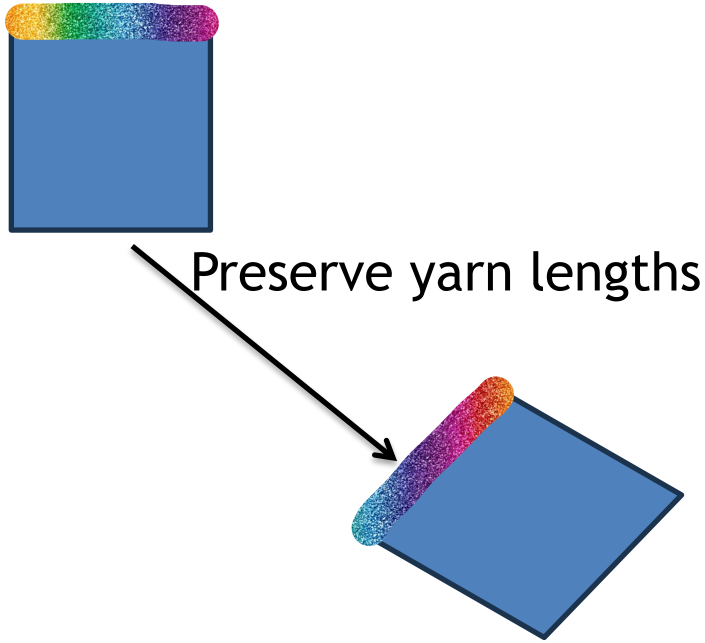
ARAP : "Isometric"

Literature for today

3. length-preserving along yarn directions
   - "Chebyshev Parameterization for Woven fabric Modeling", Oehri et al., SIGGRAPH Asia 2024







Discrete distortion energies

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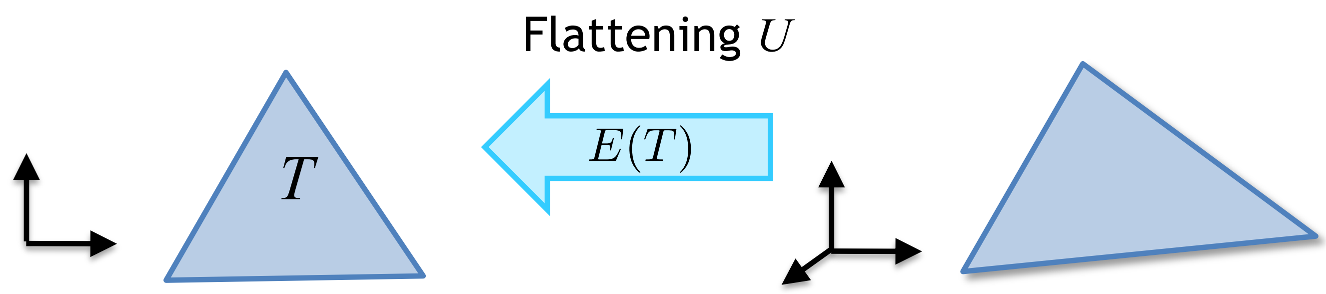
Notations

$$\underset{X \in \mathbb{R}^{2n}}{\operatorname{argmin}} \; E(X)$$

Variables: $X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^{\mathsf{T}}$

Distortion energy: $E(X) = \sum_{\text{triangles } T} A_T \, E(T)$

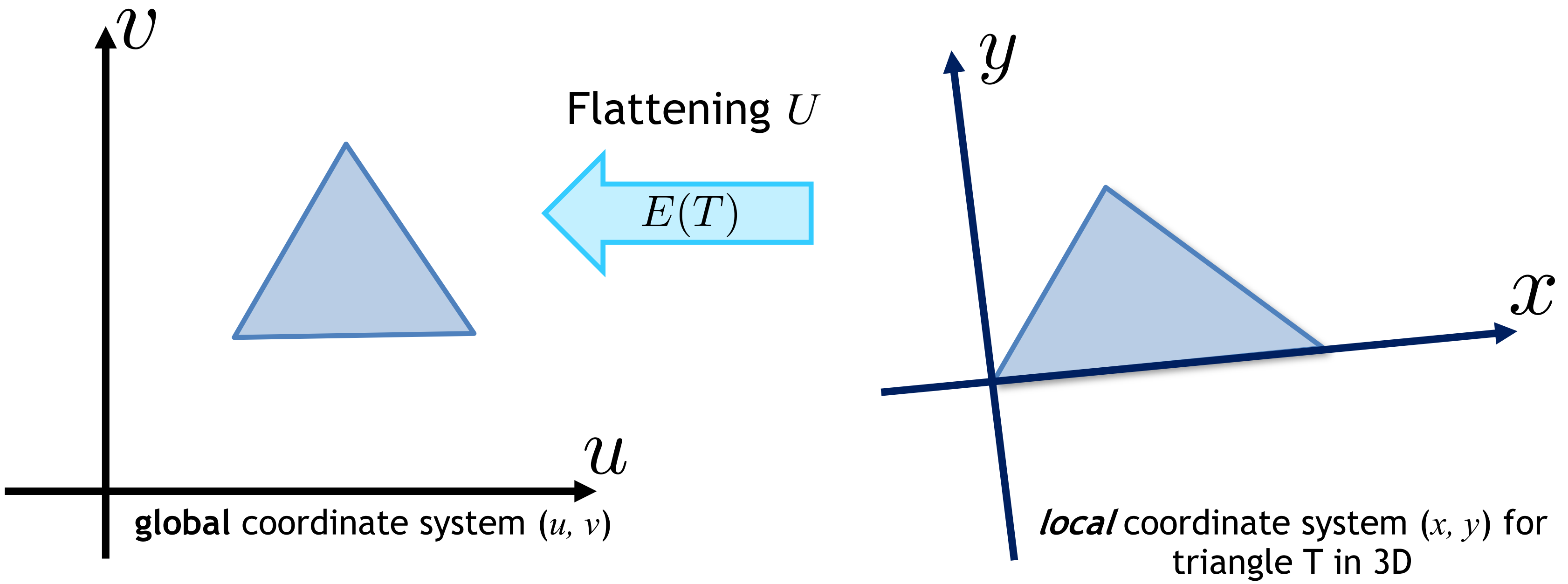
per-triangle distortion measure, value dependent on X



Local triangle basis

$$\underset{X \in \mathbb{R}^{2n}}{\operatorname{argmin}} \sum_{\text{triangles } T} A_T \, E(T)$$

$$X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^\mathsf{T}$$



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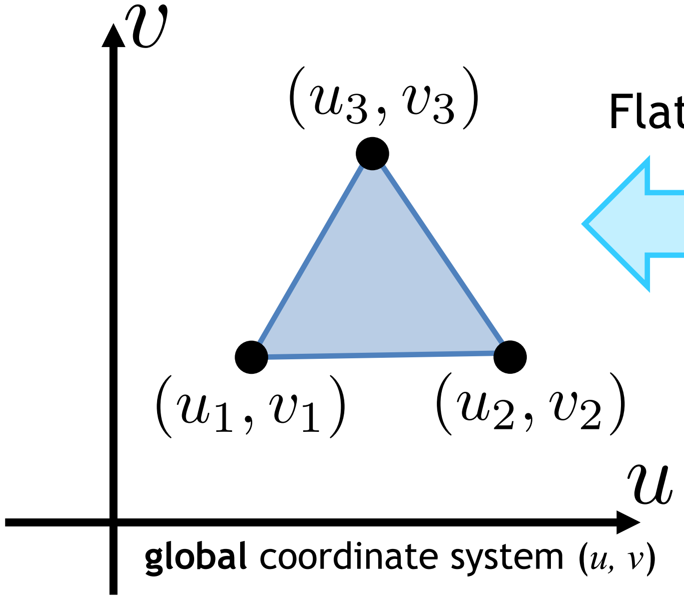
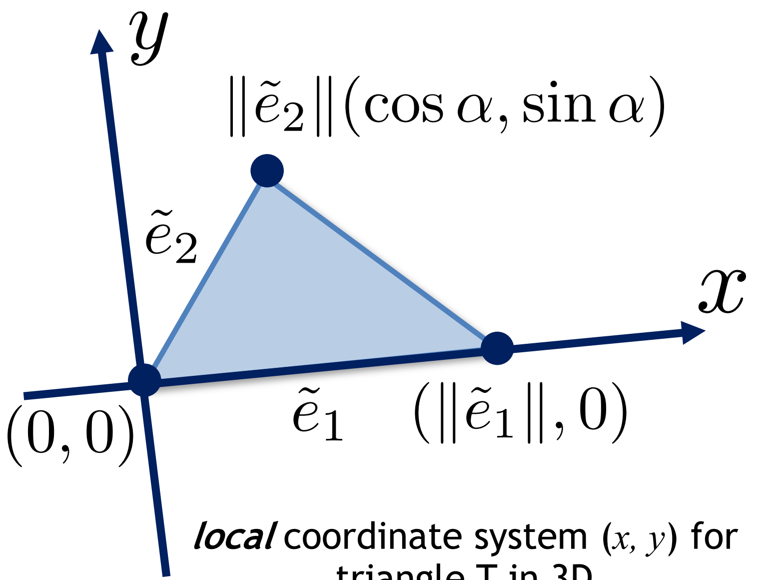


Flattening U

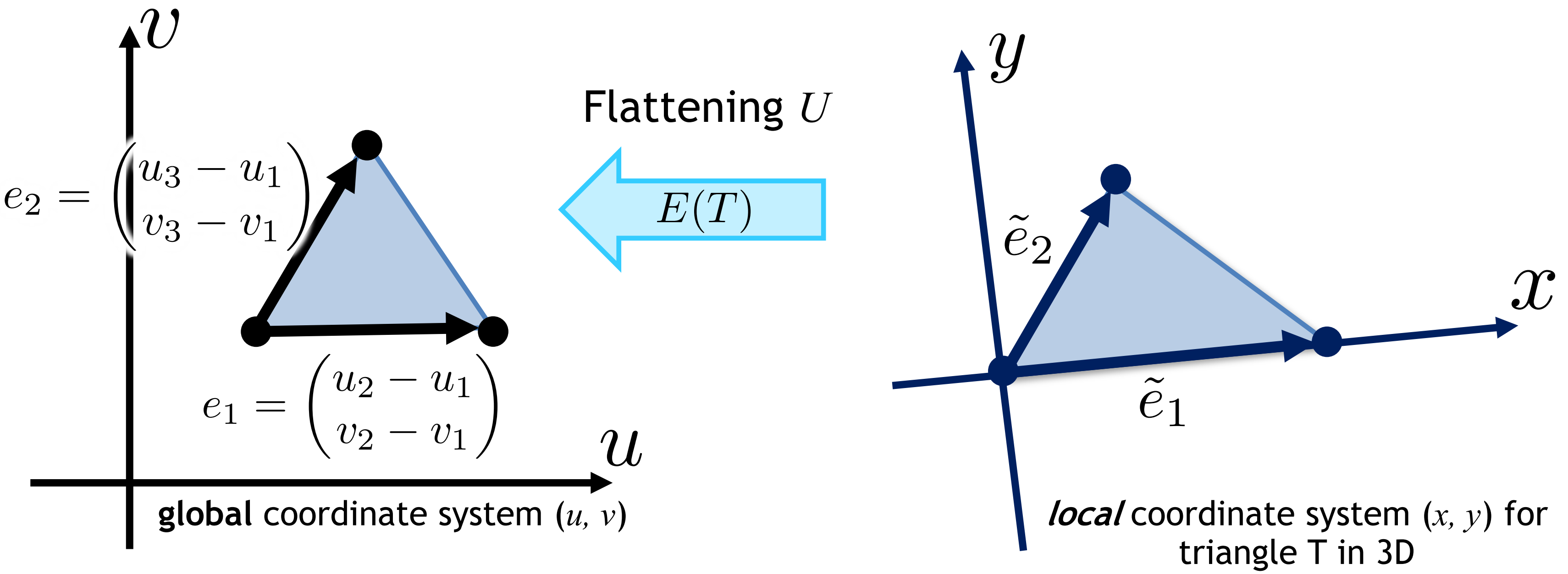
$E(T)$



Local triangle basis

$$\underset{X \in \mathbb{R}^{2n}}{\operatorname{argmin}} \sum_{\text{triangles } T} A_T E(T)$$

$$X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^\mathsf{T}$$

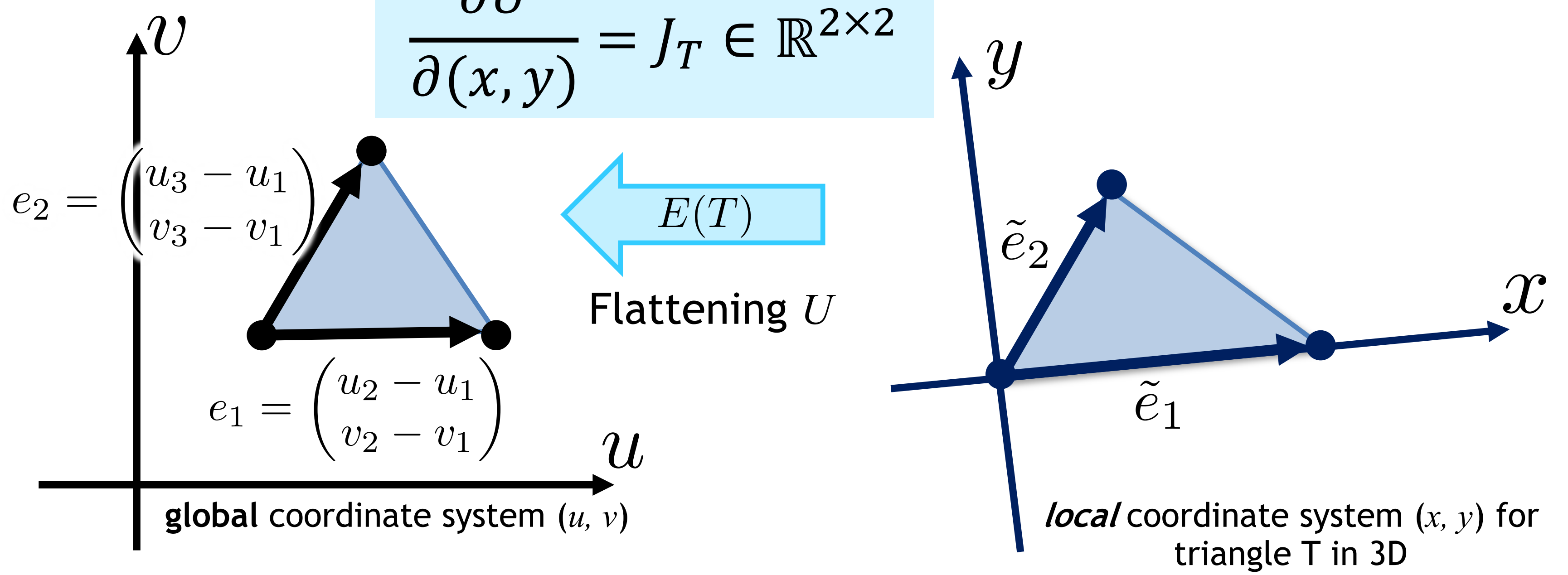


Mapping between triangles

$$\underset{X\in\mathbb{R}^{2n}}{\operatorname{argmin}} \sum_{\text{triangles } T} A_T\, E(T)$$

$$X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^\mathsf{T}$$

$$\frac{\partial U}{\partial(x,y)} = J_T \in \mathbb{R}^{2\times 2}$$



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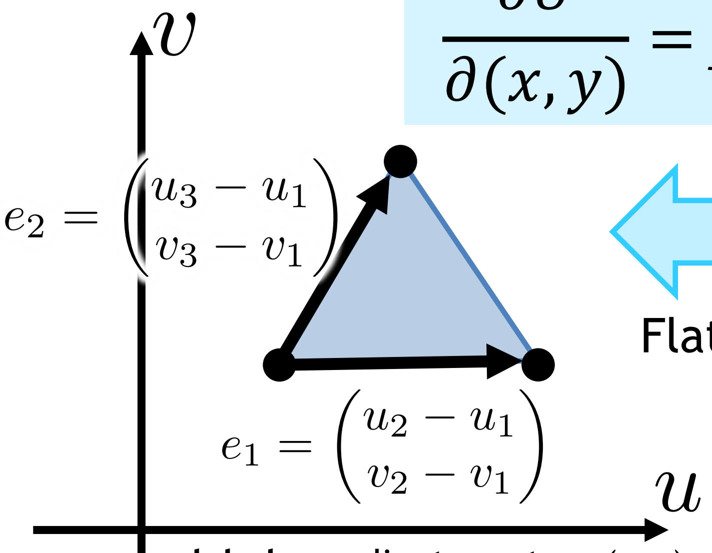
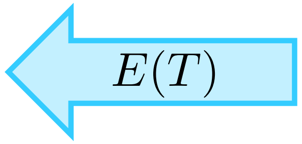
$$J_T \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix}$$

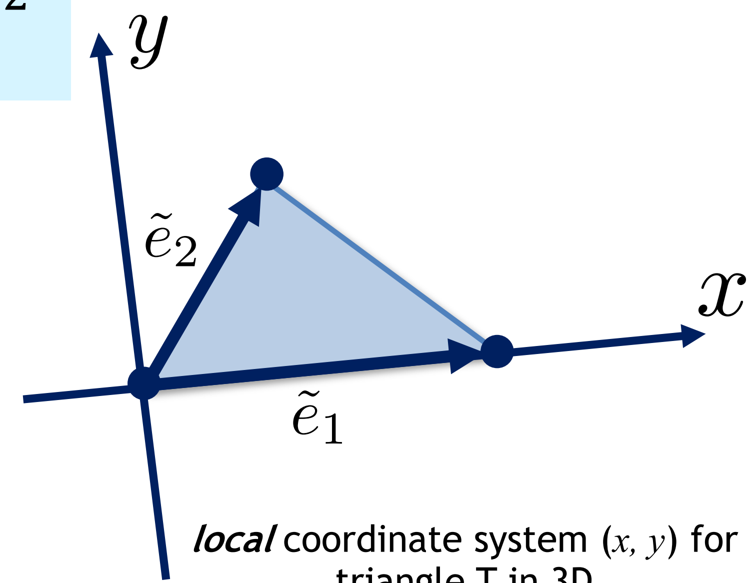
$$\frac{\partial U}{\partial(x, y)} = J_T \in \mathbb{R}^{2\times 2}$$



global coordinate system (u, v)



Flattening U



local coordinate system (x, y) for triangle T in 3D

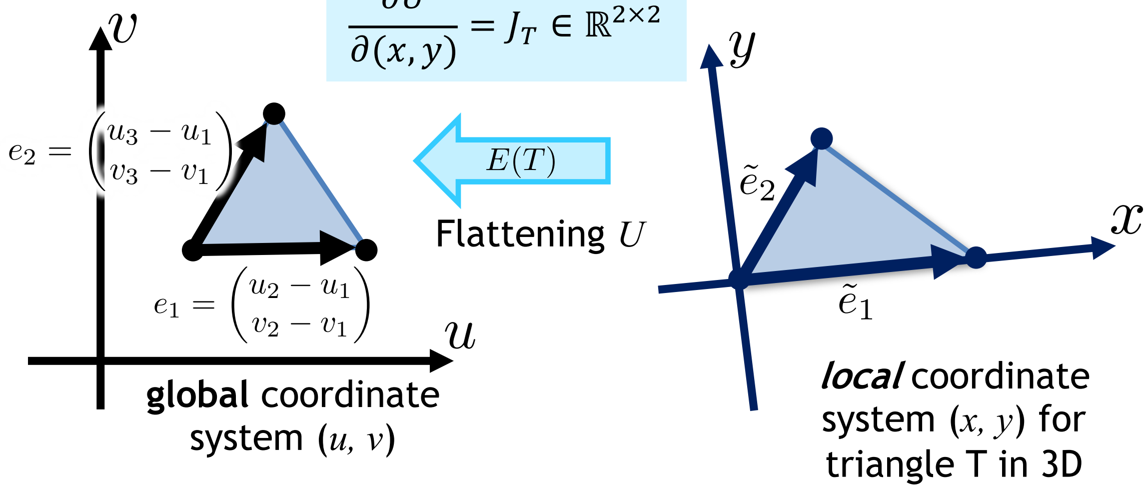
Mapping between triangles

$$\underset{X \in \mathbb{R}^{2n}}{\text{argmin}} \sum_{\text{triangles } T} A_T \, E(T)$$

$$X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^\mathsf{T}$$

$$J_T \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \Rightarrow J_T = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix}^{-1}$$

$$\frac{\partial U}{\partial (x,y)} = J_T \in \mathbb{R}^{2\times 2}$$



Mapping between triangles

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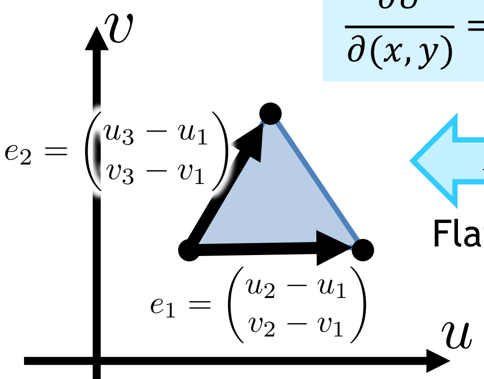
Linear in our unknowns X!

Constant 2x2 matrix, depends 3D triangle only

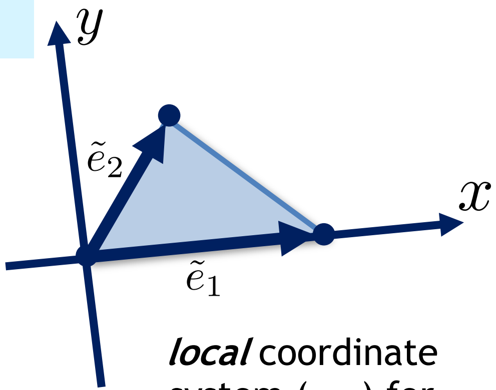
$$\frac{\partial U}{\partial(x, y)} = J_T \in \mathbb{R}^{2 \times 2}$$



global coordinate system (u, v)

$E(T)$

Flattening U



local coordinate system (x, y) for triangle T in 3D

Mapping between triangles

$$\underset{X \in \mathbb{R}^{2n}}{\operatorname{argmin}} \sum_{\text{triangles } T} A_T\, E(T)$$

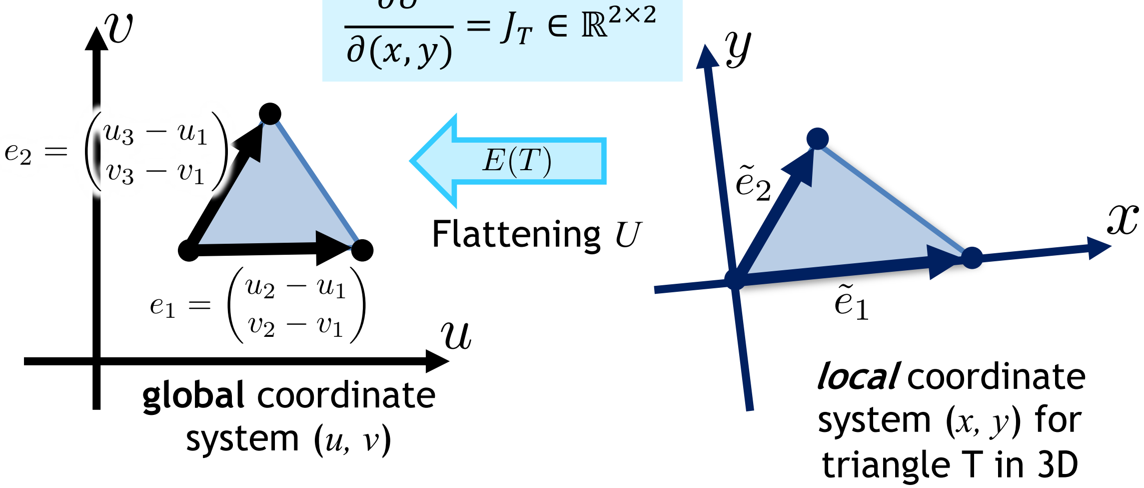
$$X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^\mathsf{T}$$

$$J_T \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \Rightarrow J_T = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix}^{-1}$$

Linear in our unknowns X! $\begin{pmatrix} u_2 - u_1 & u_3 - u_1 \\ v_2 - v_1 & v_3 - v_1 \end{pmatrix}$

Constant 2x2 matrix, depends 3D triangle only

$$\frac{\partial U}{\partial(x,y)} = J_T \in \mathbb{R}^{2\times 2}$$

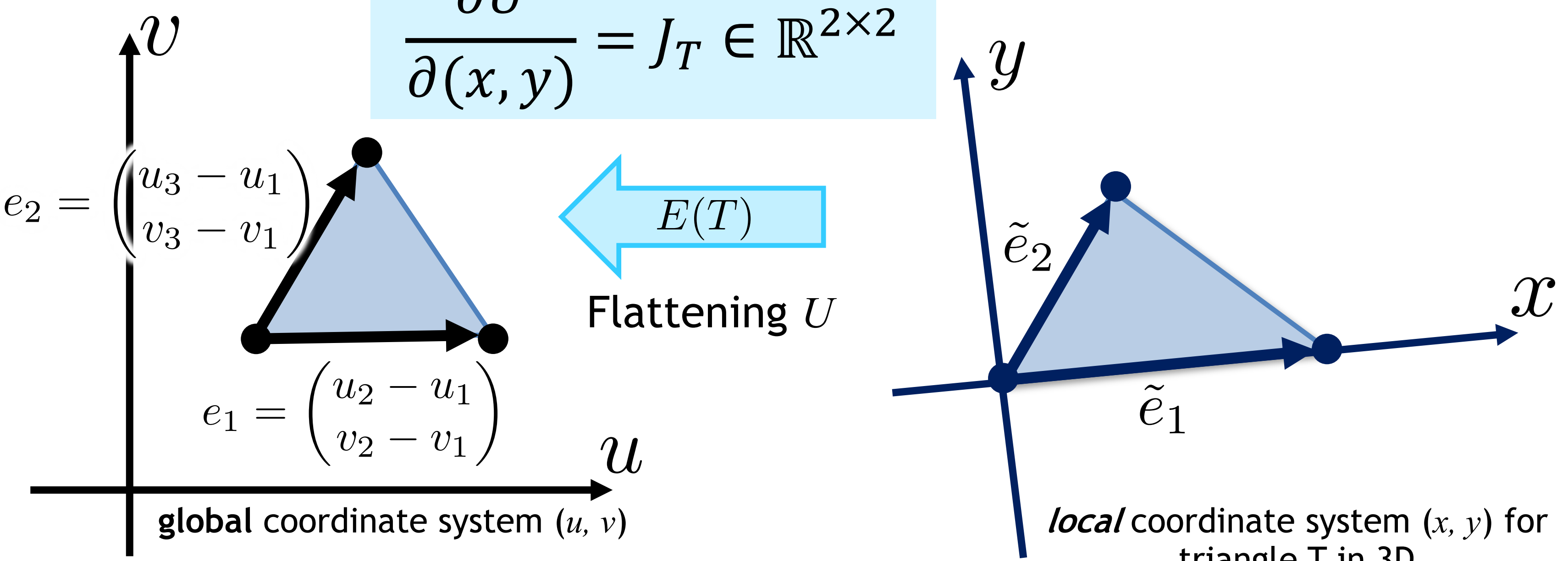


J_T **is linear** in unknowns X !

Jacobian J_T and triangle deformation

- Jacobian J_T is translation-invariant

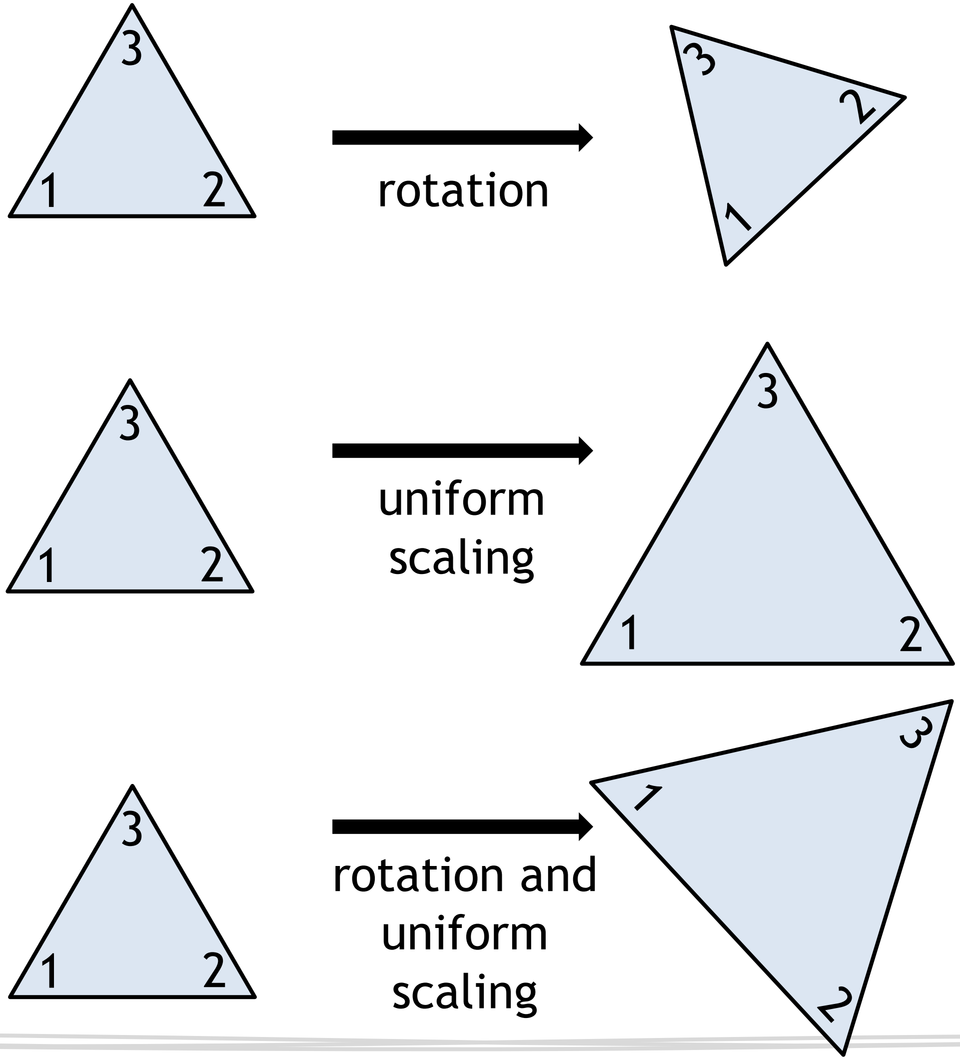
$$\frac{\partial U}{\partial(x,y)} = J_T \in \mathbb{R}^{2\times 2}$$



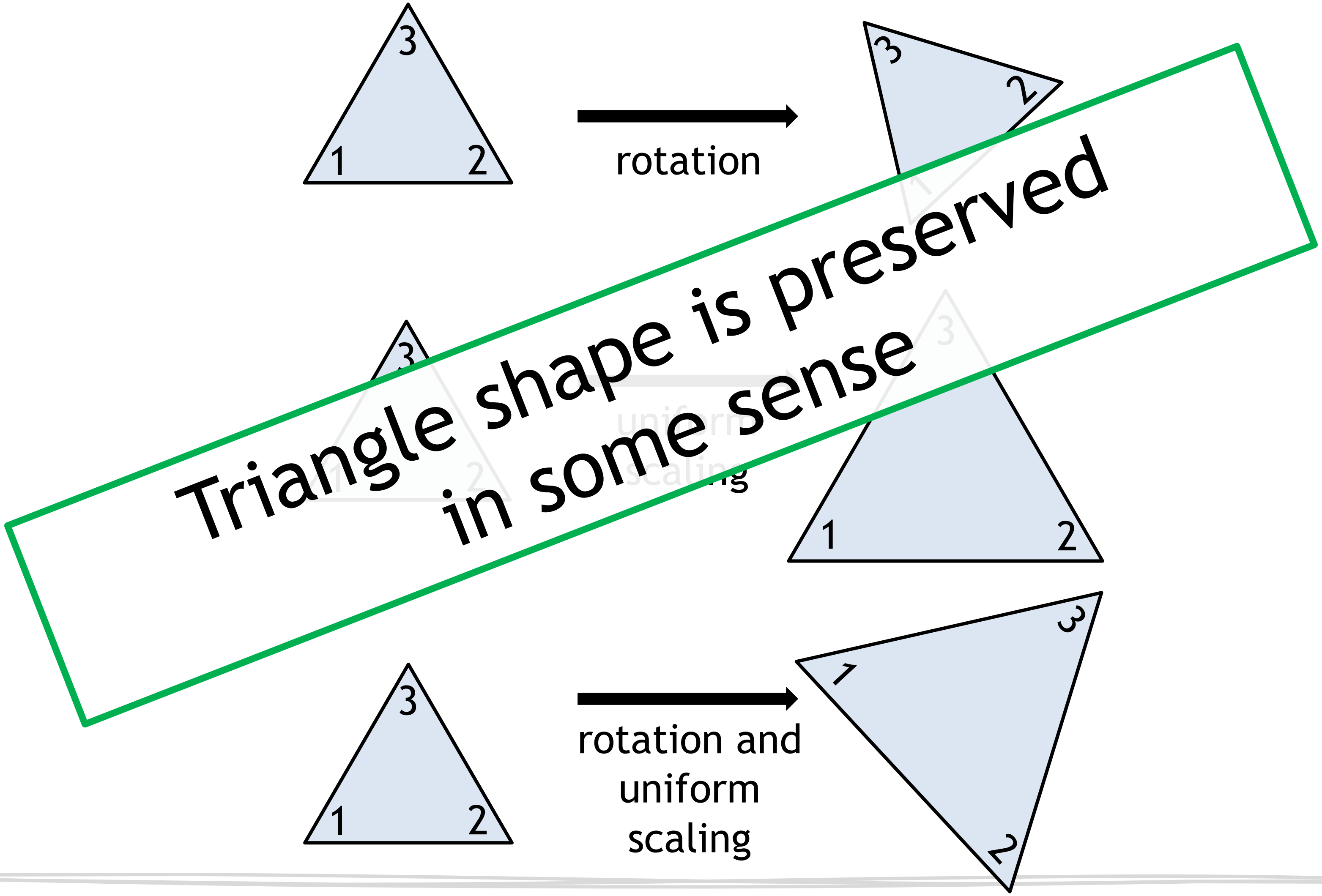
global coordinate system (u, v)

local coordinate system (x, y) for triangle T in 3D

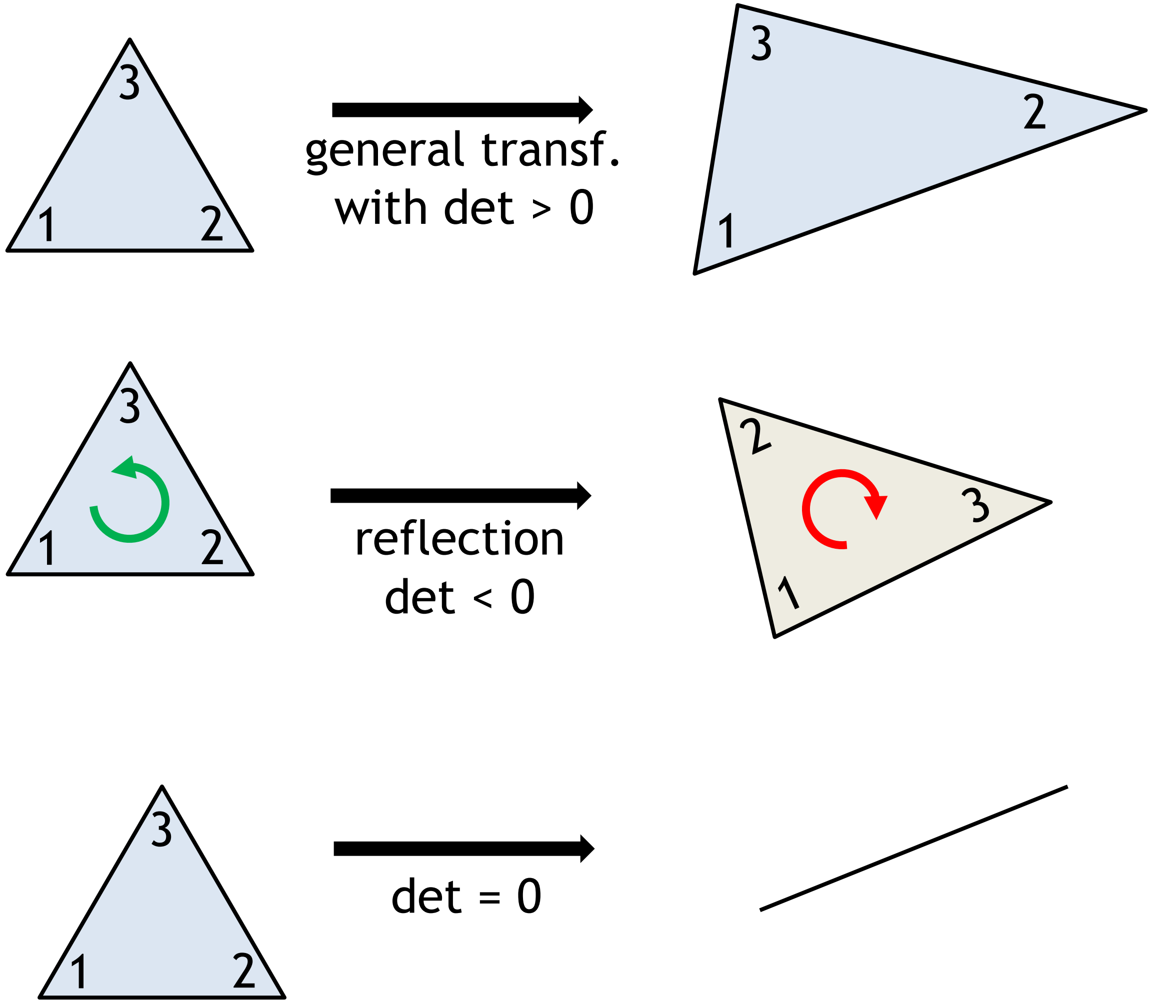
Jacobian J_T and triangle deformation



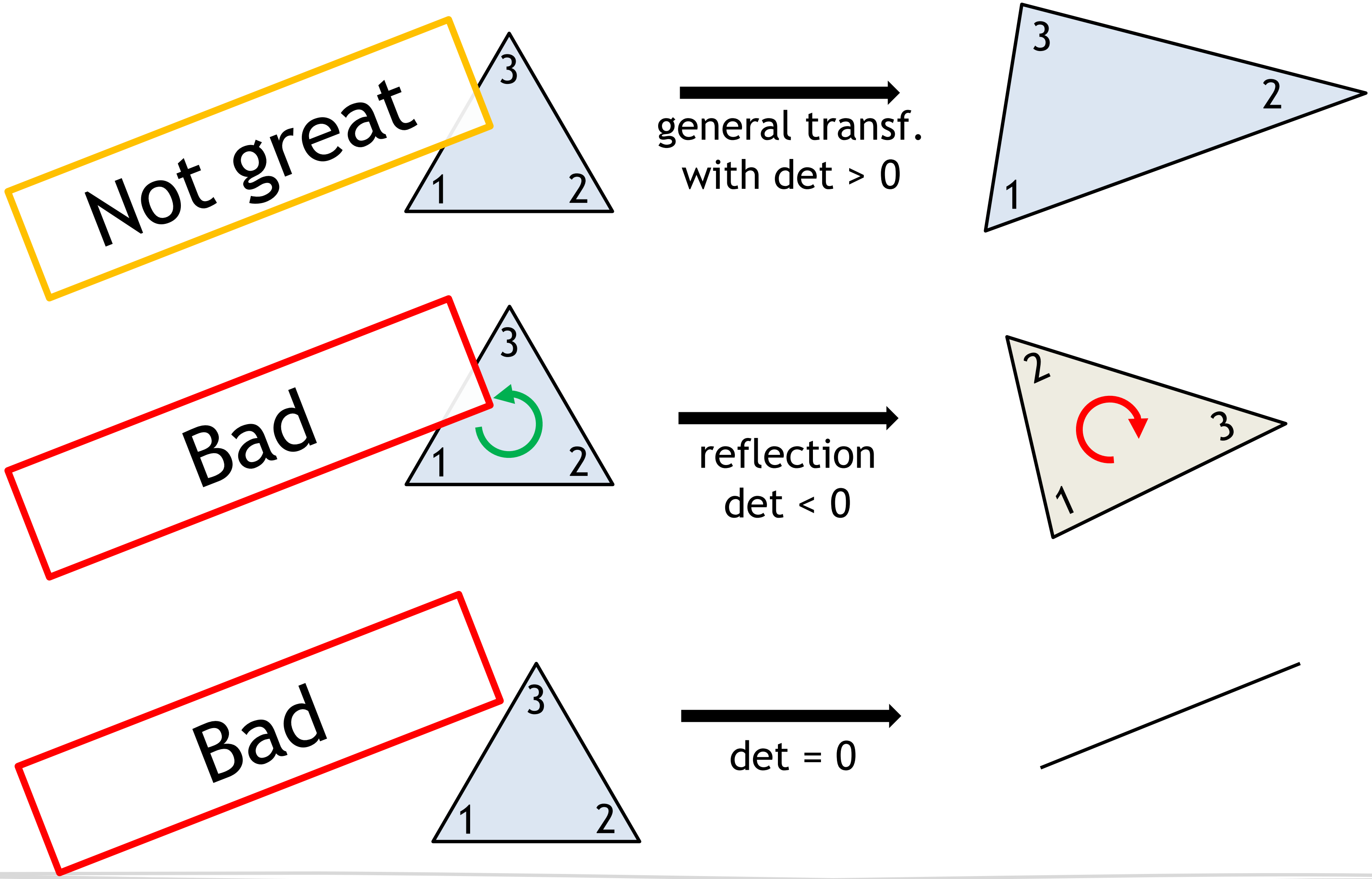
Jacobian J_T and triangle deformation



Jacobian J_T and triangle deformation



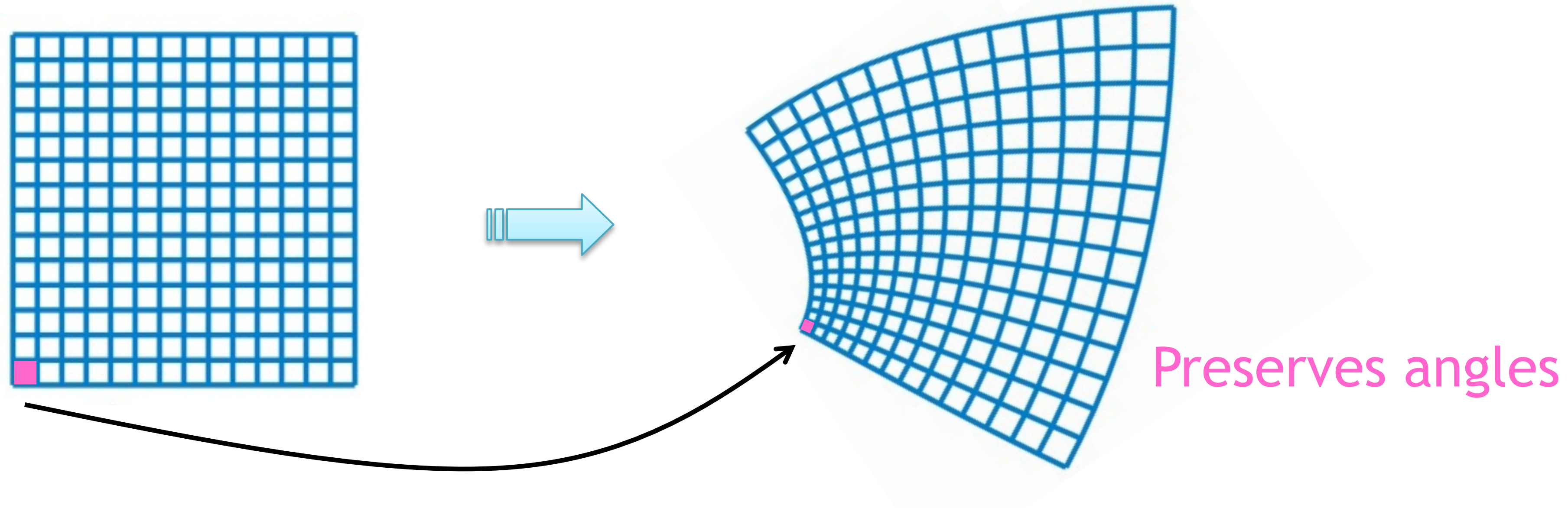
Jacobian J_T and triangle deformation



LSCM: Least Squares Conformal Maps

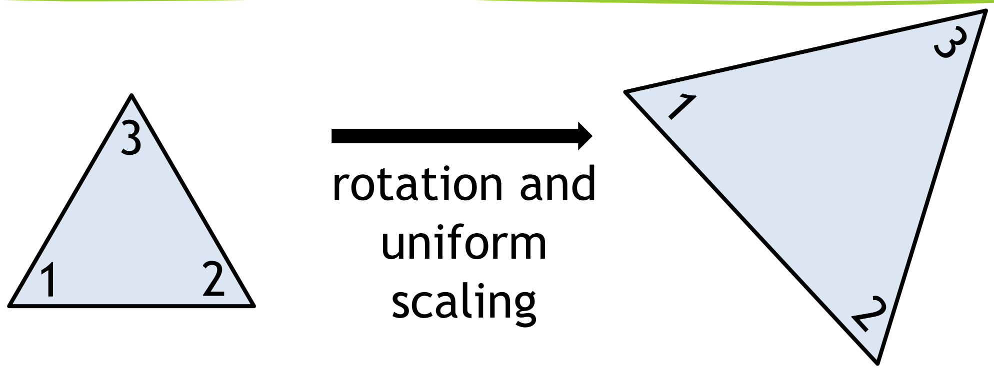
Conformal maps

- Smooth maps that preserve angles
- Straight lines are mapped to curves



Similarity = Rotation + Scale

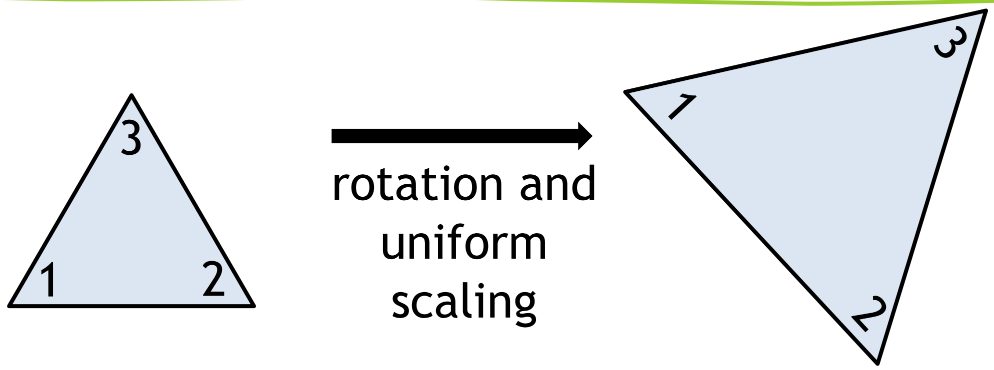
LSCM aims to preserve *triangle* angles



$$J_T = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$

$$s \in \mathbb{R}^+$$

LSCM aims to preserve angles



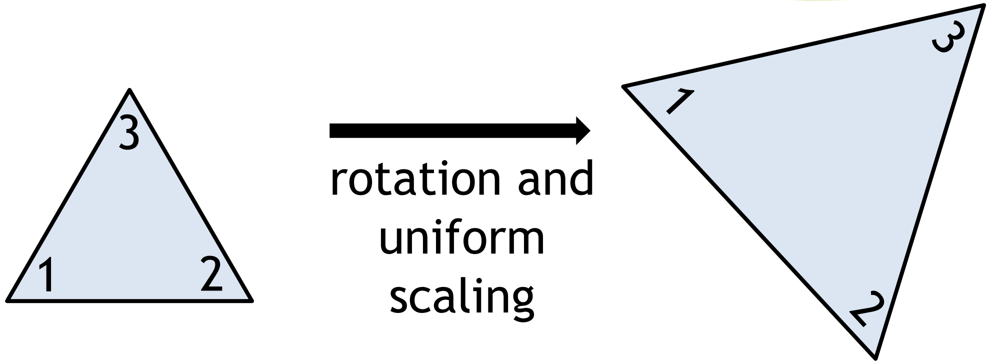
$$J_T = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} s\cos\phi & s\sin\phi \\ -s\sin\phi & s\cos\phi \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$s \in \mathbb{R}^+$

$a, b \in \mathbb{R}$

$$a^2 + b^2 = s^2(\cos^2\phi + \sin^2\phi) = s^2$$

LSCM aims to preserve angles



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$s \in \mathbb{R}^+$

$a, b \in \mathbb{R}$

$$a^2 + b^2 = s^2(\cos^2\phi + \sin^2\phi) = s^2$$

Simple, linear "template" for J_T: $J_T = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, $a, b \in \mathbb{R}$, avoid $(a, b) = (0, 0)$

Recall: J_T is linear in unknown uv values

$$\operatorname*{argmin}_{X \in \mathbb{R}^{2n}} \sum_{\text{triangles } T} A_T\, E(T)$$

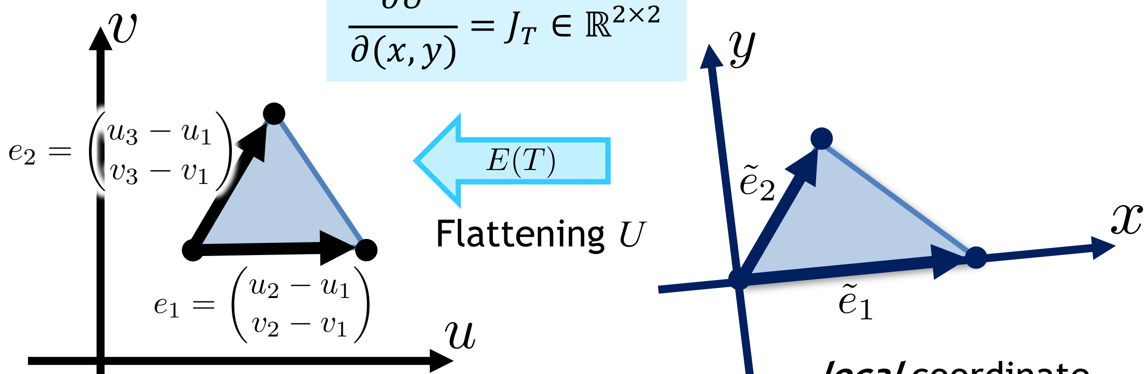
$$X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^\mathsf{T}$$

$$J_T \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \Rightarrow J_T = \underbrace{\begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix}}_{\begin{pmatrix} u_2 - u_1 & u_3 - u_1 \\ v_2 - v_1 & v_3 - v_1 \end{pmatrix}} \underbrace{\begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix}^{-1}}$$

Linear in our unknowns X!

Constant 2x2 matrix, depends 3D triangle only

$$\frac{\partial U}{\partial(x,y)} = J_T \in \mathbb{R}^{2\times 2}$$



global coordinate system (u, v)

local coordinate system (x, y) for triangle T in 3D

J_T **is linear** in unknowns X **!**

Recall: J_T is linear in unknown uv values

$$\operatorname*{argmin}_{X \in \mathbb{R}^{2n}} \sum_{\text{triangles } T} A_T \, E(T)$$

$$X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^\mathsf{T}$$

$$J_T \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \;\Rightarrow\; J_T = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix}^{-1}$$

J_T is linear in unknowns X !

$$J_T = \begin{pmatrix} j_{11}(X) & j_{12}(X) \\ j_{21}(X) & j_{22}(X) \end{pmatrix}, \quad j_{11}(X), j_{12}(X), j_{21}(X), j_{22}(X) \text{ are linear in } X$$

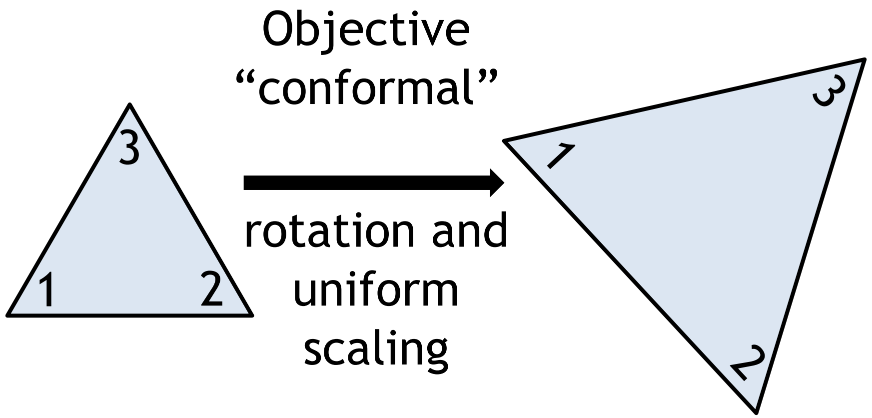
LSCM energy formulation

$$\underset{X\in\mathbb{R}^{2n}}{\operatorname{argmin}} \sum_{\text{triangles } T} A_T\, E(T)$$

$$X = (u_1, v_1, u_2, v_2, \dots, u_n, v_n)^\mathsf{T}$$

$$J_T \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \Rightarrow J_T = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix}^{-1}$$

$J_T = \begin{pmatrix} j_{11}(X) & j_{12}(X) \\ j_{21}(X) & j_{22}(X) \end{pmatrix}$, $j_{11}(X), j_{12}(X), j_{21}(X), j_{22}(X)$ are linear in X



linear "template" for J_T: $J_T = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, $a, b \in \mathbb{R}$, avoid $(a,b) = (0,0)$

LSCM energy formulation

$$\underset{X \in \mathbb{R}^{2n}}{\operatorname{argmin}} \sum_{\text{triangles } T} A_T \, E_{\text{LSCM}}(T)$$

$$X = (u_1, v_1, u_2, v_2, \ldots, u_n, v_n)^\mathsf{T}$$

$$J_T \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \;\Rightarrow\; J_T = \begin{pmatrix} | & | \\ e_1 & e_2 \\ | & | \end{pmatrix} \begin{pmatrix} | & | \\ \tilde{e}_1 & \tilde{e}_2 \\ | & | \end{pmatrix}^{-1}$$

$J_T = \begin{pmatrix} j_{11}(X) & j_{12}(X) \\ j_{21}(X) & j_{22}(X) \end{pmatrix}$, $j_{11}(X), j_{12}(X), j_{21}(X), j_{22}(X)$ are linear in X

linear "template" for J_T: $J_T = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, $a, b \in \mathbb{R}$, avoid $(a,b) = (0,0)$

$$E_{\text{LSCM}}(T) = (j_{11}(X) - j_{22}(X))^2 + (j_{12}(X) - (-j_{21}(X)))^2$$

$E_{\text{LSCM}}(T)$ is quadratic in X!

Minimizer is found by solving a sparse linear system in X.
Must avoid trivial solutions!

Null space of ∇E_{LSCM}

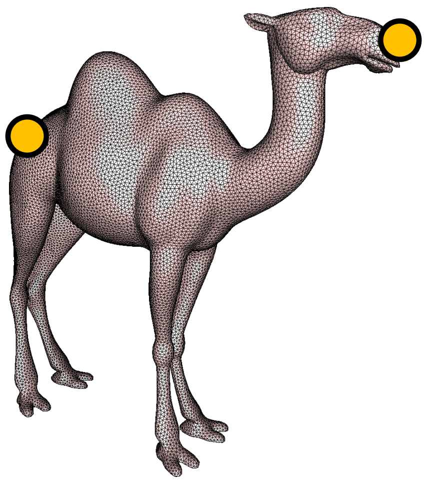
- As presented, E_{LSCM} attains its (global and only) minimum of value 0 if:
  - for a general input 3D mesh:
    all mesh vertices are assigned to a single (u,v) point
    $\rightarrow$ the null space is 2-dimensional
  - for a discrete-developable input 3D mesh (e.g. input put is already flat, or can be flattened without any distortion to any triangle). Then taking (u,v) coordinates that are a perfect unfolding of the mesh to the plane, or rotations, translations and *uniform scalings* thereof leads to zero energy.
    This is a rare and unlikely case.

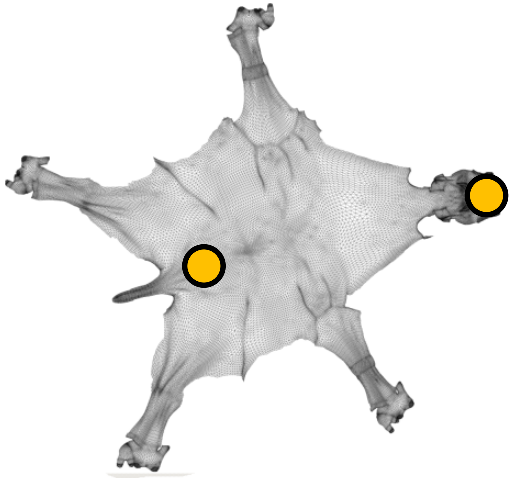
Null space of ∇E_{LSCM}

- As presented, E_{LSCM} attains its (global and only) minimum of value *0* if:
  - for a general input 3D mesh:
    all mesh vertices are assigned to a single (u,v) point
    $\rightarrow$ the null space is 2-dimensional
- This is a collapsed solution that must be avoided by fixing more than just 1 vertex (2 unknowns)!
- To avoid collapse to a single point, we need to fix the locations of 2 vertices to determine the degrees of freedom.

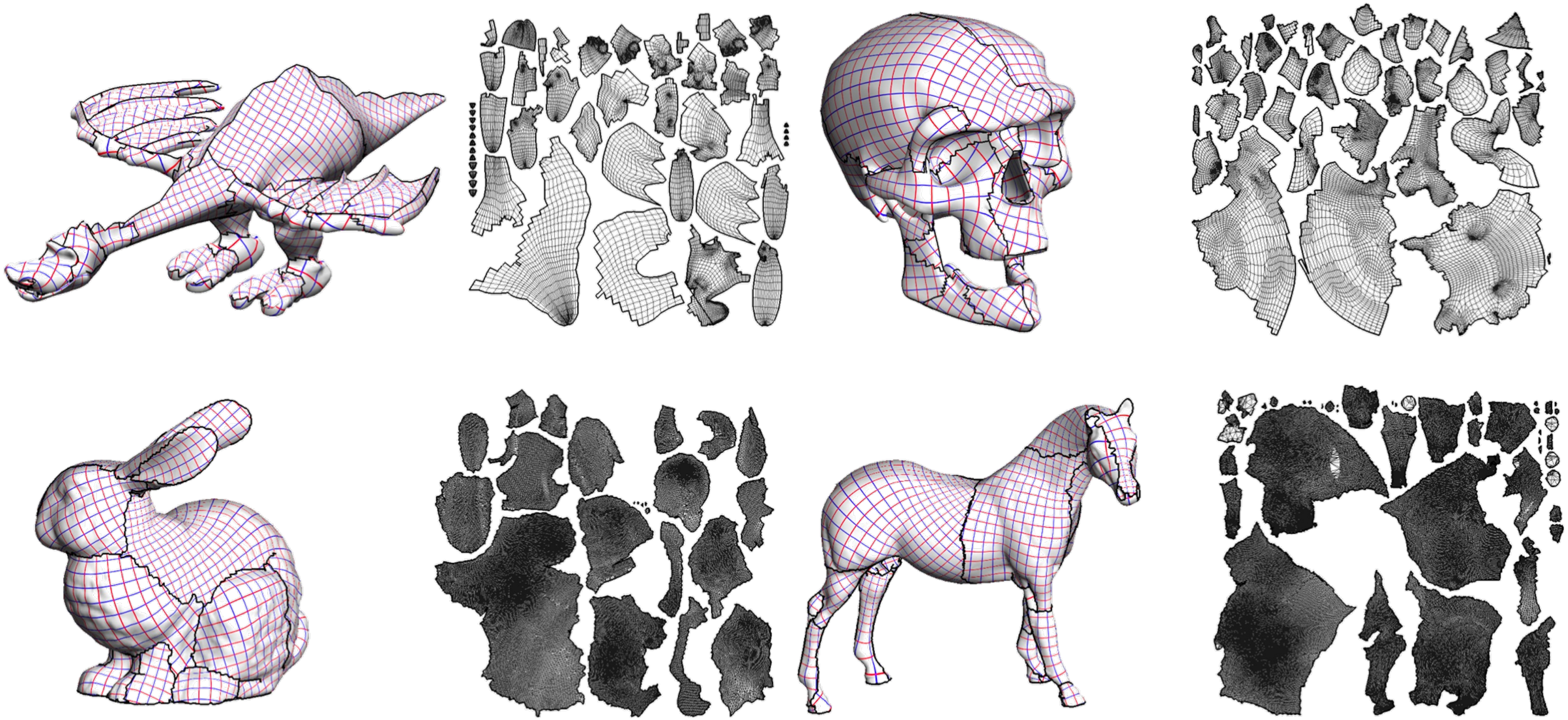
LSCM degrees of freedom

- [Levy et al. 2002] : fix 2 vertices in 2D that have the maximal shortest edge path between them.
- Leads to a sparse linear system in (2n-4) variables.
- System then has a unique and non-collapsed solution.
- The solution depends on the choice of vertices!

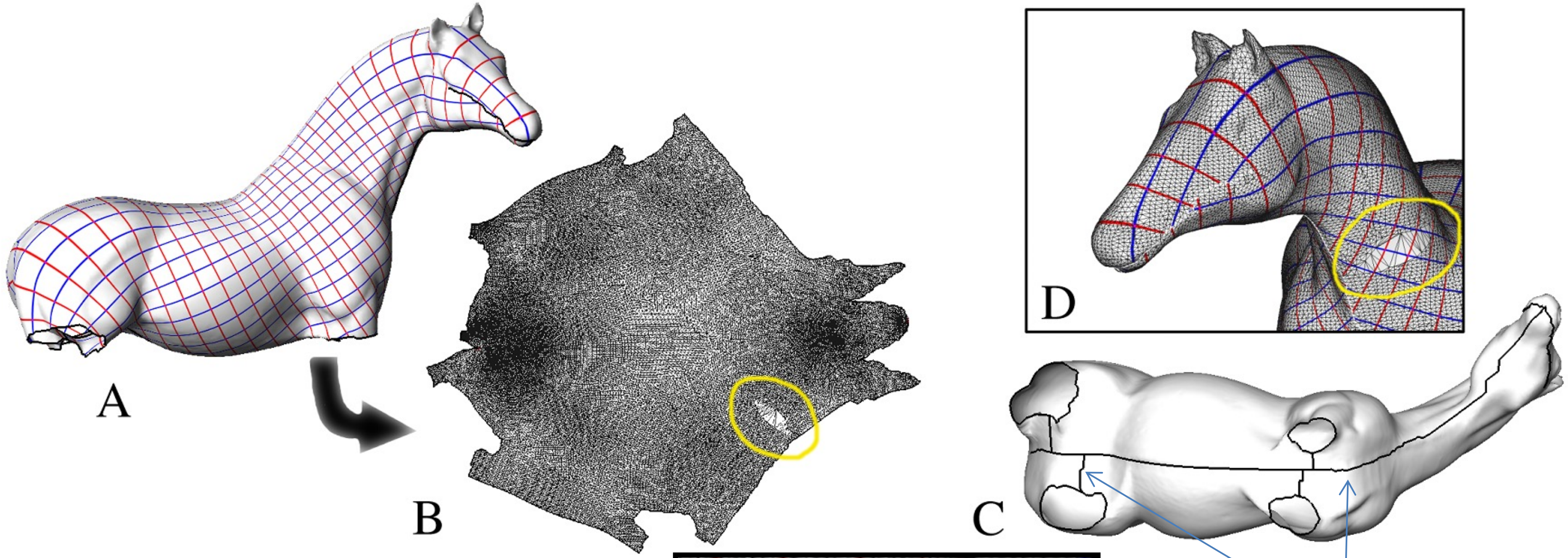




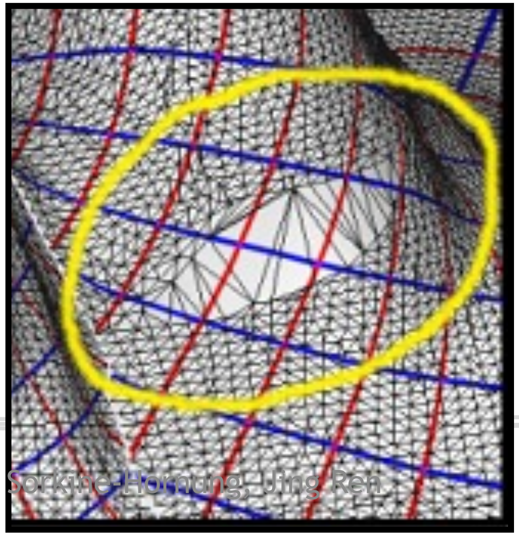
LSCM – Results



LSCM – Results



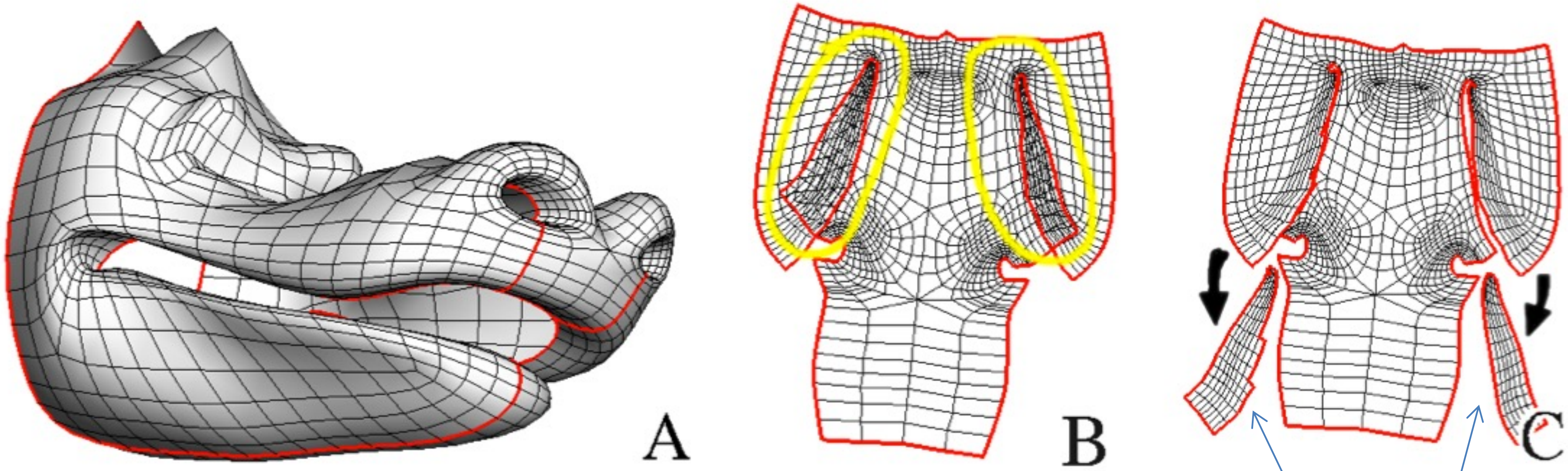
Manually placed cuts



Flipped triangles rarely occur

🡐 Robust against varying mesh density

Overlaps can still occur



Cut along overlapping boundaries

Triangle inversions and collapses can still occur (but rather rare)

- We get the J_T matrix template *only in least-squares sense*

$$J_T = \begin{pmatrix} j_{11}(X) & j_{12}(X) \\ j_{21}(X) & j_{22}(X) \end{pmatrix},\ j_{11}(X), j_{12}(X), j_{21}(X), j_{22}(X) \text{ are linear in } X$$

$$J_T = \begin{pmatrix} a & b \\ -b & a \end{pmatrix},\quad a, b \in \mathbb{R},\ \text{avoid } (a, b) = (0, 0)$$

$$E_{\mathrm{LSCM}}(T) = (j_{11}(X) - j_{22}(X))^2 + \big(j_{12}(X) - (-j_{21}(X))\big)^2$$

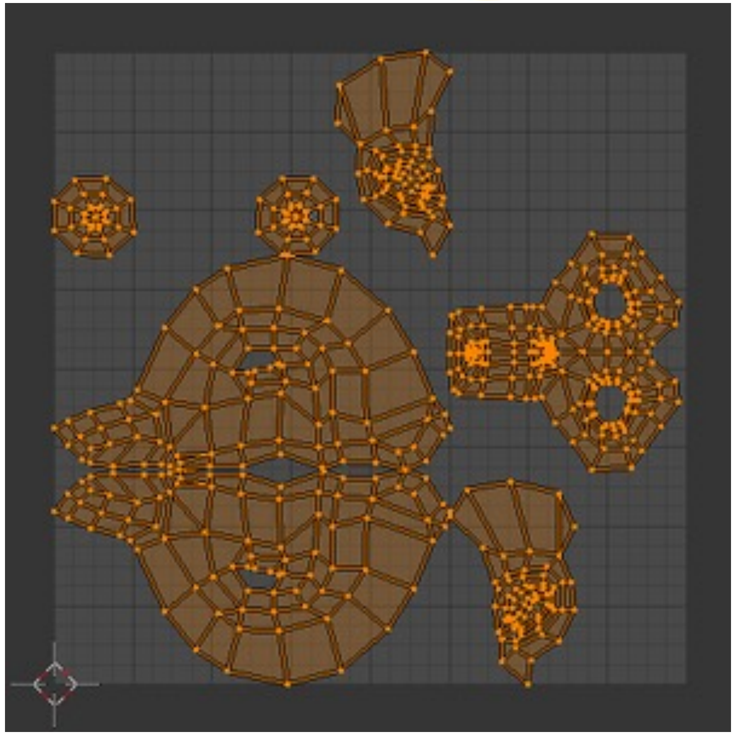
Texture mapped results

Provided in many packages

- Often called “UV-unwrapping”

- Blender
- Autodesk Maya, 3ds Max

- CGAL

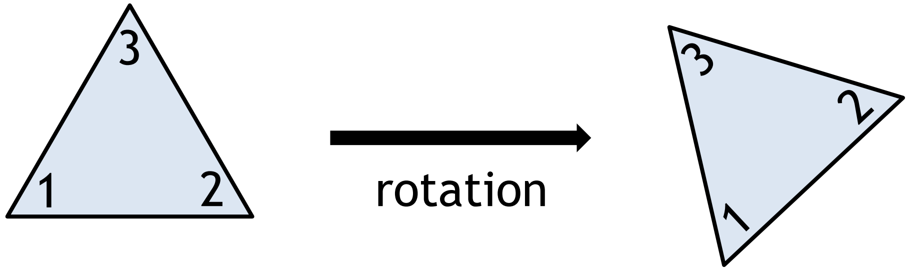


https://docs.blender.org/manual/en/latest/modeling/meshes/editing/uv.html

ARAP:
As-Isometric-As-Possible Parameterization

(Rigid)

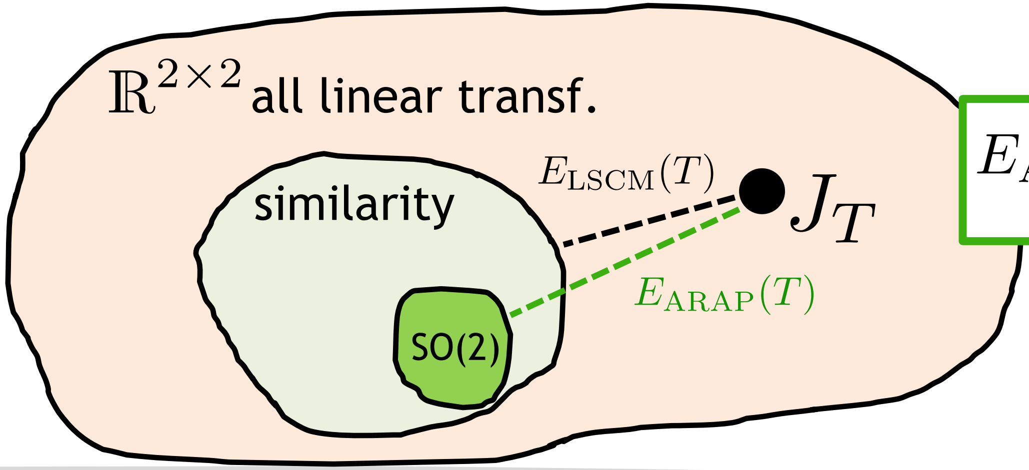
ARAP energy



$J_T \in \mathbb{R}^{2\times 2}$, linear function of X

$E_{\text{ARAP}}(T) =$ distance between J_T and a 2×2 rotation

= matrix in SO(2)



$$E_{\text{ARAP}}(T) = \min_{R_T \in SO(2)} \|J_T - R_T\|_F^2$$

Frobenius matrix norm: $\|(a_{ij})\|_F = \left(\sum_{i,j} a_{ij}^2\right)^{1/2}$

ARAP (local-global) algorithm

$$E_{\text{ARAP}}(X) = \sum_{\text{triangles } T} A(T) \min_{R_T \in SO(2)} \|J_T - R_T\|_F^2$$

- How to minimize efficiently? Local-global iterations!

1. Initialize X with LSCM or Tutte or smth
2. For each T, compute R_T using the current uv's, i.e. the current value of X
3. Fix all R_T's and solve for new X
4. If ~(stopping criterion) goto 2

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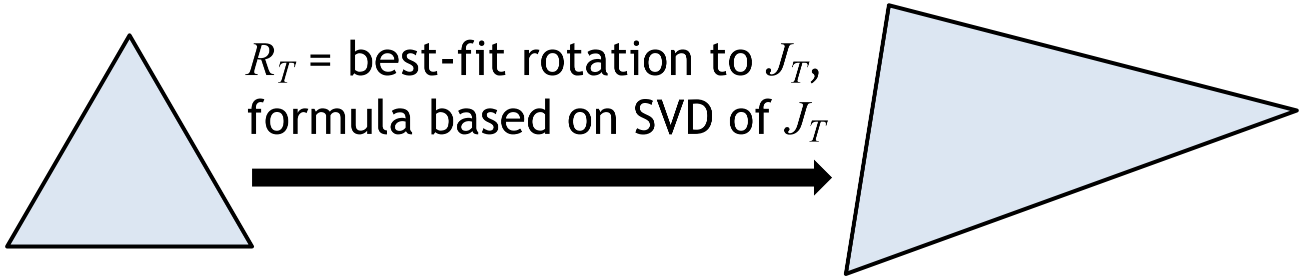

Procrustes Problem

$$R_T = \mathrm{argmin}_{R \in SO(2)} \|R - J_T\|_F$$
$$= \mathrm{argmin}_{R \in SO(2)} \|R - U\Sigma V^T\|_F$$
$$R_T = UV^T$$

ARAP (local-global) algorithm

$$E_{\text{ARAP}}(X) = \sum_{\text{triangles } T} A(T) \min_{R_T \in SO(2)} \|J_T - R_T\|_F^2$$

- How to minimize efficiently? Local-global iterations!

1. Initialize X with LSCM or Tutte or…
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ARAP (local-global) algorithm

$$E_{\text{ARAP}}(X) = \sum_{\text{triangles } T} A(T) \min_{R_T \in SO(2)} \|J_T - R_T\|_F^2$$

- $A(T)$: fixed scalar
- J_T: linear in X
- R_T: fixed

➡ Quadratic minimization in X (need to fix at least 1 vertex)

3. Fix all R_T's and solve for new X
4. If ~(stopping criterion) goto 2

ARAP (local-global) algorithm

$$E_{\text{ARAP}}(X) = \sum_{\text{triangles } T} A(T) \min_{R_T \in SO(2)} \|J_T - R_T\|_F^2$$

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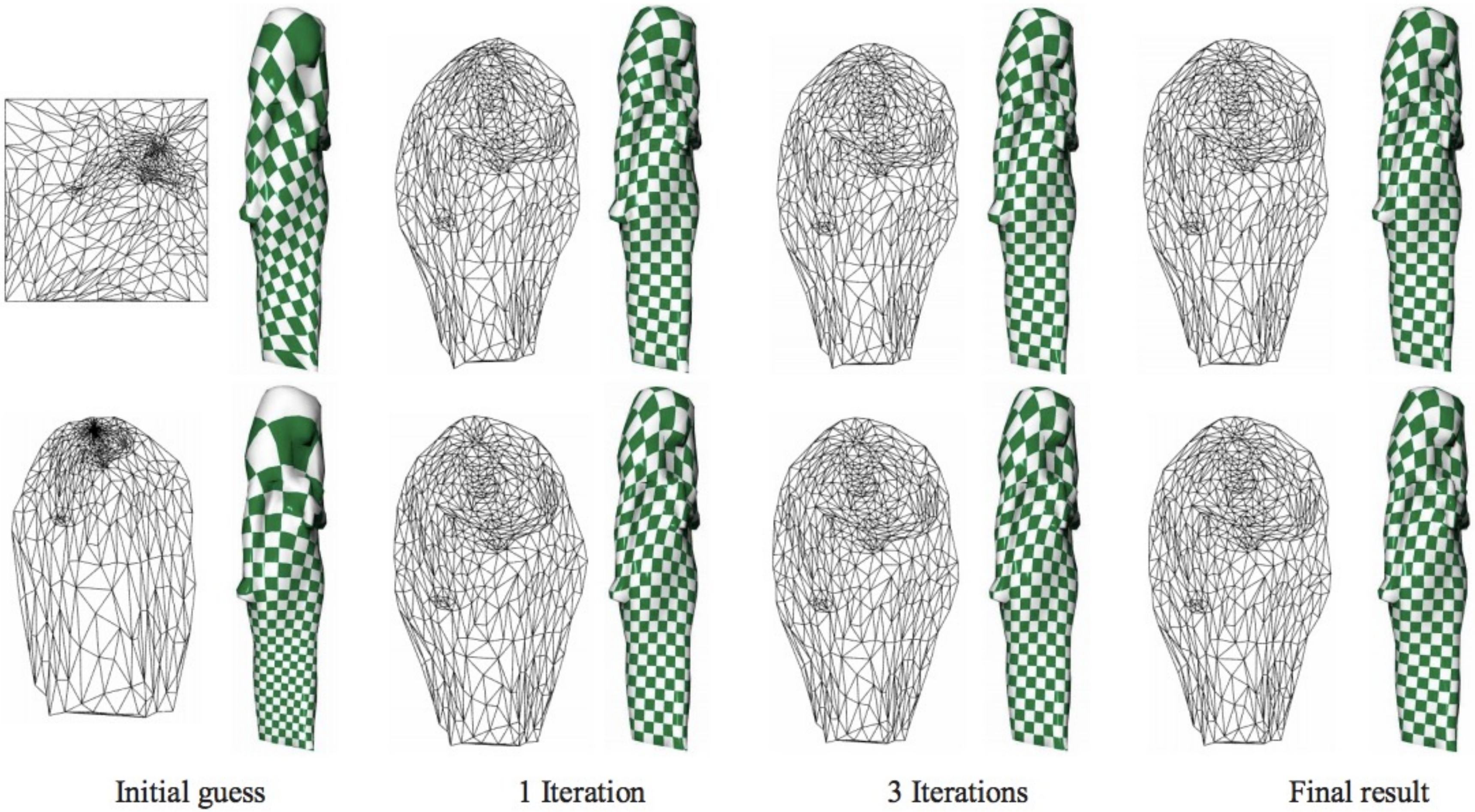
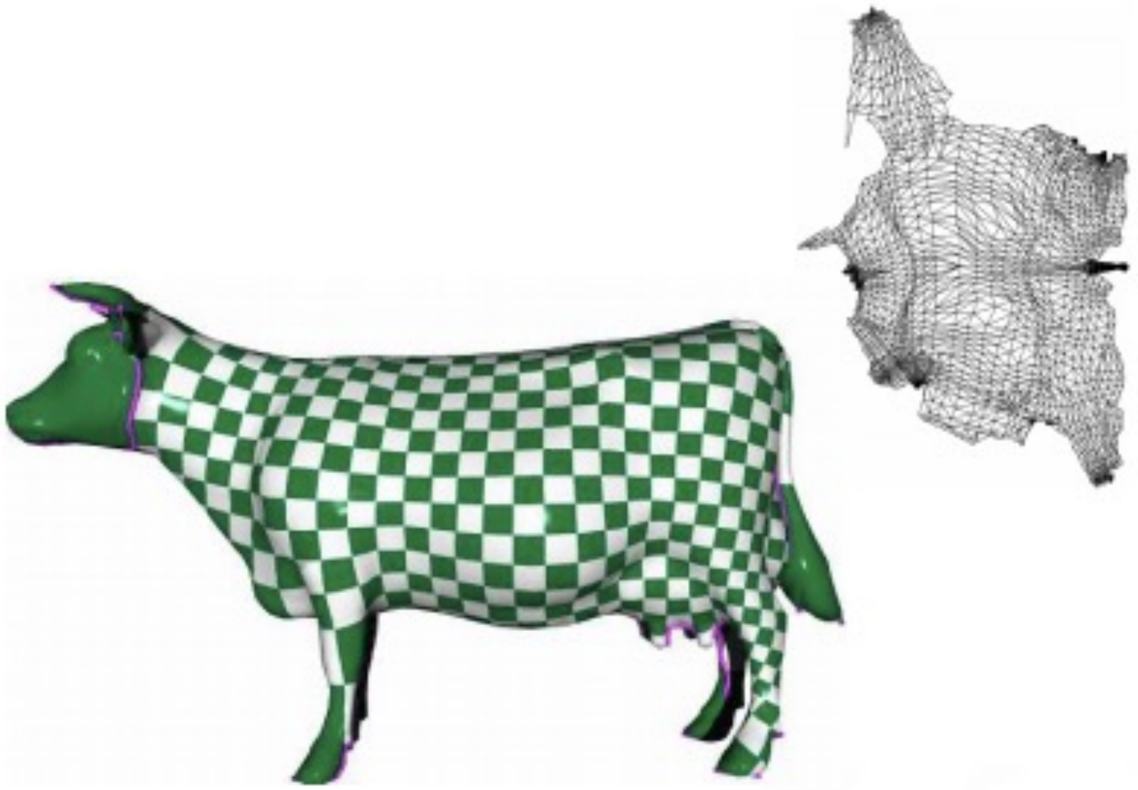
4. If ~(stopping criterion) goto 2

More details about rotation fitting and ARAP come later ☺

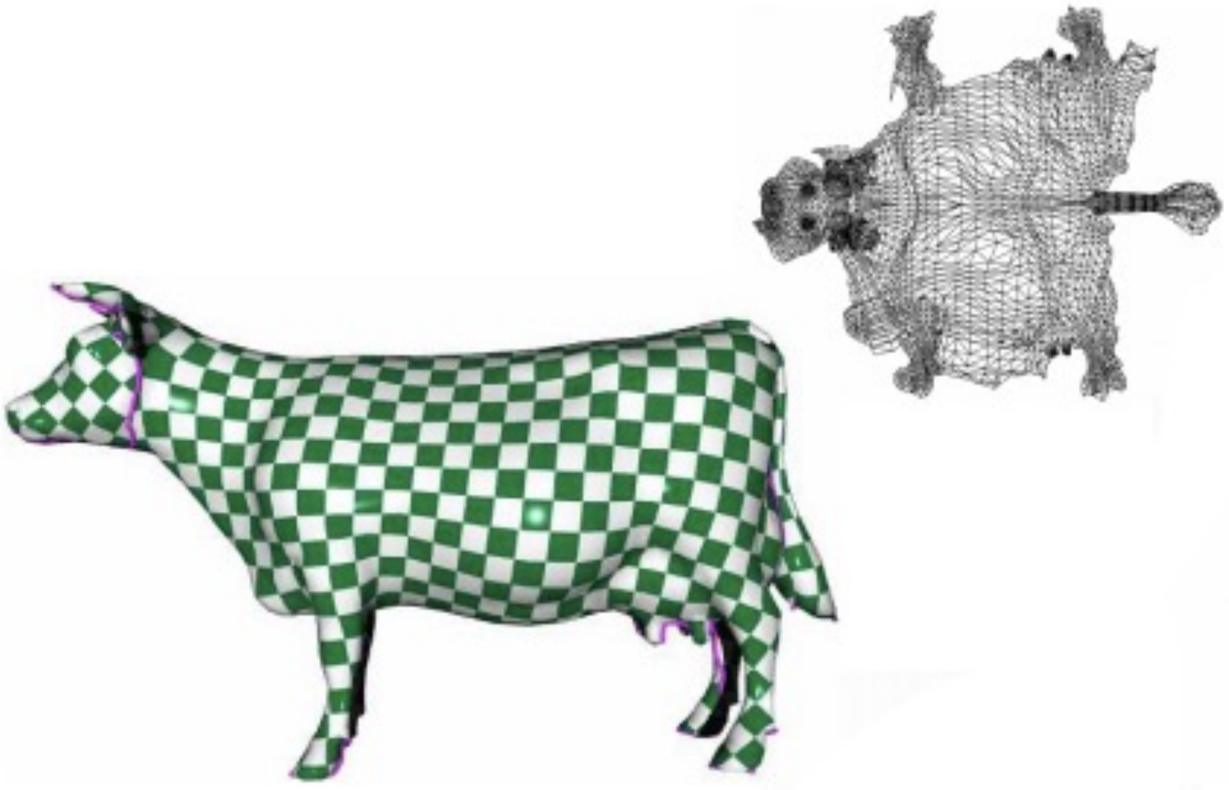
ARAP Param. – Results



ARAP Param. – Results



LSCM



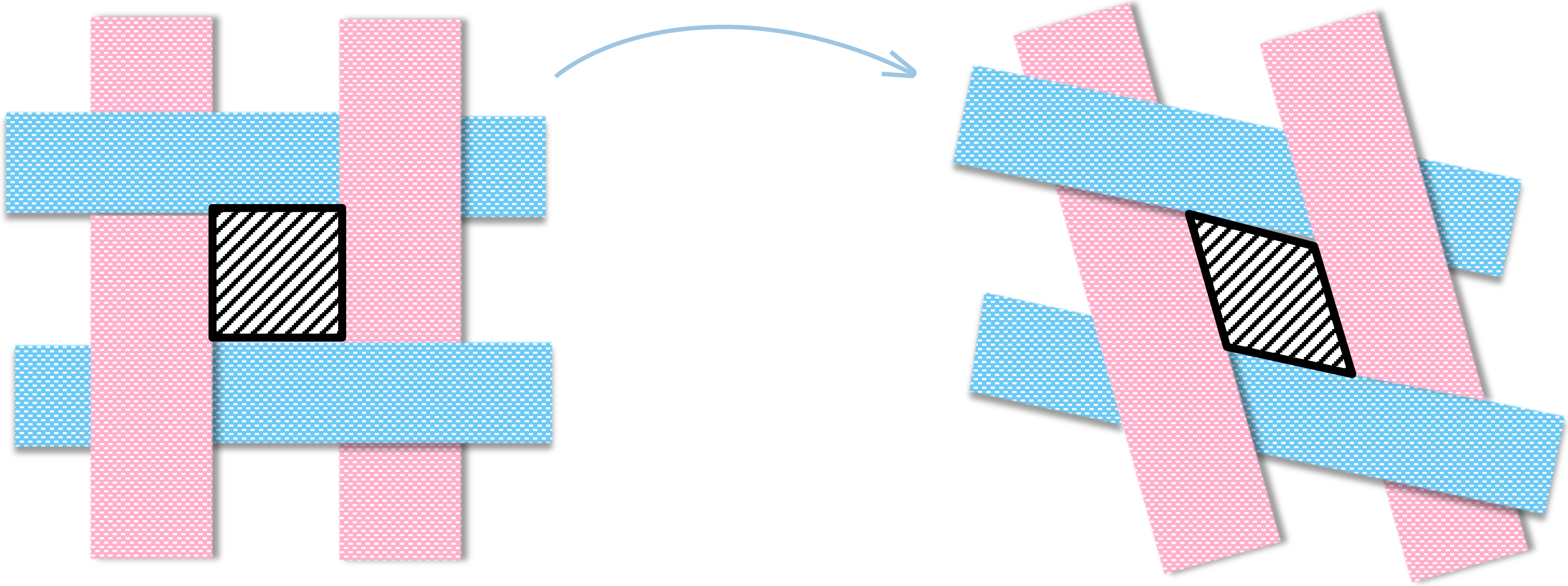
ARAP

Chebyshev Parameterzation:
preserving lengths along the yarn directions

igl

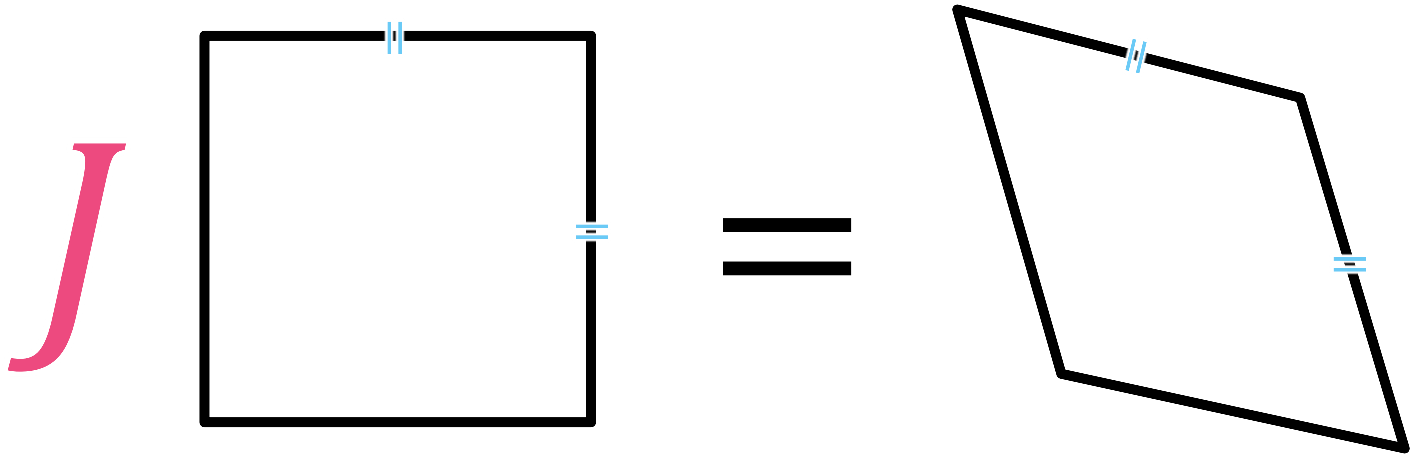
ETH zürich

Chebyshev net

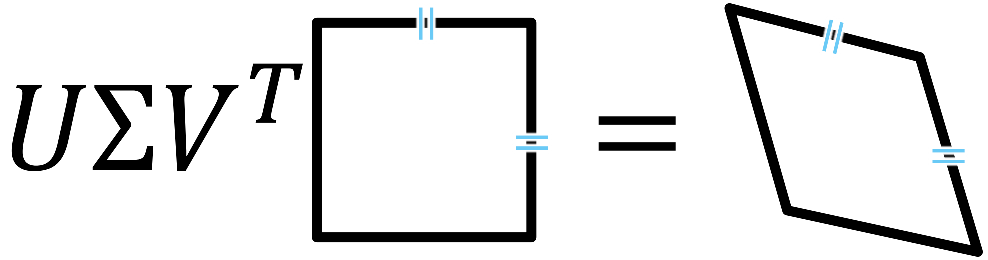


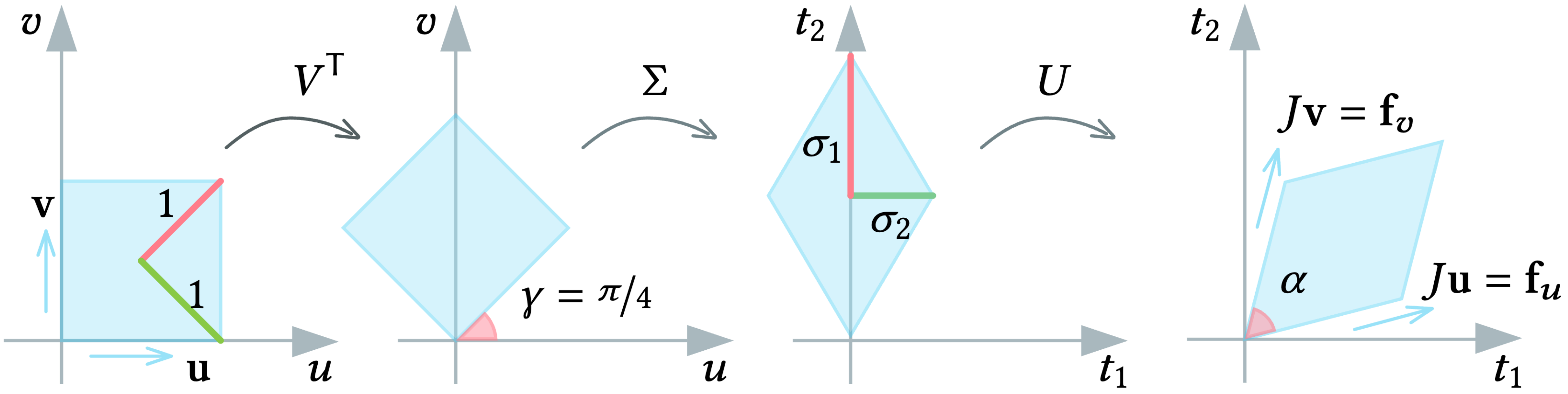
Lengths along the yarn directions are preserved → Squares are mapped to diamonds

SVD of the Jacobian of a Chebyshev net



SVD of the Jacobian of a Chebyshev net

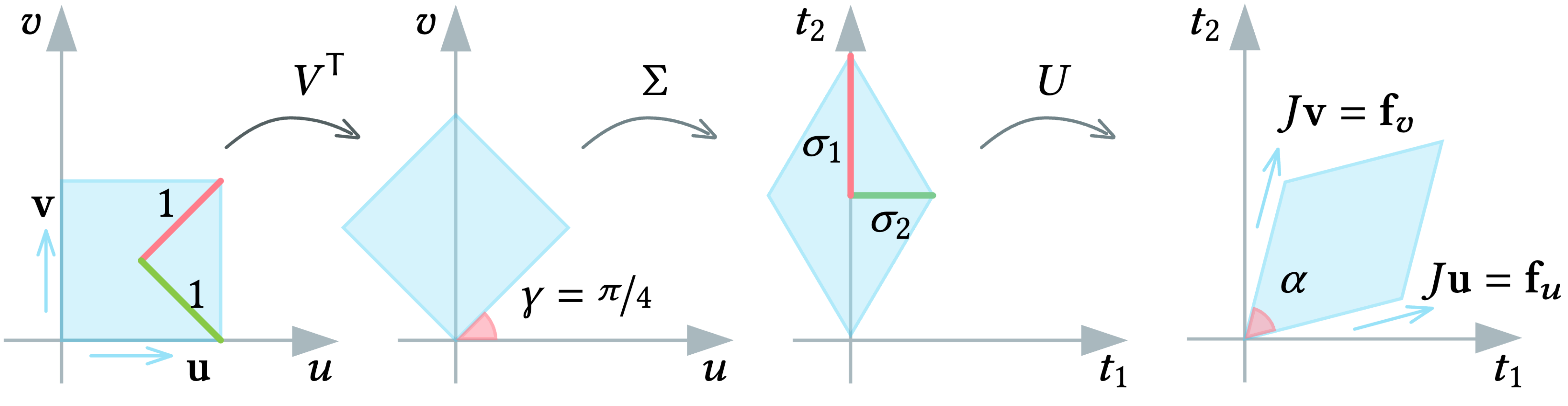




SVD of the Jacobian of a Chebyshev net

$J = U\Sigma V^T$ has a template:

- U is unknown rotation
- Singular values satisfy $\sigma_1^2 + \sigma_2^2 = 2$
- V is a known rotation (by angle $\pi/4$



Chebyshev Parameterization

$$E_{ARAP}(X) = \sum_{\text{triangles } T} A(T) \min_{R_T \in SO(2)} \| J_T - R_T \|_F^2$$

$J = U\Sigma V^T$ has a template:

- U is unknown rotation
- Singular values satisfy $\sigma_1^2 + \sigma_2^2 = 2$
- V is a known rotation (by angle $\pi/4$

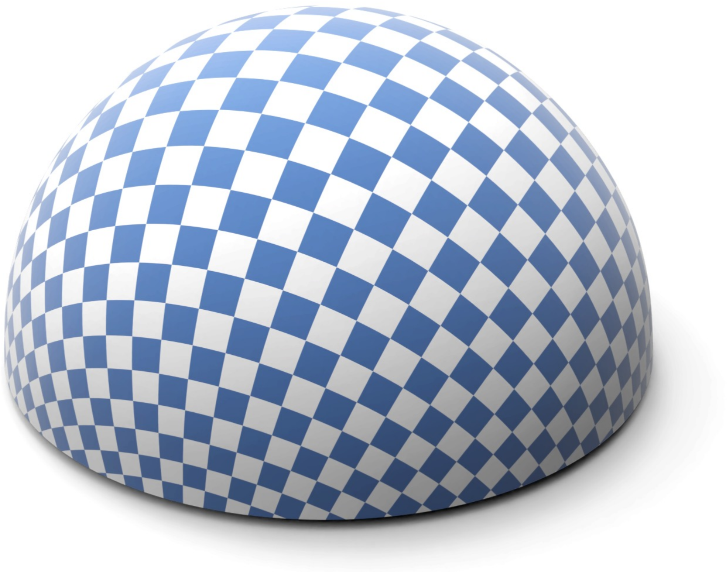
- Local-global iterations!

1. Initialize X with LSCM or Tutte or ARAP or smth
2. Local step: For each T, compute rotation and singular values using the current uv's, i.e. the current value of X
3. Global step: Fix all cheby-jacobians and solve for new X
4. If ~(stopping criterion) goto 2

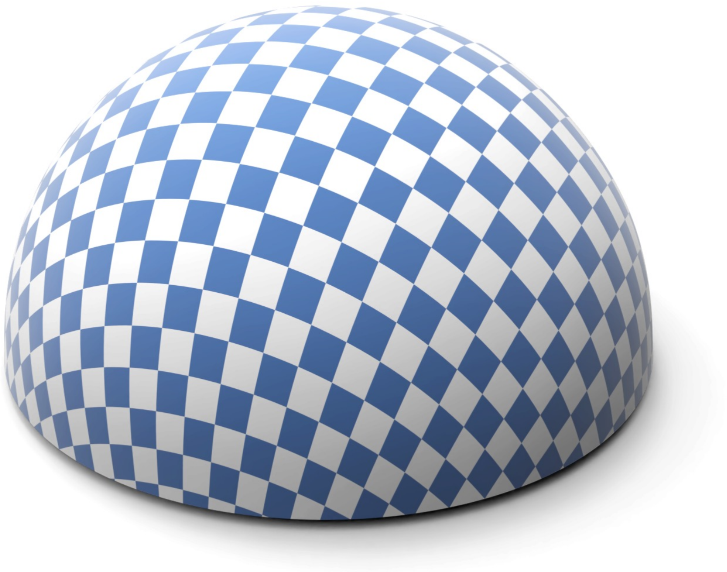
Note: the template here is derived from 2D to 3D; for parameterization we need to consider its inverse. See the paper for more details

LSCM, ARAP, vs. Chebyshev parameterization

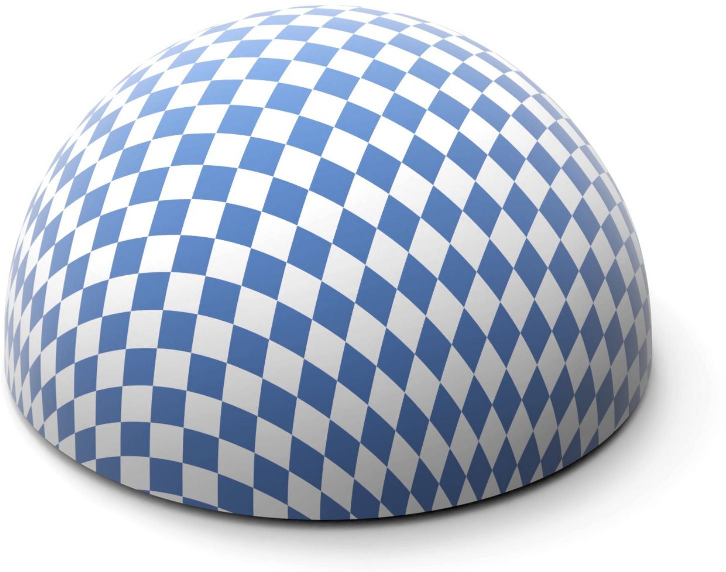
LSCM



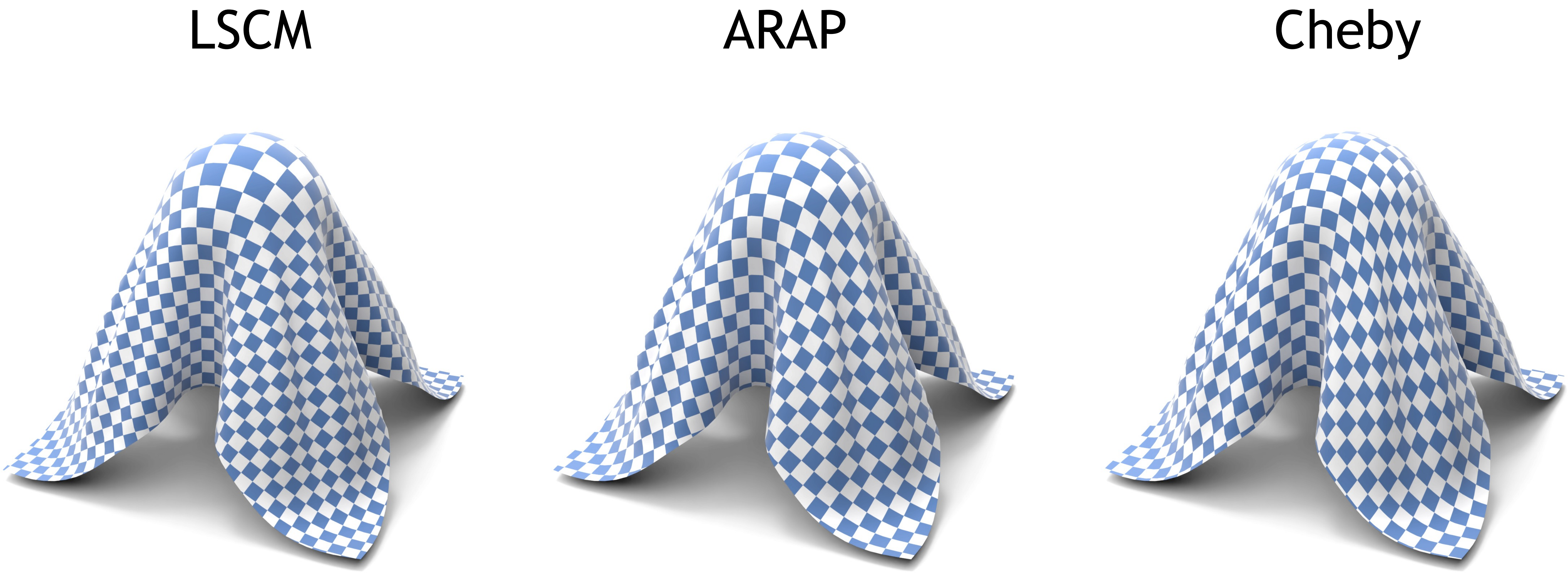
ARAP



Cheby



LSCM, ARAP, vs. Chebyshev parameterization



Remeshing

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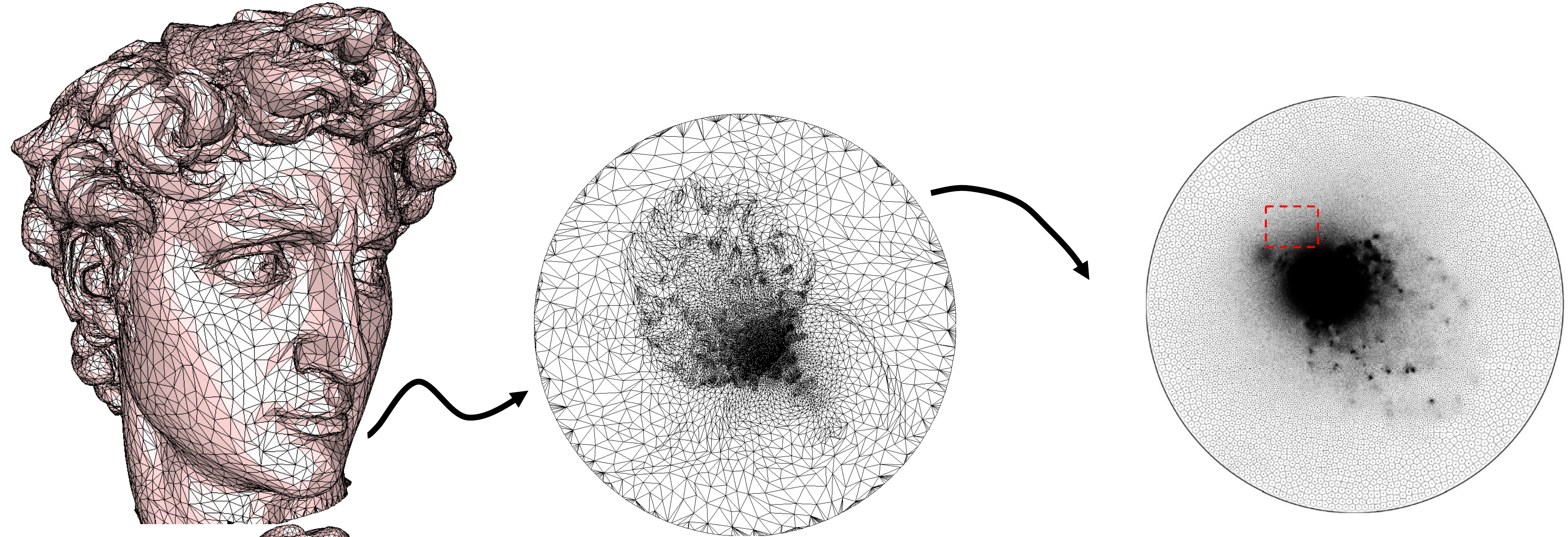
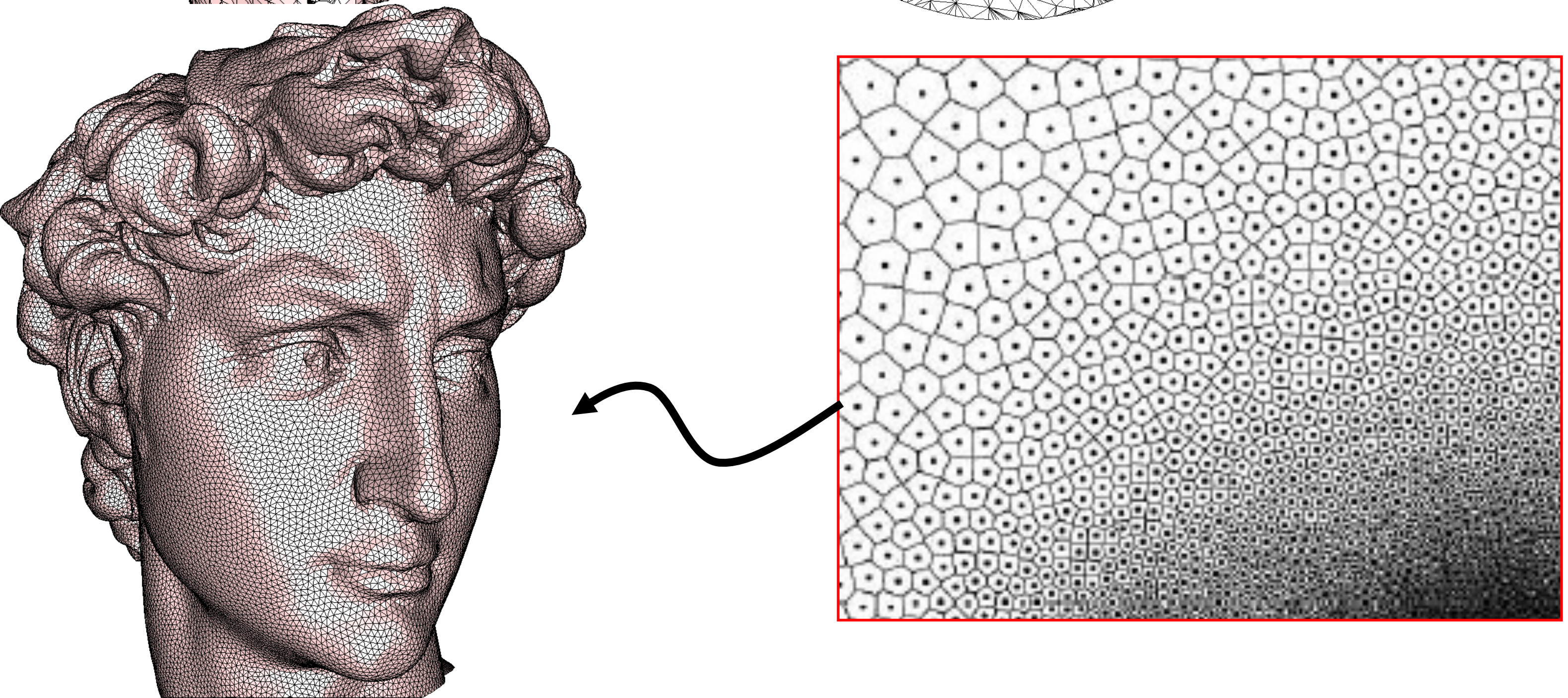
Swiss Federal Institute of Technology Zurich



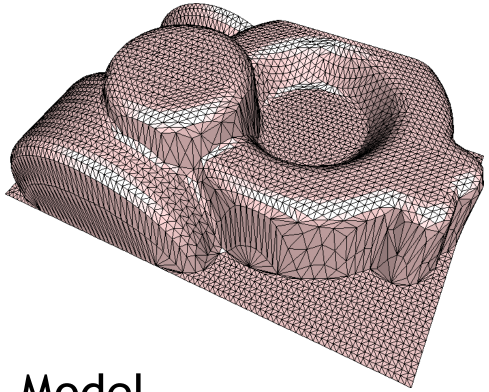


Remeshing

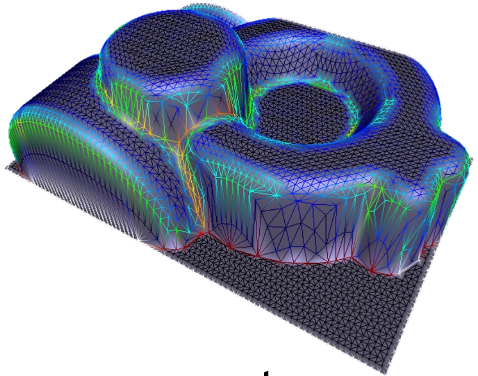
- Run parameterization
- Resample in the uv domain
  - w.r.t area distortion
- Centroidal Voronoi tessellation on the samples
- Map back to 3D surface

Interactive Geometry Remeshing

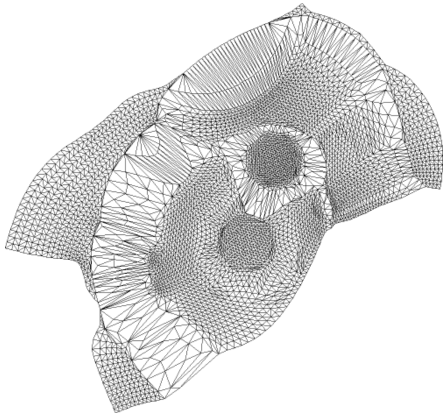
[Alliez et al., SIGGRAPH 2002]



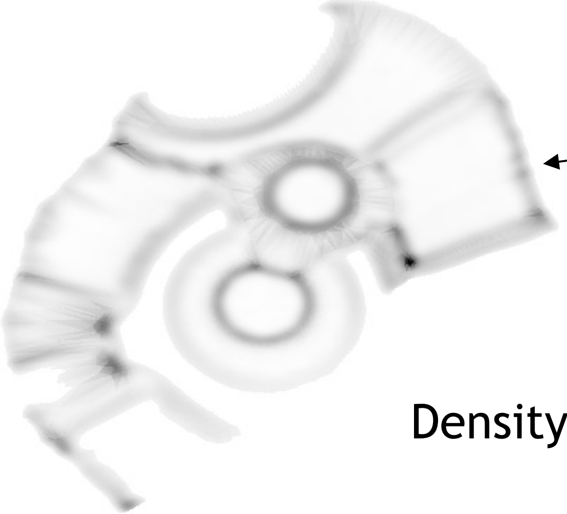
Model



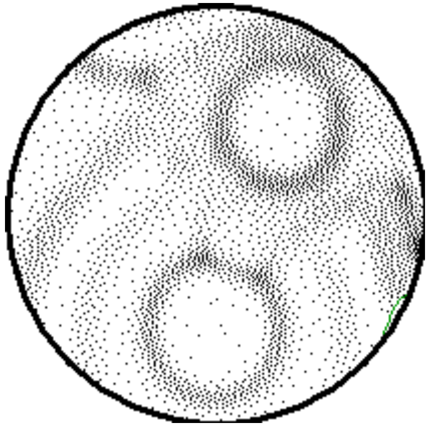
Measure curvature



Flatten it **conformally**

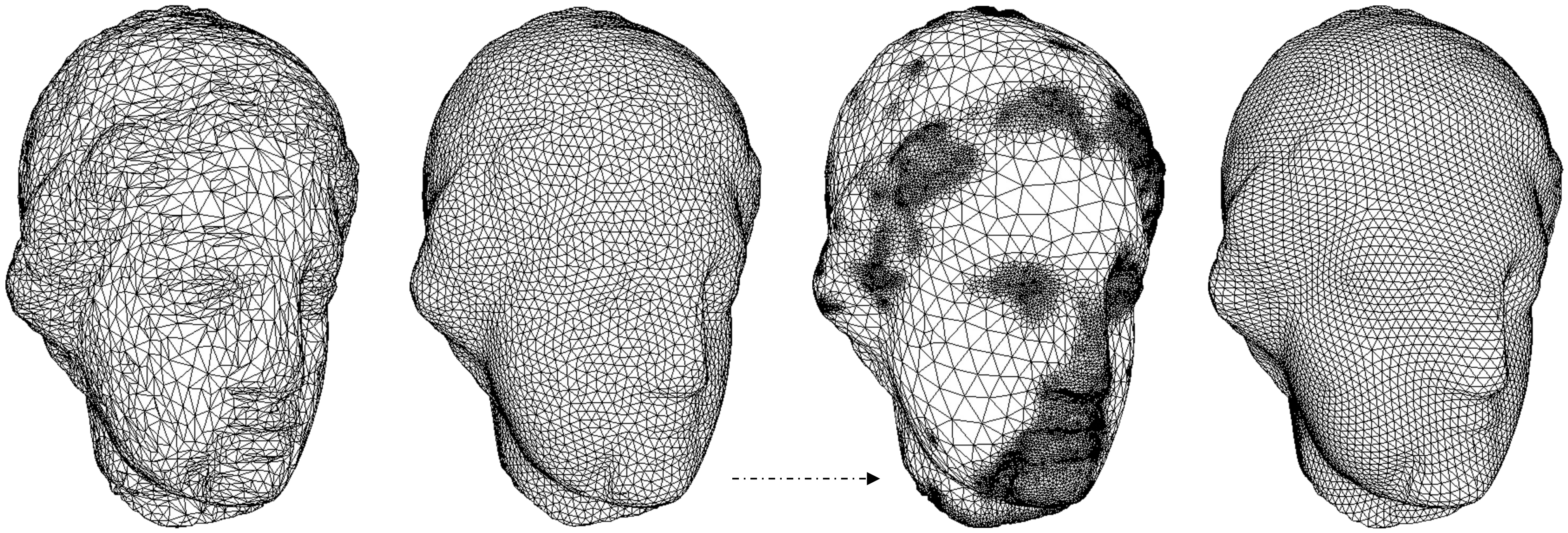


Density function in parameter space



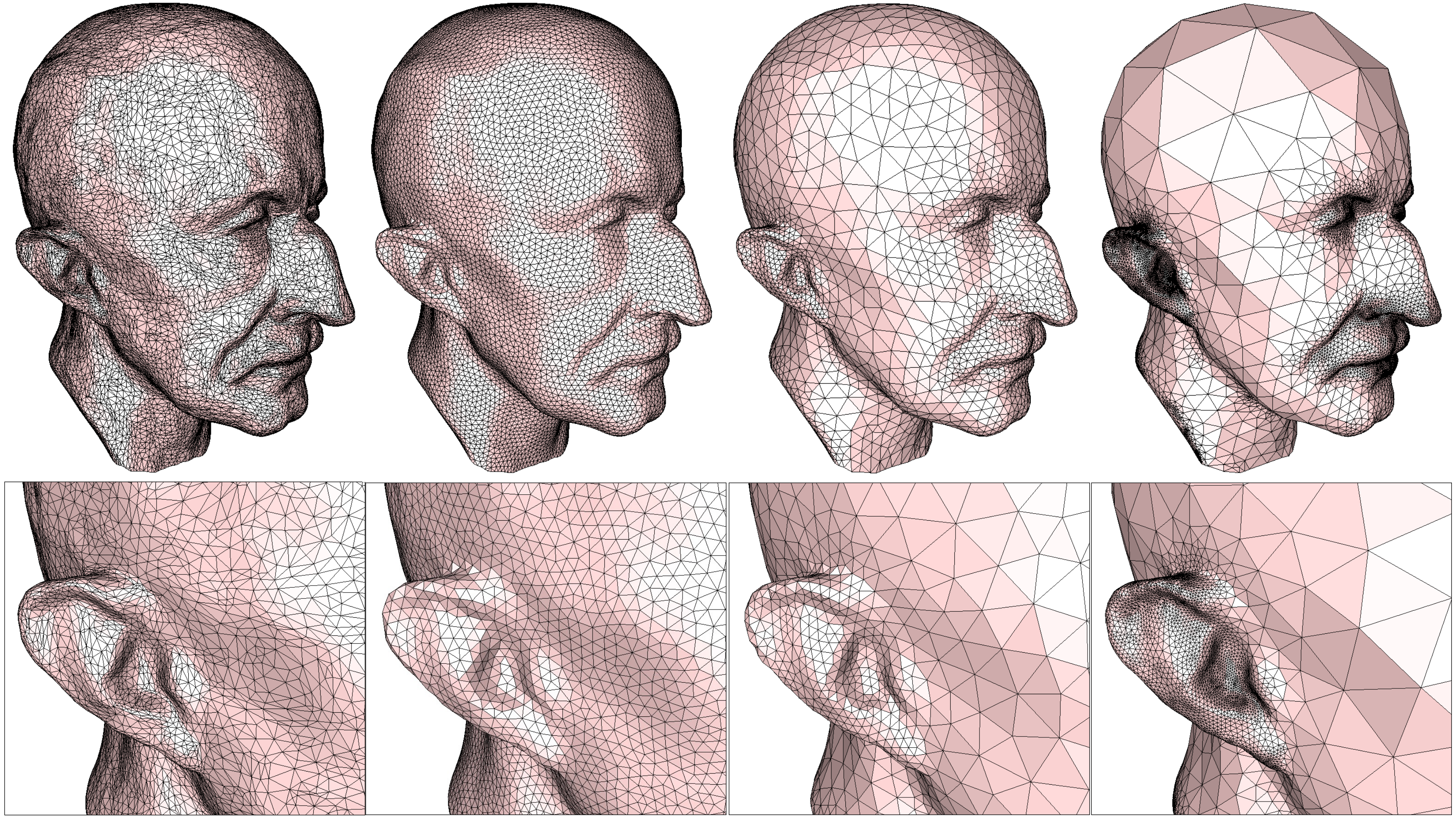
Remeshing

- Remeshing type can be adjusted to the application's needs



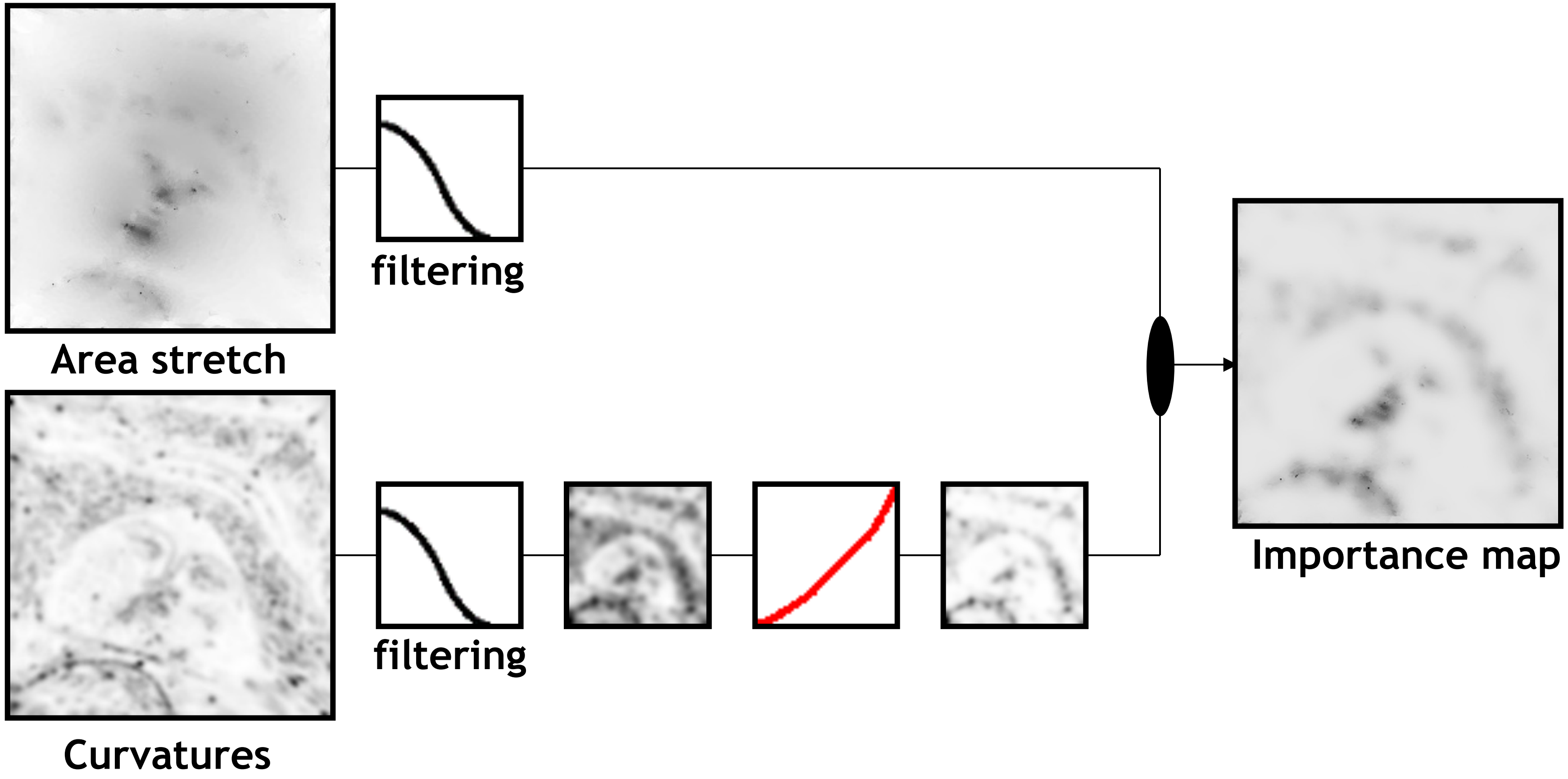
original uniform adapted semi-regular

Remeshing Examples



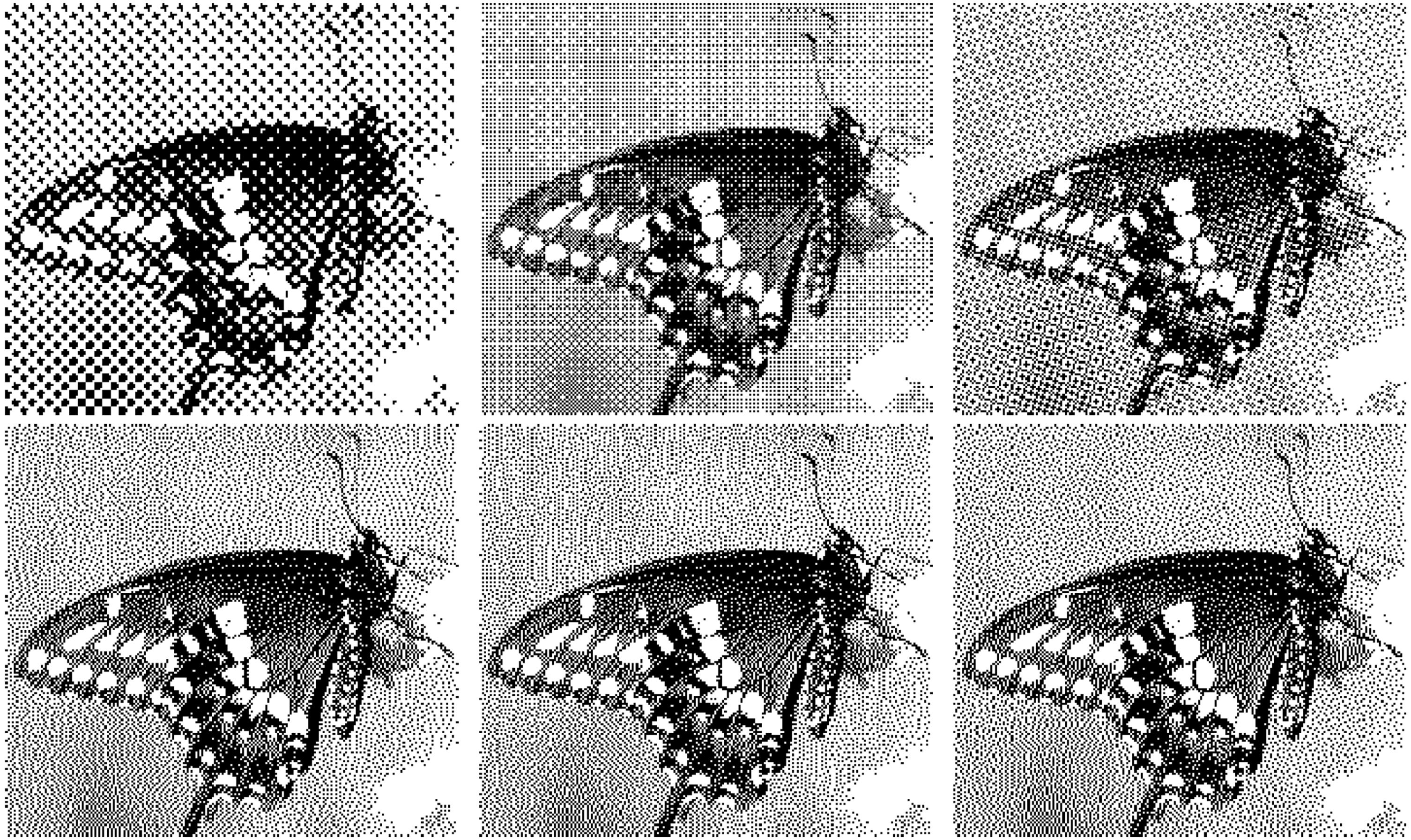
Interactive Geometry Remeshing

- Importance map created according to application needs



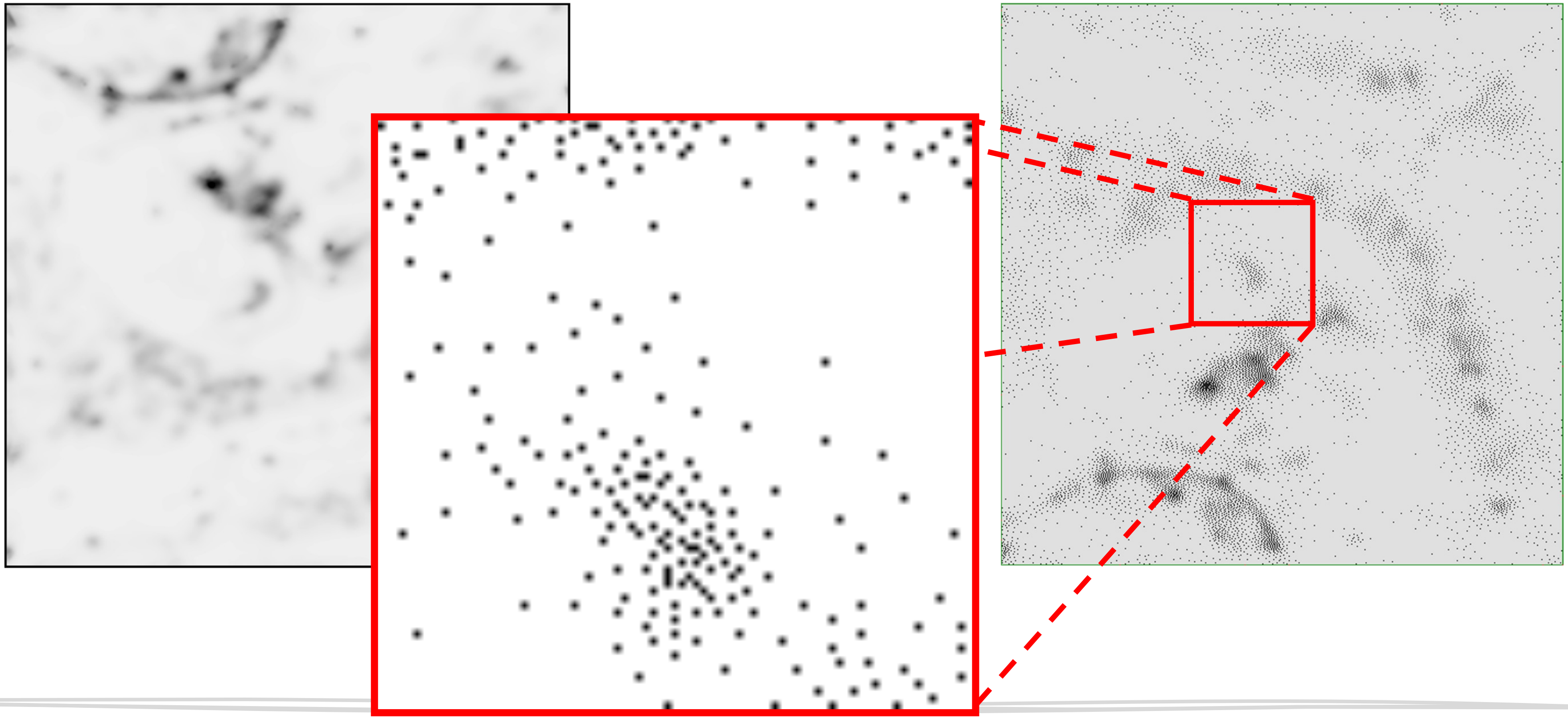
Interactive Geometry Remeshing

- Importance map is sampled by points – as in halftoning



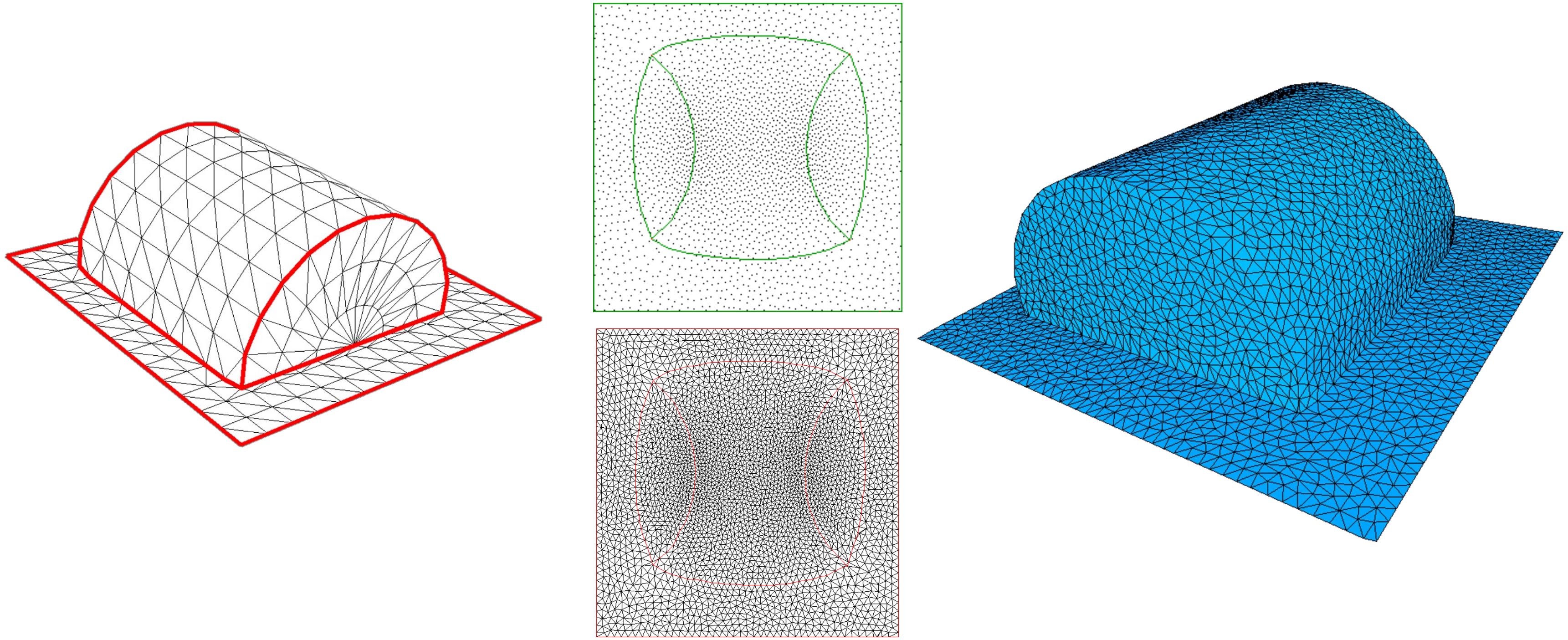
Interactive Geometry Remeshing

- Importance map is sampled by points – as in halftoning (error diffusion process)

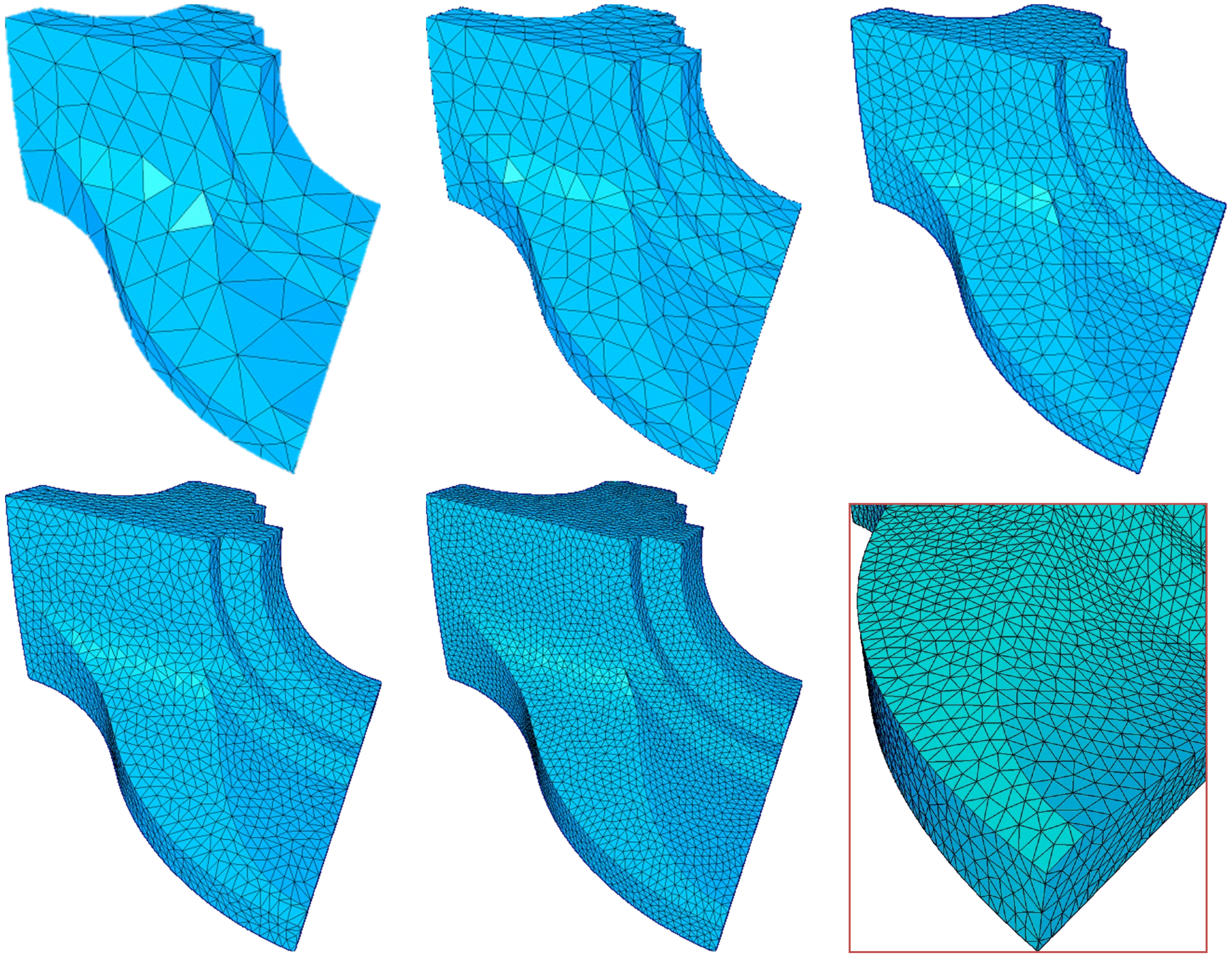


Interactive Geometry Remeshing

- Sampled points are triangulated using Delaunay
- Using the parameterization, the 2D points are lifted back into 3D

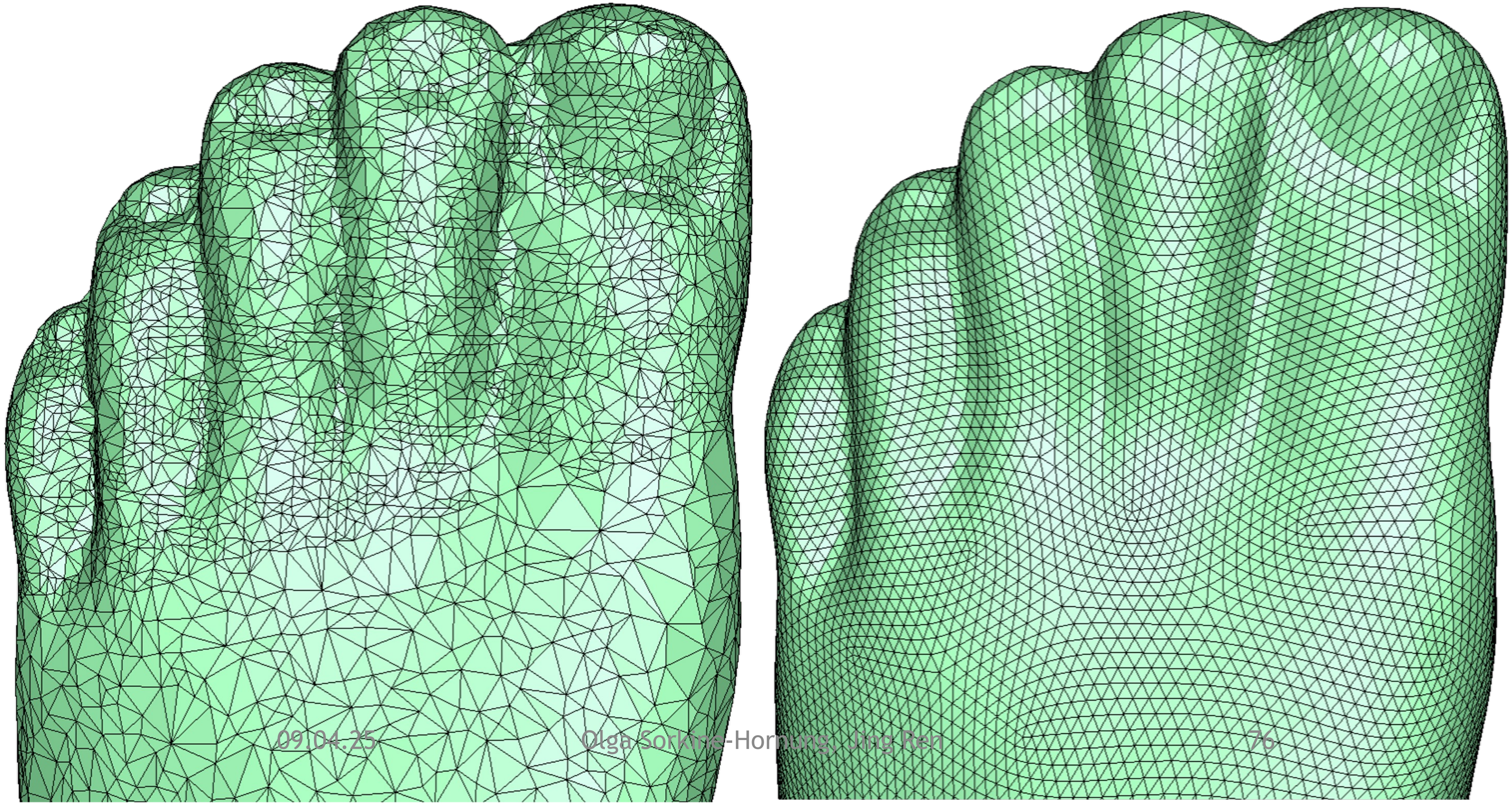


Interactive Geometry Remeshing



Interactive Geometry Remeshing

- More results



Thank You!

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